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Development of computer code and calculation of the propagation of sonic boom to the ground in a real atmosphere

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Abstract. One of the main problems in the development of a supersonic aircraft of the second generation is to ensure a safe level of the sonic boom impact on the environment. Achieving such level at the preliminary design stage is possible with the help of reliable methods for calculating the characteristics of the sonic boom due to impossibility of tube experiments at far field and expensiveness of real flight tests. This paper presents method for determining the shape of the overpressure signatures in the generalized form of an augmented Burgers equation for the case of disturbances propagation in a moving medium (atmosphere with wind). Based on the presented method, the computer code "vBoom" has been developed. The calculation results of the sonic boom characteristics (overpressure waveforms, the loudness in different metrics and the sonic boom carpet on the ground) are presented in comparison with the results obtained by NASA within the international seminar SBPW3 [1] (Sonic Boom Prediction Workshop).

1. Introduction

Currently, in many countries around the world, work is underway to develop supersonic transport (SST) of civil application. The creation of the SST is associated with solving a number of environmental problems – reduction of engine emission, noise on the ground and minimization of sonic boom impact. Due to the lack of standards restricting the impact of sonic boom on the environment, many countries have introduced a direct ban on flying over their territory at supersonic speed. Under the aegis of ICAO, a special committee on protection against the environmental impact of aviation (CAEP) was founded. The task of one of the working groups of CAEP is to develop standards for the level of sonic boom impact in environment of perspective SSTs. The lack of standards at present is largely due to a large number of affecting sonic boom propagation and attenuation factors, insufficient study of the sonic boom impact on humans, animals, buildings and structures, as well as the lack of a demonstrator aircraft implementing sonic boom reduction technologies to a safe level.

In the process of propagation of disturbances to the ground, the overpressure wave undergoes significant changes caused by atmospheric inhomogeneity, characterized by velocity profiles and wind direction, pressure and temperature, molecular effects - absorption (absorption of the sonic wave energy at the rotational an vibrational degrees of freedom of gas molecules) and dispersion (dependence of disturbance propagation velocity on frequency)[2]. Also, the overpressure wave shape on the ground is influenced by random atmospheric characteristics, such as turbulence, cloudiness, and microphysical characteristics (water content, humidity)[3]. In addition, all atmospheric parameters depend on the season (winter, spring, summer, autumn) and geographical position [4]. Thus, the shape of the overpressure wave on the ground largely depends on the real properties of the atmosphere.

Given that the overpressure wave travels immense distances of tens or even hundreds of kilometers to the ground, the study of its characteristics (signal duration and amplitude, pressure rise time in the signature, loudness, etc.) is possible only either by calculation or experimentally. Experimental studies require a flight demonstrator, which is a very expensive procedure and it is characterized by the impossibility of conducting full parametric studies of the influence of the aircraft shape on the sonic boom. Thus, numerical calculation is the only way to study the sonic boom of supersonic civil transport under development.

The process of overpressure propagation generated by an aircraft at supersonic speed is a complex problem that depends on many factors, such as the change of atmospheric parameters (pressure, density, temperature) with altitude, the presence of wind, absorption and dispersion of disturbances in the atmosphere, diffraction, atmospheric turbulence, etc. In addition, all these parameters depend on geographical location, terrain, and time of year.

