IOP Conference Series: Materials Science and Engineering

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To cite this article: Xiao Yi Wang and Xiu Yun Meng 2017 *IOP Conf. Ser.: Mater. Sci. Eng.* **187** 012005

IOP Conf. Series: Materials Science and Engineering **187** (2017) 012005 doi:10.1088/1757-899X/187/1/012005

Research on Time-series Modeling and Filtering Methods for MEMS Gyroscope Random Drift Error

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Abstract. The precision of MEMS gyroscope is reduced by random drift error. This paper applied time series analysis to model random drift error of MEMS gyroscope. Based on the model established, Kalman filter was employed to compensate for the error. To overcome the disadvantages of conventional Kalman filter, Sage-Husa adaptive filtering algorithm was utilized to improve the accuracy of filtering results and the orthogonal property of innovation in the process of filtering was utilized to deal with outliers. The results showed that, compared with conventional Kalman filter, the modified filter can not only enhance filter accuracy, but also resist to outliers and this assured the stability of filtering thus improving the performance of gyroscopes.

1. Introduction

With the rapid development of micro-electromechanical systems (MEMS) technology, MEMS gyroscope has been widely used in low-cost inertial navigation systems. It has small sizes, low cost, and light weights, but its application in high-precision navigation systems is limited due to comparatively low precision [1]. The random drift errors of MEMS gyroscopes accumulate with data integration in the inertial navigation algorithm, which will lead to large navigation errors. Therefore, effective modeling and compensation to random drift error signals of MEMS gyroscopes has been paid increasing attention to improve the measurement accuracy.

ARMA model is a dynamic linear model in random process that is currently widely used in system analysis, forecasting, identification and control. Compared with neural network and wavelet analysis, it is suitable for online real-time estimations in simple and low-cost systems. Therefore, time series analysis was used to model the random drift errors of MEMS gyroscopes in this paper.

In the filtering process, outliers will lead to serious deviations of the best estimations from the true state values of the system. Kalman filter is a linear estimator based on minimum mean square errors.

It is an effective way to use Kalman filter to suppress the random noise of gyroscopes in terms of precision enhancement. However, its filtering accuracy depends on the accuracy of the mathematical models and the statistical properties of the noise, and the presence of outliers affects the stability of the filter. Therefore, an anti-outlier adaptive Kalman filtering method was used and the simulation results demonstrated the effectiveness and feasibility of the method.

2. The time series model of MEMS gyroscope random drift error

In an autoregressive integrated moving average model, the future value of a variable is assumed to be a linear function of several past observations and random errors [2]. The general form of time series model ARMA(p,q) is expressed as follows :

$$\hat{X}_k = \varphi_1 \hat{X}_{k-1} + \varphi_2 \hat{X}_{k-2} + \cdots + \varphi_p \hat{X}_{k-p} + a_k - \theta_1 a_{k-1} - \theta_2 a_{k-2} - \cdots - \theta_q a_{k-q} \tag{1}$$

Where the $\varphi_i (i = 1, 2, ...p)$ and $\theta_j (j = 1, 2, ...q)$ are the coefficients of auto-regression and moving average respectively, which can be determined by the least-squares estimation method; p and q are integers and referred to as orders of the model; a_k is the random error at time period t . If q = 0, then (1) becomes a model of AR (p); When p = 0, the model reduces to a model of MA (q).The values of p and q can be determined by Akaike information criterion (AIC) [3].

Place a Micro Inertial Measurement Unit (MIMU) on a level surface, and collect original output signals, which are shown in Figure 1.

