Cable Tension Estimation For The Cable-stayed Bridge With Hysteresis Damping

To cite this article: An Huynh-Thai and Toan Pham-Bao 2023 *IOP Conf. Ser.: Mater. Sci. Eng.* **1289** 012009

Cable Tension Estimation For The Cable-stayed Bridge With Hysteresis Damping

An Huynh-Thai[1,2], Toan Pham-Bao[1,2,*]

[1] Laboratory of Applied Mechanics, Ho Chi Minh City University of Technology (HCMUT), 268 Ly Thuong Kiet Street, District 10, Ho Chi Minh City 700000, Vietnam
[2] Vietnam National University Ho Chi Minh City, Linh Trung Ward, Thu Duc City, Ho Chi Minh City 700000, Vietnam

[*] Corresponding author's e-mail: baotoanbk@hcmut.edu.vn

Abstract. Damping materials are popular applications for almost vibration structures; however, they have rarely been investigated in different practical experiments. That is why new approaches would be necessary to assess these problems. In this study, the mathematical model of cable is remarkable in assessing the cable tension of the cable-stayed bridge. A differential vibration equation is used to derive the cable tension considering material damping as the hysteresis phenomenon. The experimental measurements of cable vibration at Phu My Bridge are calculated to find approximate damping ratio and tension values. The selected damping model with experimental data has been collected to derive an efficient method for evaluating structure status. These values are used to assess damping efficiently in the cable-stayed bridge structures. The results presented in this paper shall help elucidate experimental procedures for characterizing damping materials. The proposed procedures are used not only for the cable-stayed bridge but also for generally cable-stayed structures.

1. Introduction

The damping vibration problems of cable structures have been applied in trafficky bridge works such as cable-stayed bridges and suspension bridges, which get attention from researchers. Presently, structural health monitoring is one of the inspection methods to estimate worked conditions and consider the damage to these bridges to detect and take fixed maintenance promptly. In this study, a mathematical model is proposed to compute the tension of the tensile cable of the cable-stayed bridge with hysteresis damping phenomenon from measured frequency.

The cable-stayed bridge includes one or more vertical concrete pillars with bases that connect many tensile cables to the desk of the bridge at the connectors. The tensile cable is a complex structure with many wires in the helical strand in one or more layers and lubricant and covered by high-density polyethylene material (HDPE). All cable connects to pillars bear on the live load from traffic status at the bridge, making cable carry elongation and vertical displacement. In contrast, the action-reaction of force in the axial cable is also called tension. The relationship between tension with damping is a popular mechanical problem for one in science and engineering. There are numerous studies for hysteresis damping widely researched by scientists all around the world, such as:

In 1989, Shimada et al. researched the cable tension through regression method and applied it to some bridges in Japan [1]. Then, Cunha et al. studied to estimate the tension with the taut string model and tested it on the cable of the Vasco da Gama cable-stayed bridge in Portugal [2]. In 2006, Dragan presented the rheological-dynamical model to control the damping parameter and compared it with another model, such as the AASHTO model and Iwan's model, by real data from previous experiments [3]. However, this model can not be practical at the site because of many necessary experimental parameters. In 2009, Maia compared viscous damping and hysteresis damping for a single degree of freedom. She proved a causal behavior of hysteresis damping because they are the solutions of an equivalent viscously damped system [4]. Besides that, Choi et al. proposed to separate the estimated tension into two parts: the pure applied axial load and the stretching force due to self-weight and re-estimate with real data at Alamillo Bridge [5]. At the same time, Labonnote et al. accomplished the prediction of material damping in Timoshenko beams, which comprises two parts, the first related to bending and the second related to shear [6]. This model was given much helpful information for damping materials, but it also needs more material and mathematical parameters from at experimental laboratory. This wasn't easy to apply for a large structure as the cable-stayed bridge, without material parameters. In 2014, Spak developed a method for determining effective homogenous parameters from straightforward cable measurements such as natural frequencies, frequency response, and mode shapes and calculated cable properties of area, density, bending stiffness, shear rigidity, and attachment stiffness [7]. The author also explained models of cable behavior are reviewed and categorized into three major classes: thin rod models, semi-continuous models, and beam models. The method can be applied to any stranded cable to determine the constituent material properties. In 2022, Mohammad Noori et al. gave a mathematical definition of hysteresis to replace vague notions [8]. In the same year, Qinglin Liu et al. compared the viscous damping model with the hysteretic damping model for single-degree-of-freedom systems [9]. After that, the authors performed the calculating analysis from three earthquake data. Mojtaba Farrokh et al. developed a novel approach to combine the learning machine and support vector machine to simulate hysteresis with several features [10]. This approach simulates hystereses with various properties, such as congruent or non-congruent, symmetric, or asymmetric problems with multiple properties. However, these methods had yet to be able to compare with tension measured by the measured device to calculate methods to estimate the accuracy of cable tension. In addition, these performed investigations have difficulty applying to do at outside, and the experimental data is not satisfied with the theory beforehand.