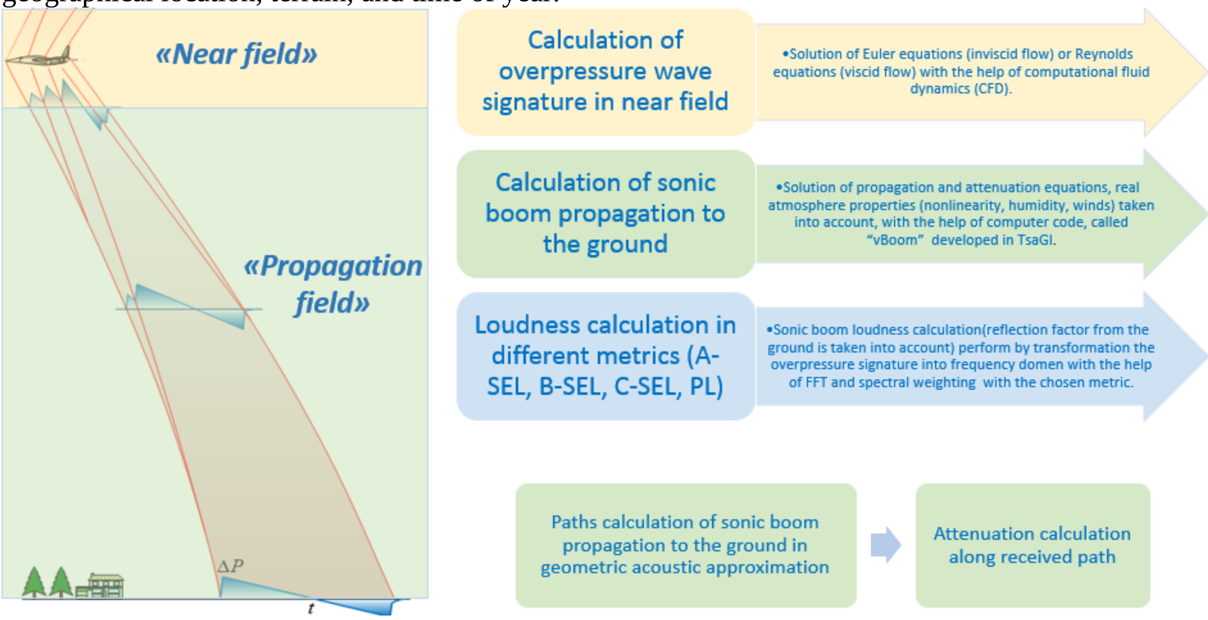

Figure 1. Procedure to evaluate characteristics of sonic boom on the ground.

At present, with an increase in computing power and the development of computational methods for numerical simulation of the flow, the following procedure is widely used to evaluate the characteristics of sonic boom on the ground [2], graphically shown in Figure 1:

1. The overpressure wave signature in the near field is determined using computational gas dynamics methods (numerical solution of the Euler equations in nonviscous formulation or the Reynolds-averaged Navier – Stokes equations with viscosity taken into account);
2. In the approximation of geometric acoustics, the problem of near field overpressure propagation to the ground is solved with regard to atmospheric attenuation. In this case, it is possible to take into account the real parameters of the atmosphere, such as humidity, temperature, wind, etc;
3. The sonic boom loudness on the ground (considering the reflection from its surface) is calculated by Fourier transform of the overpressure signature *p(t)* with further spectrum correction in accordance with the chosen metric (A, B, C, PL).

Methods describing sonic boom propagation and attenuation in windy atmosphere are based on fundamental conservation laws (dynamics equation) and equation of state. The ambient parameters will be denoted by the subscript «0», and acoustic disturbances - by the superscript «’». The second-order (in accordance with a system of orders introduced by Cleveland [5]) acoustic equations for a weakly inhomogeneous moving viscous heat-conducting medium have the form:

$$\begin{cases} \frac{d\rho'}{dt} + \rho_0 \nabla \boldsymbol{v'} = -\boldsymbol{v}' \cdot \nabla\rho_0 - \nabla \cdot (\rho' \boldsymbol{v}') - \rho' \nabla \cdot \boldsymbol{v_0} \\ \rho_0 \frac{d\boldsymbol{v}'}{dt} + \nabla p' = -\rho' \frac{d\boldsymbol{v}'}{dt} - \rho_0 (\boldsymbol{v}', \nabla)\boldsymbol{v}' + (\lambda_0 + 2\mu_0)\nabla(\nabla, \boldsymbol{v}') + \frac{\rho'}{\rho_0}\nabla p_0 - \rho_0 (\boldsymbol{v}', \nabla)\boldsymbol{v_0} \\ \rho_0 T_0 \left(\frac{ds'}{dt} + (\boldsymbol{v}', \nabla)s_0\right) = \kappa_0 \nabla^2 T' \end{cases} \tag{1}$$

where μ, λ are shear and dilatation viscosity coefficients (the terms were introduced, for example, in [6]) respectively, κ is the thermal conductivity coefficient in the Fourier's law of heat conduction.