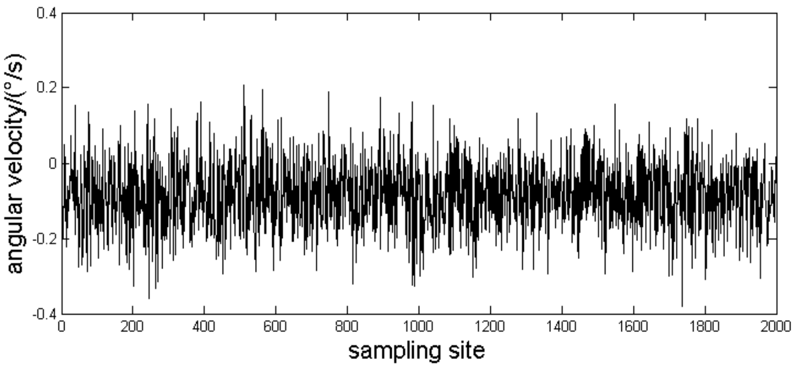

Figure 1. The static output of a MEMS gyroscope

Only when the random series have the characteristics of zero mean, stationarity, and normality, time-series model can be established [4].So in order to build time-series model of drift error, the data collected need to be preprocessed. After remove outliers, constant and trends component, the remaining data upon testing fulfill the requirement of timing model. The model parameters and AIC values obtained by calculation are shown in table 1.

Table 1. The model parameters and AIC values of random drift error

	AR(1)	AR(2)	AR(3)	ARMA(1,1)	ARMA(2,1)
φ_1	0.2134	0.2178	0.2182	0.1085	- 0.2219
φ_2	—	- 0.0207	- 0.0240	—	0.0721
φ_3	—		0.0149	—	—
θ_1	—	—	—	0.1098	0.4403
AIC	-2.1085	-2.1089	-2.1092	-2.1090	- 2.1091

The AIC values show no significant difference in different model. Considered the real-time demand for a low cost system, AR(1) model is chosen to establish the time-series model, which is expressed as:

$$x_k = \phi x_{k-1} + w_k \tag{2}$$

where x_k is the output of the timing model, $\phi = 0.218$, White noise $w_k \sim NID(0, \sigma_w{}^2)$.

3. The improved algorithm of Kalman filter

3.1 The establishment of filtering model

When process noise is colored noise and observation noise is white noise, the method of amplifying state variables is used to establish state space model. Set $X_k = [r_k, x_k]^T$, r_k is the real angular rate of gyroscope, x_k is the output of the timing model. Then state space model is given as

$$\begin{bmatrix} r_k \\ x_k \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & \phi \end{bmatrix} \begin{bmatrix} r_{k-1} \\ x_{k-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w_{k-1} \tag{3}$$

$$Z_k = [1, 0] \begin{bmatrix} r_k \\ x_k \end{bmatrix} + v_k \tag{4}$$

That is,

$$X_k = AX_{k-1} + \Gamma W_{k} \tag{5}$$

$$Z_k = H X_k + V_1 \tag{6}$$

Where $A = \begin{bmatrix} 1 & 1 \\ 0 & \phi \end{bmatrix}$, $\Gamma = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$, $W_{k-1} = w_{k-1}$, $V_{k-1} = v_{k-1}$, $H = [1, 0]$. The process noise w_k and observation noise v_k are uncorrelated, zero mean. Their variance values are Q_k and R_k respectively. Q_k is set as the residuals covariance of AR(1). R_k is set as the covariance of actual measurement data.

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3.2 Adaptive Kalman filter

When affected by the working environment, the statistical characteristics of noise may change. The conventional kalman filter needs to be improved as the performance of Kalman filter depends on the accuracy of the model established [5]. An adaptive Kalman filter, using Sage-Husa filtering algorithm to adjust the statistical characteristics of process noise and measurement noise on-line is described as follows [6] [7]:

$$\hat{\mathrm{X}}_{\mathrm{k|k-1}} = \mathrm{A}\hat{X}_{k-1} + \mathrm{q}_{k-1} \tag{7}$$

$$P_{k|k-1} = AP_{K-1}A' + \Gamma_{k|\mathrm{k}-1}Q_{k-1}\Gamma_{k|\mathrm{k}-1}' \tag{8}$$