The following section proposed a mathematical model for the beam model, which was extended viscous damping for cable vibration, to appraise the equivalent damping parameter known as hysteresis damping.

2. Background knowledge and method

Through traffic, the cable will be elongated, appear to tension on the tensile cable, and damping phenomenon. This mechanical phenomenon makes the influence on vibration of the structure. The Euler–Bernoulli beam theory is extended for the cable as a beam model with flexural rigidity, mass per unit of length, the equivalent damping parameter, and general tension has been used and shown in Figure 1. Assuming that during suffered load, the cross sections of the cable do not change. The differential equation of motion for cable vibration $\mathrm{u}(\mathrm{x},\mathrm{t})$ is given by:

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 u(x,t)}{\partial x^2}\right] - N(x)\frac{\partial^2 u(x,t)}{\partial x^2} + \rho(x)\frac{\partial^2 u(x,t)}{\partial t^2} + c_{eq}\frac{\partial u(x,t)}{\partial t} = q(x,t) \quad (1)$$

where: flexural rigidity EI, mass per unit of length ρ, the equivalent damping coefficient c_{eq}, general tension N and $q(x,t)$ is the external excitation force of vibration.

Owning to model simplification for free vibrations, the external excitation is clearly assumed to be $q(x,t) = 0$, $EI(x)$, $\rho(x)$, and $N(x)$ is simplified constant for cable, so that:

$$EI\frac{\partial^4 u(x,t)}{\partial x^4} - N\frac{\partial^2 u(x,t)}{\partial x^2} + \rho\frac{\partial^2 u(x,t)}{\partial t^2} + c_{eq}\frac{\partial u(x,t)}{\partial t} = 0 \quad (2)$$

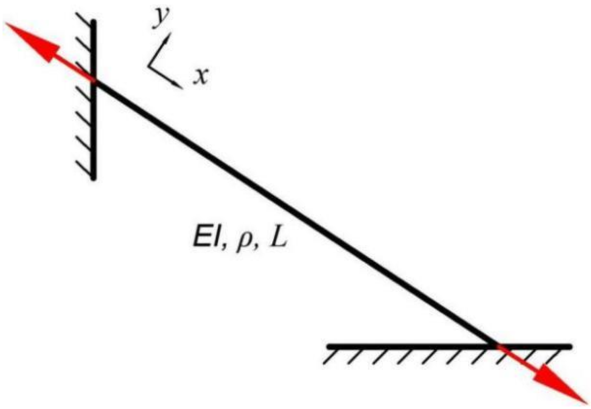

Figure 1. A mechanical model of an inclined cable.

Using the separation of variables u(x, t)=W(x)T(t) with W(x) and T(t) are eigenfunctions [11], as following:

$$\begin{cases} \frac{EI}{\rho}\frac{d^4W(x)}{dx^4} - \frac{N}{\rho}\frac{d^2W(x)}{dx^2} - \omega^2 W(x) = 0 \\ \frac{-d^2T(t)}{dt^2} - \frac{c_{eq}}{\rho}\frac{dT(t)}{dt} - \omega^2 T(t) = 0 \end{cases} \tag{3}$$

where: $\lambda = \omega^2$ is the eigenvalue of the differential equation.

$$\begin{cases} W(x) = \sum_{i=1}^{4} C_i e^{s_i x} = C_1 e^{s_1 x} + C_2 e^{s_2 x} + C_3 e^{s_3 x} + C_4 e^{s_4 x} \ (a) \\ T(t) = D_1 e^{t\left(\frac{\sqrt{C_{eq}^2 - 4\omega^2\rho^2} - c_{eq}}{2\rho}\right)} + D_2 e^{t\left(\frac{-\sqrt{C_{eq}^2 - 4\omega^2\rho^2} - c_{eq}}{2\rho}\right)} \ (b) \end{cases} \tag{4}$$

where: $s_i = \pm\sqrt{\frac{N}{2EI} \mp \sqrt{\frac{N^2}{4E^2I^2} + \frac{\rho\omega^2}{EI}}}$; C_1, C_2, C_3, C_4 and D_1, D_2 is the integral constants of these differential equations.