The second-order equation of state for a moving weakly inhomogeneous medium takes the form:

$$\frac{dp'}{dt} = {c_0}^2 \frac{d\rho'}{dt} + K \frac{d^2\rho'}{dt^2} + \frac{{c_0}^2}{\rho_0} \frac{B}{2A} \frac{d{\rho'}^2}{dt} + \rho' \boldsymbol{v_0} \cdot \nabla {c_0}^2 + \boldsymbol{v}' \cdot ({c_0}^2 \nabla\rho_0 - \nabla p_0) + \boldsymbol{v}' \cdot ({c_0}^2 \nabla\rho' - \nabla p'),$$

where $K = \frac{\kappa_0}{\rho_0 c_p}(\gamma - 1) + {c_0}^2 \sum_\nu \frac{m_\nu \tau_\nu}{1 + \tau_\nu \frac{d}{dt}}$, and operator $\frac{d}{dt} = \frac{\partial}{\partial t} + \boldsymbol{v_0} \cdot \nabla$.

2. Sonic Boom Propagation

Within the approximation of geometric acoustics [7, 8], the medium parameters are assumed to be functions of "slow" coordinates and wave time, i.e. functions of the form $\Phi\big(\varepsilon\boldsymbol{r}, t - \phi(\boldsymbol{r})\big)$, where $\phi(\boldsymbol{r})$ is the eikonal satisfying the equation

$$(\nabla\phi)^2 = \frac{\Omega^2}{{c_0}^2}, \qquad \Omega = 1 - \boldsymbol{v}_0 \cdot \nabla\phi$$

and describing the geometry of the leading edge of the wave. It is also useful to introduce $\boldsymbol{q} = \nabla\phi$. Under the assumptions made, the geometry of propagation of disturbances is defined by a system of equations [7, 9]:

$$\begin{cases} \frac{d\boldsymbol{r}}{ds} = \frac{\boldsymbol{v}_0 + \frac{v^2}{\Omega}\boldsymbol{q}}{v_r} \\ \frac{d\boldsymbol{q}}{ds} = -\frac{1}{v_r}\left(\frac{\Omega}{c_0}\nabla c_0 + q_\alpha \nabla {v_0}_\alpha\right) \end{cases} \tag{2}$$

Disturbances propagate along the curvilinear coordinate s (natural curve parameter) with the velocity $\boldsymbol{v}_r = \boldsymbol{v}_0 + \frac{{c_0}^2}{\Omega}\boldsymbol{q}$, $v_r = |\boldsymbol{v}_r|$.

The system of equations (2) can be solved numerically using left-handed first-order derivative approximation method.

3. Sonic Boom Attenuation

The generalization of augmented Burgers equation [5] on case of moving medium, which performed on the base of the system of equations (1) gives the following equation:

$$\frac{\partial p'}{\partial s} = p' \frac{1}{\mathcal{B}} \frac{\partial \mathcal{B}}{\partial s} + \frac{\beta}{2\rho_0 {c_0}^3}\left(\frac{c_0}{v_r}\right)\Omega \frac{\partial {p'}^2}{\partial t'} + \frac{\mathscr{b} + \mathcal{R}}{2\rho_0 {c_0}^3}\left(\frac{c_0}{v_r}\right)\Omega^2 \frac{\partial^2 p'}{\partial {t'}^2} \tag{3}$$

where $\beta = \frac{B}{2A} + 1$ is coefficient of nonlinearity, $\mathscr{b} = \lambda_0 + 2\mu_0 + \frac{\kappa_0}{c_p}(\gamma - 1)$ is thermoviscous coefficient, $\mathcal{R} = \rho_0 {c_0}^2 \sum_\nu \frac{m_\nu \tau_\nu}{1 + \tau_\nu \Omega \frac{\partial}{\partial t'}}$, $\mathcal{B} = \sqrt{\frac{\rho_0 \Omega {c_0}^2}{A_0 v_r}}$

