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Elastic Analysis of Steel-Concrete Composite Beams with Partial Interaction

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Abstract. The paper presents an exact analytical method for the elastic analysis of steel-concrete composite beams with partial interaction. Accepting the basic assumptions of the Newmark analytical model and adopting the axial force in the concrete slab as the main unknown, the second order nonhomogeneous differential equation of the steel-concrete composite element with partial interaction is derived. Further, the complete solutions for simply supported and fixed-ended composite beams subjected to concentrated and uniform loads respectively, are developed. The solution of the homogeneous equation is determined by imposing proper Dirichlet or Neumann boundary conditions depending on the static scheme of the element. The particular solutions are then derived for the considered loading conditions. It is shown that the internal axial force in concrete slab associated to composite beams with partial interaction can be expressed as a fraction of the axial force in concrete slab under full interaction through a non-dimensional function $f(\alpha L)$ which takes into account the connection's stiffness, the mechanical properties and also the length of the element. Moreover, the solutions are included in a flexibility-based approach to derive the force-displacement relations of the beam element with partial interaction. For the resulted 2-noded beam-column element with 6DOF, the stiffness matrix is derived, showing that the partial composite action may be included at the element level by means of a series of correction factors applied to the standard full-interaction stiffness matrix coefficients. A numerical example is provided to demonstrate the accuracy and performance of the proposed method. Within the elastic range, the predicted load-midspan deflection curve is in very good agreement with both experimental and other numerical results retrieved from international literature. A parametric study was conducted to investigate the influence of the shear connection degree on the beam's midspan deflection and the results were compared with those computed by using code provisions.

1. Introduction

In the last decades, composite steel-concrete members have seen widespread use as parts of the structural system of multistorey buildings and bridges. A common type of composite beams is represented by a concrete slab supported by a cold formed steel profile interconnected with stud shear connectors. In this case, best use of concrete and steel materials is being made, as concrete is efficient in compression while the steel component is effective in tension. Under service load conditions, the ductile shear connectors allow relative longitudinal slippage between the connected components, therefore at cross sectional level, a discontinuity occurs in the strain distribution diagram at the steel-concrete interface, as shown in Figure 1b. The evaluation of slip effects on both strength and stiffness properties is of current concern in the specialized literature although significant research has been reported in the past decades [1-3]. Analytical procedures based on the key assumptions of Newmark's model [4] have been proposed for the static behaviour of steel-concrete composite beams with partial interaction, terms that is defined by nonzero interlayer slip. In this paper, the second order differential equation of the steel-concrete element is derived adopting the basic hypothesis of the above-mentioned model. The equation is formulated in terms of axial force in the concrete slab under partial interaction and the closed-form solution is derived for two different loading scenarious. The solutions are then included in a flexibility-based approach to derive the force-displacement relations of the beam element with partial interaction. The elastic analysis of steel-concrete composite beams with interlayer slip is highly significant because it can be used to validate the code provisions related to deflection estimation and, moreover it provides specific information that may be cast within the framework of more complex inelastic analysis. Furthermore, the solutions presented herein may be used to enhance the analysis procedure presented in [1, 2] by considering an exact expression of the axial force in the concrete which explicitly includes the influence of the connection stiffness, the degree of shear connection, the cross sectional properties, the shear connectors spacing (uniform or triangular) and the element's length. In this way, the analysis method takes advantage of using only one 2-noded beam-column element with 6 DOF that can account for the main variables that dominate the structural behaviour of members and framed structures.

2. Differential equation of steel-concrete composite beams with partial interaction

The governing differential equation of steel-concrete composite beams with partial (incomplete) interaction is derived adopting the basic assumptions of Newmark's model [4]:

(1) linear elastic behaviour of steel, concrete and shear connectors;
(2) steel and concrete layers are continuously connected with constant shear modulus;
(3) plane sections remain plane after flexural deformation (Bernoulli hypothesis);
(4) frictional effects, uplift and shear deformations are neglected;
(5) displacements and rotations are small.