Based on circular damping frequency and natural circular frequency, the following relationship was determined:

$$\omega_d = \omega_n\sqrt{1 - \zeta^2} \tag{5}$$

$$f_n = \frac{f_d}{\sqrt{1 - \left(\frac{c_{eq}}{4\pi\rho f_n}\right)^2}} \tag{6}$$

where: ζ is the damping ratio, f_n is the natural frequency and f_d is the damping frequency.

The equivalent damping was determined such as:

$$c_{eq} = 4\pi\rho\sqrt{f_n^2 - f_d^2} \tag{7}$$

From Equation 4, the critical damping parameter is derived

$$\left(\frac{c_{eq}}{\rho}\right)^2 - 4\omega^2 = 0 \tag{8}$$

$$c_{eq}^{cri} = 4\pi\rho f_n \tag{9}$$

According to the assumptions, the boundary conditions were expressed by the pinned support, such as $W(x) = 0$ and $W^{"}(x) = 0$ at both ends ($x = 0, x = L$). Whereas Equation 4 was substituted as following:

$$[X][C_1 \quad C_2 \quad C_3 \quad C_4]^T = 0 \tag{10}$$

where [X] is the coefficient matrix of simultaneous equations $X = e^{s_i x}$. If the determinant of the coefficient matrix is zero, there is no solution to be zero in the equation, as known $det(X) = 0$.

In case of the pinned support for two end of beam, $sL = n\pi$, the tension will be estimated by the fomular, as folowing:

$$N = \rho\left(\frac{2Lf_n}{n}\right)^2 - EI\left(\frac{n\pi}{L}\right)^2 \tag{11}$$

3. Results and discussion

A measured experiment was performed on several cables of the Phu My bridge, one of the cable-stayed bridges in Ho Chi Minh City, at the beginning of 2017 as shown in Figure 2. The accelerometers accomplished the measurement for some cables C2101, C2106, and C2113. These devices are used in this continuous experiment within 10 minutes with 100 samples per second. The measured devices were directly installed on the coating of cables at 2.75 meters from the bridge deck in height. After using mathematical transformation, a dataset of measurement frequency was simultaneously collected. From these data, the first frequency modes of each cable are calculated (Figures 3 to 5), and Table 1 shows the material properties of each cable.

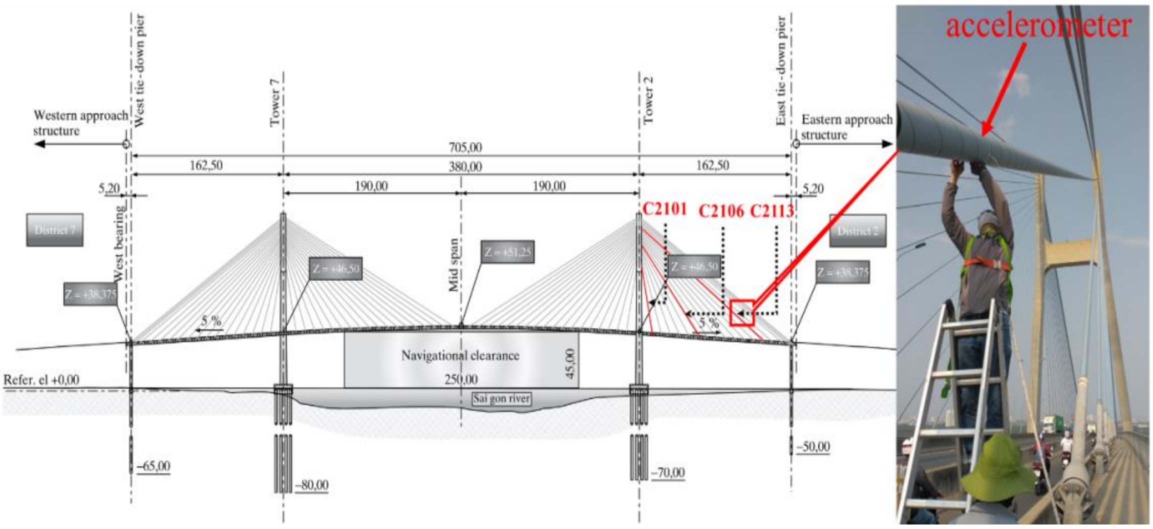

Figure 2. Measurement of cable vibration in the Phu My bridge.