For the numerical solution of equation (3), as a rule, a transfer to a dimensionless form is made by introducing dimensionless pressure $P = P_{ref} p'$ and dimensionless variables:

dimensionless coordinate

$$d\sigma = \frac{1}{\bar{x}} ds, \qquad \bar{x} = \frac{\rho_0 {c_0}^2 v_r}{\beta \omega_0 \Omega P_{ref}},$$

where $\bar{x}$ is the length of discontinuity formation [10], dimensionless dispersion parameter

$$C_\nu = \frac{m_\nu \tau_\nu {\omega_0}^2 \Omega^2}{2 v_r} \bar{x},$$

thermoviscous absorption parameter (Gol'berg number),

$$\Gamma = \frac{2\rho_0 {c_0}^2 v_r}{\mathscr{b} {\omega_0}^2 \Omega^2 \bar{x}},$$

dimensionless relaxation time of the ν – th process

$$\theta_\nu = \omega_0 \Omega \tau_\nu,$$

dimensionless time

$$\tau = \omega_0 t',$$

dimensionless ambient pressure, density, sound speed and relative pressure

$$P_0 = \frac{p_0}{P_{ref}}, \qquad D_0 = \frac{\rho_0 R T_{ref}}{P_{ref}}, \qquad c = \frac{c_0}{\sqrt{R T_{ref}}}, \qquad \Pi = \frac{P_{ref}}{2\rho_0 {c_0}^2}$$

As a result, we arrive at the equation:

$$\frac{\partial P}{\partial \sigma} = P \frac{\partial P}{\partial \tau} + \frac{1}{\Gamma} \frac{\partial^2 P}{\partial \tau^2} + \frac{1}{\mathcal{B}} \frac{\partial \mathcal{B}}{\partial \sigma} P + \sum_\tau \frac{C_\nu \frac{\partial^2}{\partial \tau^2}}{1 + \theta_\nu \frac{\partial}{\partial \tau}} P \tag{4}$$

The numerical procedure for solving equation (4) is to solve each of the following equations separately:

$$\frac{\partial P}{\partial \sigma} = P \frac{\partial P}{\partial \tau} \tag{4.1}$$

$$\frac{\partial P}{\partial \sigma} = \frac{1}{\Gamma} \frac{\partial^2 P}{\partial \tau^2} \tag{4.2}$$

$$\frac{\partial P}{\partial \sigma} = \sum_\tau \frac{C_\nu \frac{\partial^2}{\partial \tau^2}}{1 + \theta_\nu \Omega \frac{\partial}{\partial \tau}} \mathrm{P} \tag{4.3}$$

$$\frac{\partial P}{\partial \sigma} = \frac{1}{\mathcal{B}} \frac{\partial \mathcal{B}}{\partial \sigma} P \tag{4.4}$$

Problems (4.1)-(4.4) are solved sequentially so that the solution of each problem is the initial data for solving the next one (an operator-splitting method). This procedure continues iteratively until the ground surface is reached. The convergence to the solution of equation (4) and the accuracy of this approach are described in more detail in [11]. The first term is the allowance of nonlinear effects in the sound propagation in the atmosphere. The second term reflects the effects of classical thermoviscous attenuation due to viscous and heat-conducting properties of the air. The third term describes relaxation processes (excitation of vibrational degrees of freedom of molecules), occurring in the air under the influence of a sonic boom wave. In order to simplify the task of accounting for relaxation processes in air, the air itself is represented as a mixture of nitrogen and oxygen. The fourth term is the change in density of acoustic energy with a change of the ray tube area

4. Computer Code «vBoom»

The procedure for calculating the sonic boom propagation and attenuation implemented in the "vBoom" program code is shown schematically in Figure 2. A specific feature of the calculation algorithm is the determination of the sonic boom zone at its initial stage, finding the boundary azimuth angles of the sonic boom carpet by means of dichotomy. At large azimuth angles, the sonic boom rays are reflected from the lower layers of the atmosphere without reaching the ground surface, followed by complete attenuation of disturbances in its upper layers. For the boundary angles found, the corresponding overpressure waveform in the near-field is calculated by linear interpolation. The further calculation procedure is carried out in a quasi-one-dimensional formulation with sequential determination of the propagation path and attenuation of the sonic boom disturbances at each step of the iteration procedure. To speed up the finding of a solution, the mesh spacing is selected based on the condition of maintaining the dimensionless parameter σ in the range of 0.001-0.05, which guarantees the validity of using a separate solution of equation (4) and the effectiveness of sonic boom propagation modeling [12].