$$K_k = P_{k|k-1}H_k'\left[H_k P_{k|k-1}H_k' + R_{k-1}\right]^{-1} \tag{9}$$

$$e_k = Z_k - \hat{Z}_k = Z_k - H_k\hat{\mathrm{X}}_{\mathrm{k|k-1}} \tag{10}$$

$$\hat{X}_k = \hat{\mathrm{X}}_{\mathrm{k|k-1}} + K_k(e_k - r_{k-1}) \tag{11}$$

$$P_k = \left[I - K_k H_k\right]P_{k|k-1} \tag{12}$$

The time varying noise estimator is described as follows:

$$\mathrm{d}_{k-1} = (1-\mathrm{b})/(1-\mathrm{b}\wedge\mathrm{k}) \tag{13}$$

$$\mathrm{r}_k = (1-\mathrm{d}_{k-1})\mathrm{r}_{k-1} + \mathrm{d}_{k-1}\left[Z_k - H_k\hat{\mathrm{X}}_{(\mathrm{k|k-1})}\right] \tag{14}$$

$$\mathrm{q}_k = (1-\mathrm{d}_{k-1})\mathrm{q}_{k-1} + \mathrm{d}_{k-1}\left[\hat{X}_k - \mathrm{A}\hat{X}_{k-1}\right] \tag{15}$$

$$R_k = (1-d_{k-1})R_{k-1} + d_{k-1}*[[Z_k - H_k\hat{X}_{(k|k-1)} - r_k][Z_k - H_k\hat{X}_{(k|k-1)} - r_k]^T - H_k P_{k|k-1}H_k^{\ T}] \tag{16}$$

$$Q_k = (1-\mathrm{d}_{k-1})Q_{k-1} + \mathrm{d}_{k-1}[K_k[Z_k - H_k\hat{\mathrm{X}}_{(\mathrm{k|k-1})} - \mathrm{r}_k][Z_k - H_k\hat{\mathrm{X}}_{(\mathrm{k|k-1})} - \mathrm{r}_k]^T K_k^T + P_k - \mathrm{A}P_{k|k-1}A^T] \tag{17}$$

b is the forgetting factor and usually take 0.95 ~ 0.99.

3.3 Algorithm of anti-outliers

Outliers in measurements can result in divergence of filtering result, so it is desirable to improve the filter to have the effects of anti-outliers. During the Kalman filtering process, innovation e_k possess the characteristic of orthogonality [8]. Then according to equation (10),

$$E(Z_k Z_k^{\ T}) = E(\hat{Z}_k\hat{Z}_k^{\ T}) + E(e_k e_k^{\ T}) = H_k\hat{\mathrm{X}}_{(\mathrm{k|k-1})}\hat{\mathrm{X}}^T_{(\mathrm{k|k-1})}H_k^{\ T} + H_k P_{k|k-1}H_k' + R_{k-1} \tag{18}$$

Set $J = H_k\hat{\mathrm{X}}_{(\mathrm{k|k-1})}\hat{\mathrm{X}}^T_{(\mathrm{k|k-1})}H_k^{\ T} + H_k P_{k|k-1}H_k' + R_{k-1}$ and ε is supposed to be a disturbance caused by the calculation error and other factors. When observation Z_k satisfies $E(Z_k Z_k^{\ T}) \in [J-\varepsilon, J+\varepsilon]$, we can draw a conclusion that Z_k isn't an outlier; Conversely, it is. When outliers exit in measurements, orthogonality will be destroyed. An activation function is needed to keep the orthogonality of innovation after Z_k has been considered an outlier to assure the stability of filter, which can be defined as follows [9],

$$f(r)=\begin{cases} 1 & (m\ge r) \\ m/r & (m<r) \end{cases} \tag{19}$$

Where $m=\sqrt{J}, r=|Z_k|$, Then the state estimation equation (11) can be replaced by

$$\hat{X}_k=\hat{\mathrm{X}}_{\mathrm{k|k-1}}+K_k(f(r)Z_k-H_k\hat{\mathrm{X}}_{\mathrm{k|k-1}}-r_{k-1}) \tag{20}$$

4. Result

Take the static output of a MEMS gyroscope as shown in Figure 1 as observations of the filters. The results of filtering for random drift error are shown in Figure 2.