Table 1. The material properties of the cable [12].

Cable no	No. of strand	Density mass (kg/m)	Length (m)	Inertia (m^4)	Elastic modulus (N/m^2)
C2101	27	31.860	63.584	8.363×10^{-6}	
C2106	35	41.300	94.998	1.372×10^{-5}	1.97×10^{11}
C2113	45	53.100	157.468	2.178×10^{-5}	

-

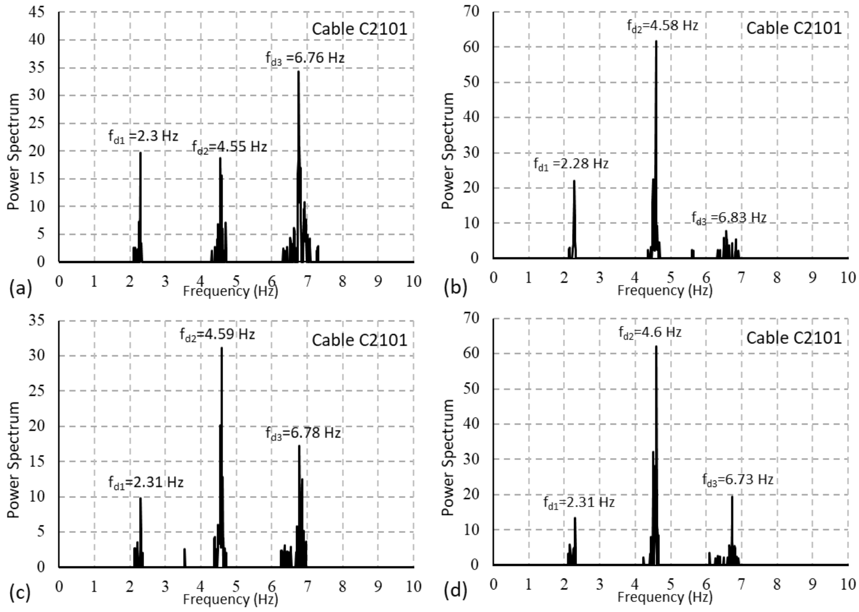

Figure 3. The power spectrums of the cable C2101.

a) 1st time, b) 2nd time, c) 3rd time, d) 4th time

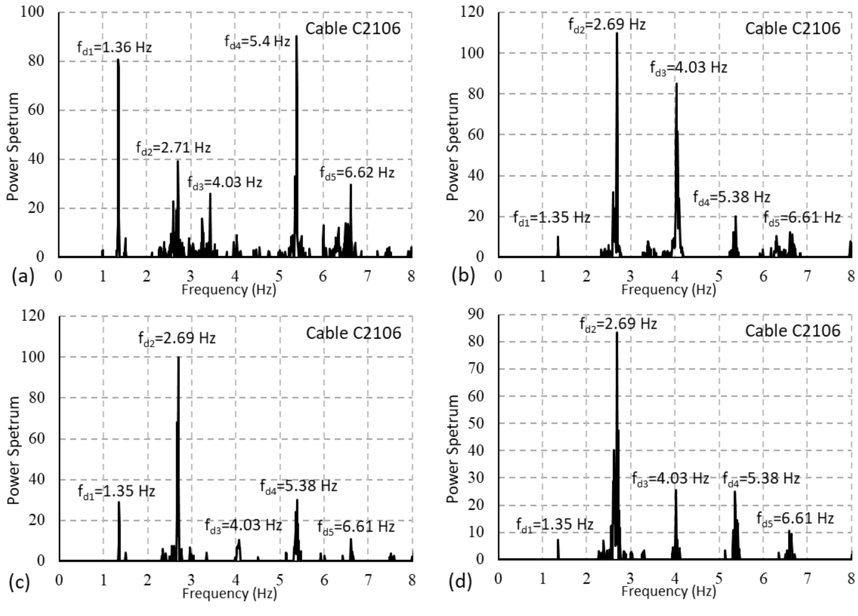

Figure 4. The power spectrums of the cable C2106:

a) 1st time, b) 2nd time, c) 3rd time, d) 4th time

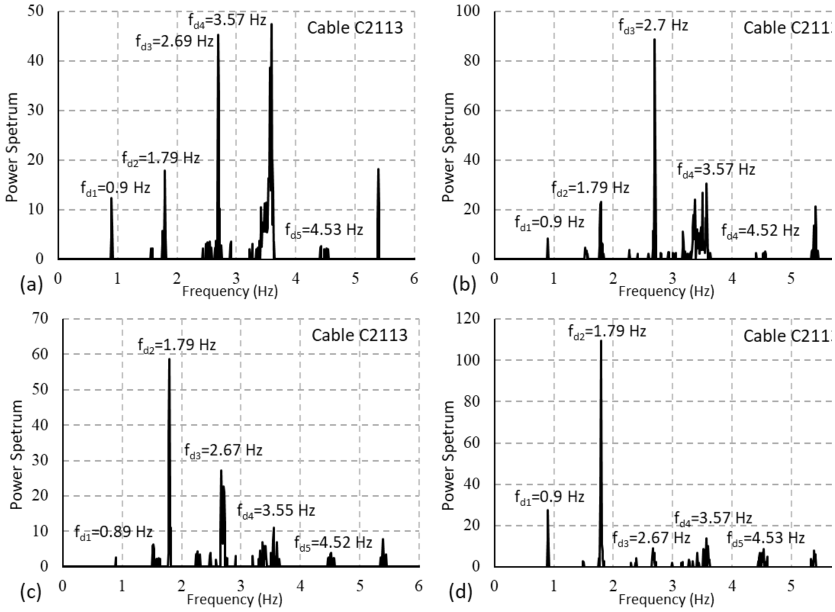

Figure 5. The power spectrums of the cable C2113:

a) 1st time, b) 2nd time, c) 3rd time, d) 4th time

After using Equation 6, the tension value was comparatively performed in Table 2.

Table 2. The cable tension result.

Cable no	Tension Force (*kN*)						
	Mode	1st time	2nd time	3rd time	4th time	Cunha's method [2]	Shimad's method [1]
C2101	1	2782.684	2853.967	2807.276	2767.636	2083.472	2686.125
	2	2770.618	2841.902	2795.210	2755.571	2150.616	
	3	2750.509	2821.792	2775.101	2735.461	2245.670	
C2106	1	2667.035	2654.839	2660.007	2639.351	2623.984	2689.264
	2	2658.167	2645.971	2651.139	2630.484	2591.175	
	3	2643.388	2631.192	2636.360	2615.704	2617.422	
	4	2622.696	2610.500	2615.668	2595.013	2623.984	
	5	2596.093	2583.897	2589.065	2568.410	2623.984	
C2113	1	4285.249	4318.906	4243.007	4276.56	4345.457	4241.226
	2	4280.125	4313.782	4237.884	4271.437	4329.399	
	3	4271.586	4305.243	4229.345	4262.898	4203.125	
	4	4259.632	4293.289	4217.390	4250.943	4125.013	
	5	4244.261	4277.918	4202.020	4235.573	4377.282	

According to this result, the cable tension has a change for each mode. Method of this study is also compared with both Cunha's popolar method [2] and Shimada's regression method [1] shown in Table 2.

In the case of long cables, the tension values between the study and the two above methods give mutually equivalent results. However, in the case of short cables, the tensions of the study are similar to Shimada's least squares results and deviate significantly from Cunha's taut string model. These deviations are due to the effect of the short cable's flexible rigidity and damping. The assumption of the hysteresis damping model, the equivalent damping parameter, depends on the inverse proportionality of the circular frequency of vibration [11]. The regression model was applied to estimate hysteretic factors.

$$c_{eq} = \alpha + \frac{h}{\omega} \tag{12}$$

where c_{eq}, ω are the independent and dependent variables of the regression; h, α are the hysteresis damping factor and constant damping factor.