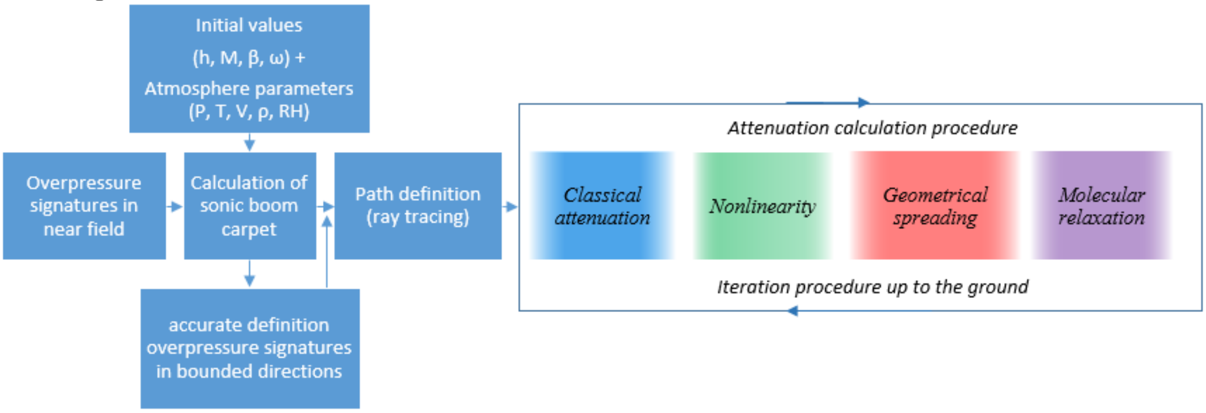

Figure 2. Procedure for calculating the propagation and attenuation of a sonic boom

5. Sonic boom calculation results of C25P and NASA X-59

The calculation results (sonic boom carpet, overpressure signature at ground level and loudness in different metrics) of the sonic boom propagation and attenuation are presented for the C25P aerodynamic layout (Figure 3a) and NASA X-59 prototype demonstrator of sonic boom reduction technology in the C609 layout (Figure 3b). The pressure distributions in the near field for C25P and NASA X-59 are given as the initial data at a distance of three aircraft lengths from the aircraft axis for different azimuth angles φ with increments of 10° and 2° correspondingly. Initial conditions for calculation of sonic boom characteristics for C25P are next: Mach number is 1.6, cruise flight altitude is 15760 meters eastward. The same parameters for NASA X-59 are next: Mach number is 1.4, cruise flight altitude is 16459.2 meters eastward The parameters of the real atmosphere were obtained by measuring the corresponding parameters (temperature , pressure and wind) by altitude at several points over the US territory and are shown in Figure 4. Linear interpolation is used to find the values of atmospheric parameters at an arbitrary altitude.

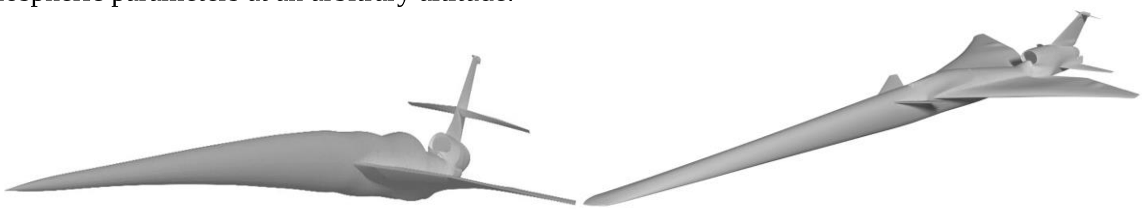

Figure 3. Aerodynamic layouts of C25P and NASA X-59 (C609)