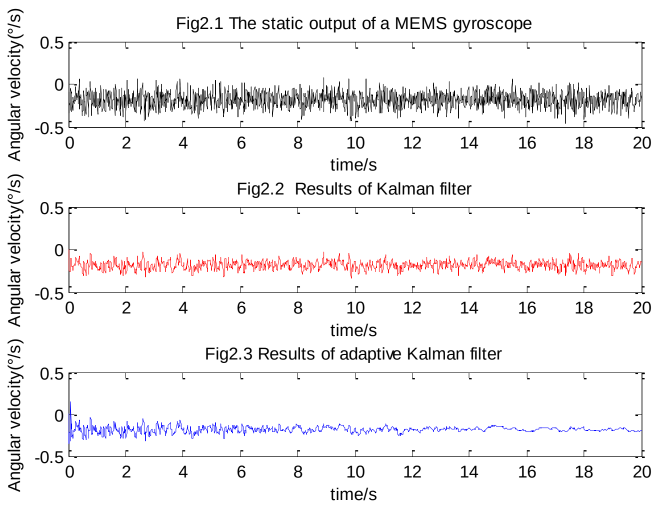

Figure 2. Results of static experiments

As can be seen from Figure 2, compared with conventional Kalman filter, filtering accuracy of adaptive Kalman filter has improved obviously. The mean and standard deviation values of noise before and after filtering are shown in table 2.

Table 2. Mean and standard deviation of noise before and after filtering

	mean（°/s）	Standard deviation（°/s）
random drift error	-0.184049	0.0863
After Kalman filter	-0.183968	0.048196
After adaptive Kalman filter	-0.184041	0.034918

The mean remains unchanged before and after filter because Kalman filter estimation is a method of unbiased estimation. The standard deviation of signals after adaptive Kalman filter is relatively smaller than after Kalman filter, which showed that improved kalman filter has a better filtering effect for random drift error.

For achieving the dynamic simulation and viewing the effect of eliminating outliers, add a sinusoidal signal and isolated, continuous outliers to the static output shown in Fig.1 as observations of the filters. The results of filtering are shown in Figure3.

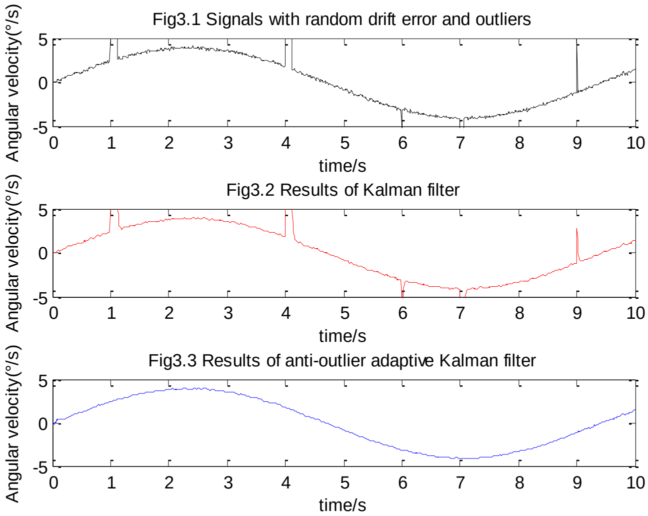

Figure 3. Results of dynamic simulation

As we can see from Figure 3, anti-outlier adaptive Kalman filter can eliminate outliers and decrease noise, which can improve the using precision of gyroscopes.

5. Conclusion

In this paper, enhancing the accuracy and stability of conventional Kalman filter for random drift error of MEMS gyroscope is the research topic，and the achievements are summarized as following: 1. According to time series analysis methods, the time series model of MEMS gyroscope has been established; 2. To improve the accuracy of the filter, an adaptive Kalman filter has been used; 3. An anti-outliers function for identifying outliers and an activation function as the weight to each measurement for modifying the orthogonal property of innovation sequence have been used to eliminate negative influences on performances of filter caused by outliers.

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