The result of the analysis is performed for three cables with coefficient of correlation in Table 3. In this study, the hysteretic damping factor is an assumption as a unit of length and dependence on time. However, the data from Figures 3 to 5 and Table 2 are also shown that a little of the quantity of frequency modes can determine. The shortage of modernized devices makes it difficult for the stability of value.

Table 3. Analysis results of regression.

Cable no	α (*N.s/m*)	*h* (*N/m*)	*r*
C2101	852	-10442	0.795
C2106	461	-3257	0.840
C2113	383	-2028	0.896

To influentially visualize the hysteresis model, the damping factor, and critical parameter change for each measurement time. However, these data were noisy due to errors. As shown in Figure 6-8, the higher frequency, the more stable the hysteresis damping factor. The correlation coefficient r is quite good enough to conclude that the correlation is strong. Moreover, the value of the hysteretic damping factor has a difference, due to different dimensions of each cable such as diameter, length, and number of taut, etc. The shorter the cable, the greater the hysteretic damping factor. The damping of short cable C2101 is 2 times greater than long cable. This phenomenon proves that the damping parameter affect the dynamic properties of the cable. Therefore, it is necessary to consider the effect of damping in the cable tension estimation.

Last but not least, the procedure for the analysis is also shown in Table 4, performing the analyzed procedure using the measured frequency with material properties and analyzing between cable tension and hysteresis damping phenomenon. In this way, the data collection and analysis have been unified. In the future, the data will be used to apply machine learning.

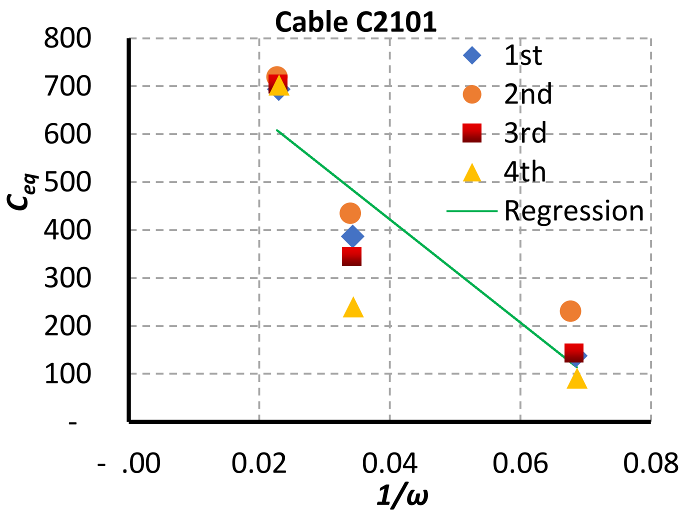

Figure 6. The relationship between equivalent damping parameters and circular frequency for cable C2101.

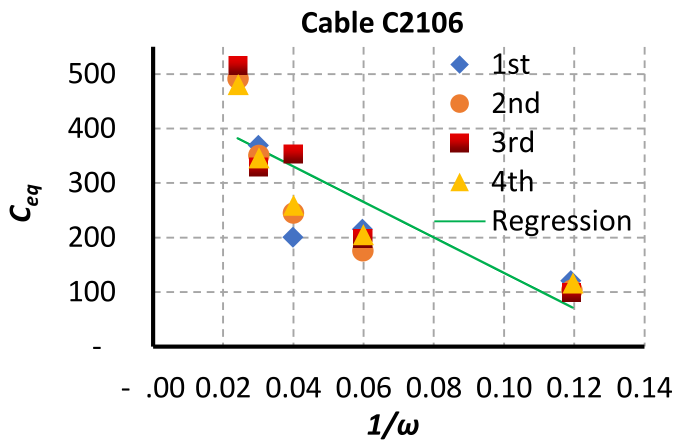

Figure 7. The relationship between equivalent damping parameters and circular frequency for cable C2106.