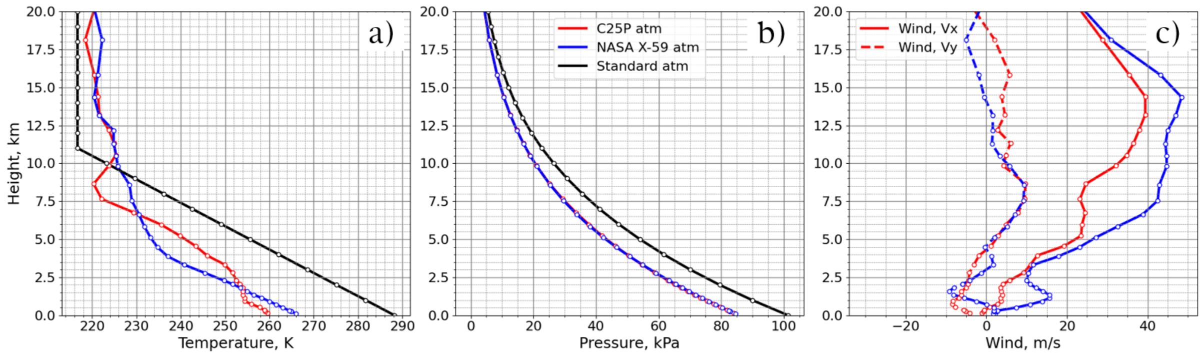

Figure 3. Atmosphere temperature (a), pressure (b) and winds profiles (c)

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The comparison of the calculation results of the overpressure signatures as for C25P as for NASA X-59 at the ground during propagation in a standard atmosphere without wind (Figure 4a,b) with NASA results (sBoom) in branch of SBPW3 shows complete correspondence of amplitude and pressure rise time in the signature for all considered azimuth angles. In this case, the equation (4), describing the attenuation of disturbances in the atmosphere, at $\Omega = 1$ is reduced to the well-known extended Burgers equation, originally obtained by Cleveland [5]. The calculation in conditions of the real atmosphere with wind (Figure 5a,b) shows agreement in wave amplitudes but there are insignificant time axis distortion. In addition, the amplitude of overpressure signatures at the ground due to propagation in the standard atmosphere is higher than in the real atmosphere by 3-4 Pa.

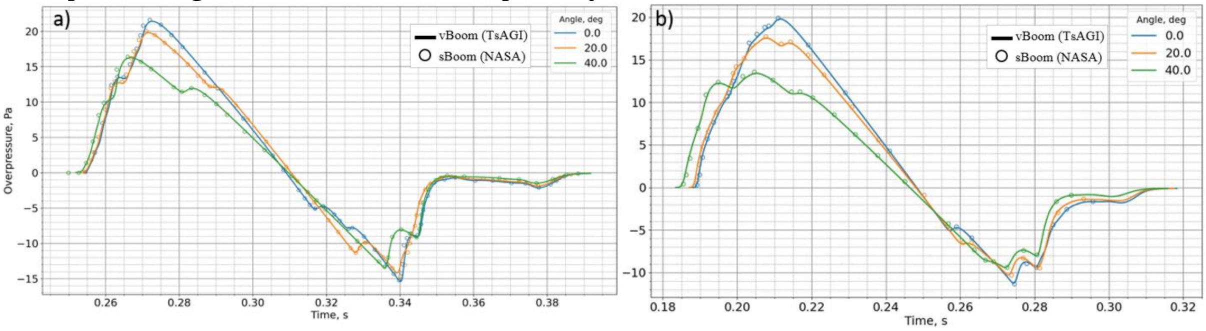

Figure 4. Overpressure signatures at the ground for C25P (a), and NASA X-59 (b) during propagation in condition of standard atmosphere without wind

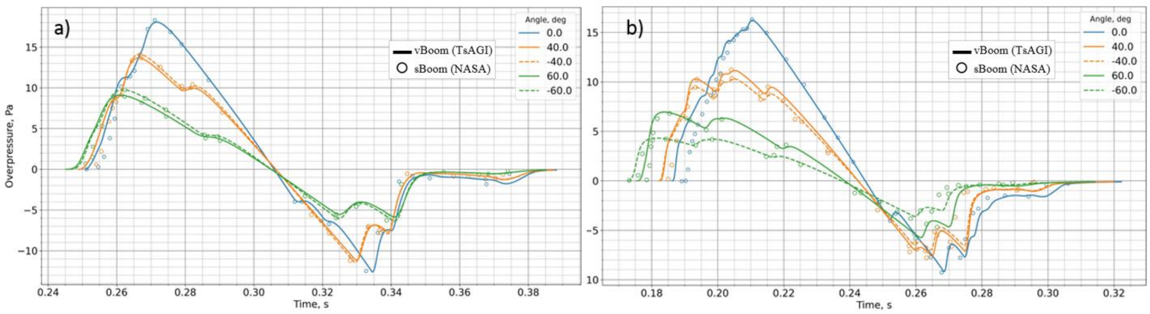

Figure 5. Overpressure signatures at the ground for C25P (a), and NASA X-59 (b) during propagation in condition of real windy atmosphere

The calculation results of the sonic boom loudness in different metrics as for C25P as for NASA X-59 (Figure 6a, b) demonstrate the non-monotonic nature of the change in its value with an azimuth angle φ. The maximum discrepancy in the calculation results falls on large φ and in a standard atmosphere is about 1 dB, and about 2 dB in the real atmosphere (Figure 7a, b). In addition, the sonic boom loudness in the standard atmosphere is higher than in the real atmosphere by about 1.2 dB in the PL metric.

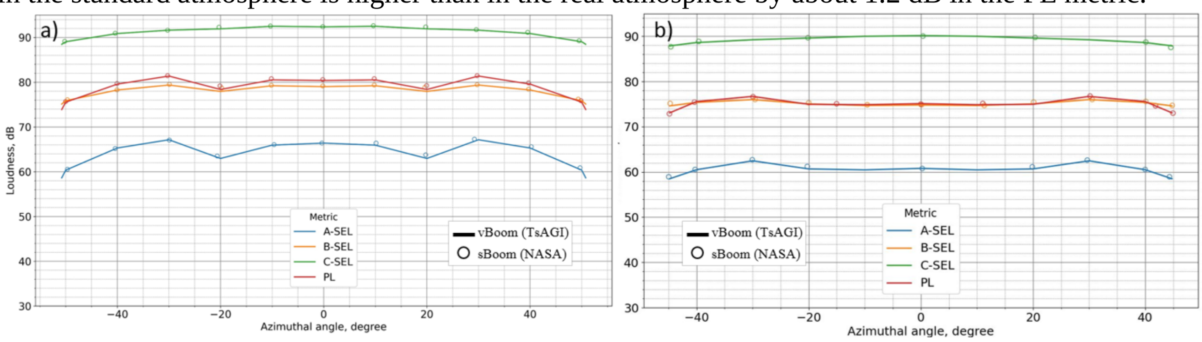

Figure 6. Loudness in different metrics (A-SEL, B-SEL, C-SEL, PL) for C25P (a), and NASA X-59 (b) during propagation in condition of standard atmosphere without wind

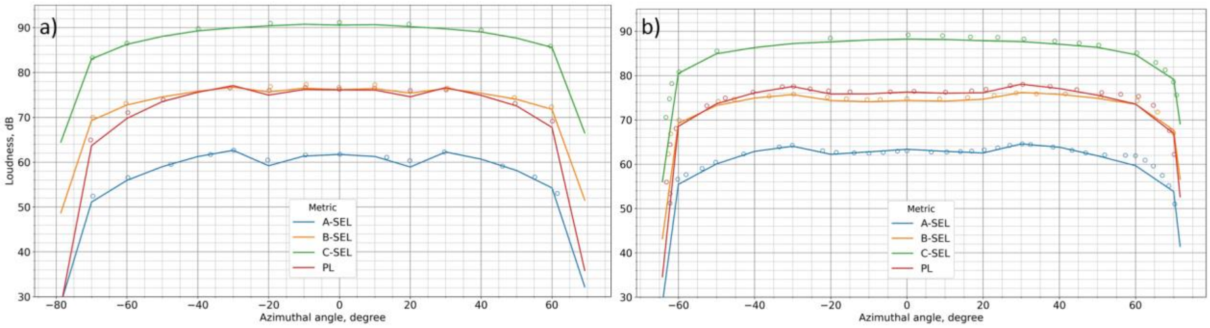

Figure 7. Loudness in different metrics (A-SEL, B-SEL, C-SEL, PL) for C25P (a), and NASA X-59 (b) during propagation in condition of real windy atmosphere

The comparison of sonic boom carpet during disturbances propagation on the ground at conditions of standard atmosphere (Figure 8a) for NASA X-59 (C609) demonstrator shows good agreement with NASA results even after use the first order approximation scheme of equations (2). The comparison of the results in condition of real windy atmosphere demonstrate significant discrepancy in calculation of bordered positions of sonic boom carpet. The bordered positions correspond to high values of azimuthal angels and received discrepancies is connected with the accuracy of bordered azimuth definition. However, the widths of sonic boom carpets remains approximately the same (161-162 km).