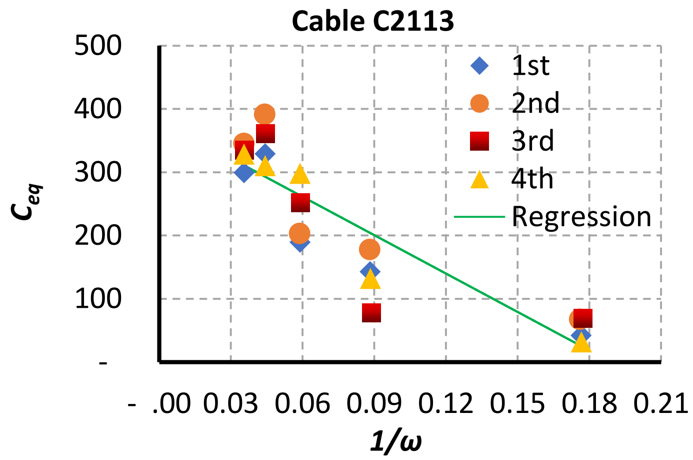

Figure 8. The relationship between equivalent damping parameters and circular frequency for cable C2113.

Table 4. The analysis procedure.

Step	Description	Parameter
1	Collect the material properties of the cable.	E, I, ρ, L
2	Measure on cable vibration in the cable-stayed bridge.	f_d
3	Calculate the natural frequency from Equation (6) with ith and jth mode.	f_n
4	Using Equation (7) to determine the equivalent damping parameter.	c_{eq}
5	Evaluate critical equivalent damping parameter Equation (7).	c_{eq}^{cri}
6	From the natural frequency and Equation (11) assess cable tension.	N
7	Using nonlinear regression model to estimate hysteresis and constant damping factors by Equation (12).	α, h
8	Analysis relationship between equivalent damping parameter, tension and circular frequency.	
9	Collect sample data sets to support damping condition diagnosis in the future	

4. Conclusions

In conclusion, a proposed mathematical equation is to assess the mechanical influence of hysteresis damping on the tension of the cable-stayed bridge. These collected results make full use of predicting the future condition of the bridge. The calculated results are, respectively factors of hysteresis damping through the nonlinear regression. The calculated results are natural frequency, tension, and equivalent damping parameter of the regression model including the hysteresis damping factor and constant damping factor. This proposal provides a simple procedure to evaluate parameter which influences the damping vibration of the cables on the cable-stayed bridge. The tension of cables was computed at operating conditions on the live load from traffic status. A point to be noted is also that, this proposal was simplified, it is necessary to use modernized devices to have more trusted data.

Acknowledgement
We acknowledge Ho Chi Minh City University of Technology (HCMUT), VNU-HCM, for supporting this study.

References

[1] Shimada, T., Kimoto, K. and Narui, S. 1989 Study on estimating tension of tied hanger rope of suspension bridge by vibration method, Proc. *JSCE*, **404**(I-11), pp. 455-458
[2] Álvaro Cunha, Elsa Caetano 1999 Dynamic measurements on stay cables of cable-stayed bridge using an interferometer laser system *Experimental Techniques* **23** *38-43*
[3] Milašinović DD 2007 Rheological–dynamical analogy: Prediction of damping parameters of hysteresis damper *International Journal of Solids and Structures* **44**(22-23) 7143-7166
[4] Maia N 2009 Reflections on the hysteretic damping model *Shock and Vibration* **16**(5) 529-542.
[5] Choi DH, Park WS and Nassif H 2013 New Procedure for Estimating Cable Force in Cable-stayed Bridge (No. 13-2407)
[6] Labonnote N, Rønnquist A and Malo KA 2013 Modified hysteretic damping model applied to Timoshenko timber beams *Computers & Structures* **121** 22-31
[7] Spak KS 2014 *Modeling cable harness effects on spacecraft structures* Doctoral dissertation (Virginia Polytechnic Institute and State University)
[8] Noori M and Altabey WA 2022 Hysteresis in Engineering Systems *Applied Sciences* **12**(19) 9428
[9] Farrokh M, Ghasemi F, Noori M, Wang T and Sarhosis V 2022 An Extreme Learning Machine for the Simulation of Different Hysteretic Behaviors *Applied Sciences* **12**(23) 12424
[10] Liu Q, Wang Y, Sun P and Wang D 2022 Comparative Analysis of Viscous Damping Model and Hysteretic Damping Model *Applied Sciences* **12**(23) 12107
[11] Rao, Singiresu S. 2019 Vibration of continuous systems Second edition. John Wiley & Sons Ltd.
[12] Nam H., Nghia N. T. 2011 Estimation of Cable Tension Using Measured Natural Frequencies *Procedia Engineering* **14** 1510-1517