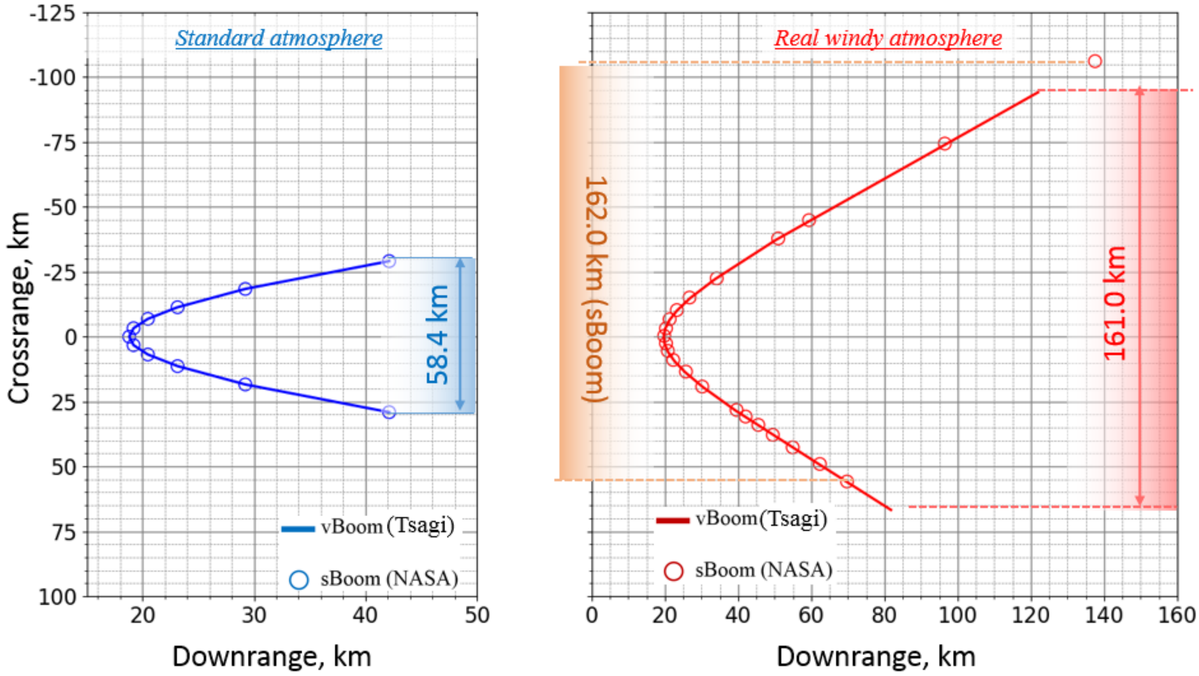

Figure 8. Sonic boom carpet at the ground for NASA X-59 during propagation in condition of standard without wind (a) and real windy atmosphere(b)

6. Conclusion

The methods of sonic boom propagation to the ground and attenuation along received paths are based on the fundamental conservation laws of continuum mechanics. The generalized form of augmented Burgers equation for the case of propagation in moving medium is presented at this paper to describe the attenuation of sonic boom disturbances in an inhomogeneous atmosphere with wind. For the case of the numerical resolution of obtained Burgers equation the computer code «vBoom» has been developed. The code validation was done based on the SBPW3 data (in particular, the calculation results of overpressure signatures, the loudness in various metrics and the sonic boom carpet on the ground). The

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calculation results as for aerodynamic layout C25P as for prototype of NASA X-59 (C609) demonstrator in a standard atmosphere without wind are in complete agreement with the NASA («sBoom») results. However, in a real atmosphere with wind there isn't difference in calculation of the overpressure wave amplitude, but there is significant time axis distortion of the wave shape, near 2dB discrepancy in loudness and differences in calculation of bordered positions of sonic boom carpet near 10 km.

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