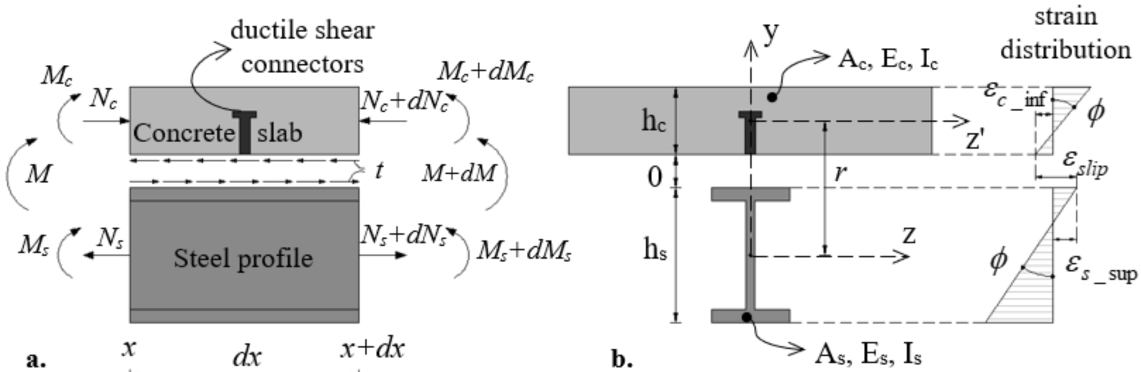

Figure 1. **a**. Infinitesimal steel-concrete composite element; **b**. Strain distribution.

Consider the force distribution within the differential element shown in Figure 1a. By expressing the horizontal equilibrium in both components, the following relations can be written:

$$\frac{dN_c}{dx} = -t; \qquad \frac{dN_s}{dx} = -t \tag{1}$$

in which N_c and N_s are the internal axial forces in concrete slab and steel profile, respectively, whereas t is the interface shear flow and can be computed as:

$$t = \frac{P_c}{i_c} \tag{2}$$

where, P_c is the shear force acting on a single shear connector and i_c is the constant spacing between connectors. Under assumption (1), the shear force P_c can be computed as a function of shear stud stiffness K and corresponding slip s:

$$P_c = K \cdot s \tag{3}$$

Combining relations (1), (2) and (3), the interlayer slip can be expressed as:

$$s = -\frac{i_c}{K}\frac{dN_c}{dx} \tag{4}$$

Within the partial interaction conditions, a discontinuity occurs in the strain distribution diagram at the steel-concrete interface as shown in Figure 1b. The slip between steel and concrete component is quantified in terms of slip strain, ε_{slip}, as follows:

$$\varepsilon_{slip} = \varepsilon_{c\_inf} - \varepsilon_{s\_sup} \tag{5}$$

where ε_{c_inf} and ε_{s_sup} are the strains within the concrete slab and steel profile at the interface and can be computed as the sums of axial and bending components:

$$\begin{aligned} \varepsilon_{c\_inf} &= -\frac{N_c}{E_c A_c} + \frac{M_c}{E_c I_c}\frac{h_c}{2} \\ \varepsilon_{s\_sup} &= \frac{N_s}{E_s A_s} - \frac{M_s}{E_s I_s}\frac{h_s}{2} \end{aligned} \tag{6}$$

In the above relations, $E_{c(s)}A_{c(s)}$ and $E_{c(s)}I_{c(s)}$ denote the axial and flexural stiffnesses of concrete and steel components calculated with respect to their geometric centroid. Moreover, under the above assumptions, the curvature can be defined as:

$$\varphi = \frac{M_c}{E_c I_c} = \frac{M_s}{E_s I_s} = \frac{M_c + M_s}{E_c I_c + E_s I_s} \tag{7}$$

Combining relations (5), (6) and (7) and knowing from equilibrium that N_c is equal to N_s, the slip strain becomes:

$$\varepsilon_{slip} = -N_c\left(\frac{1}{E_c A_c} + \frac{1}{E_s A_s}\right) + \frac{M_c + M_s}{E_c I_c + E_s I_s} r \tag{8}$$

in which r is the distance between the geometric centroids of steel and concrete components. Expressing the total bending moment with respect to the steel profile centroid as:

$$M = M_c + M_s + N_c \cdot r \tag{9}$$

the slip strain can be further written as:

$$\varepsilon_{slip} = -N_c\left(\frac{1}{E_c A_c} + \frac{1}{E_s A_s}\right) + \frac{M - N_c \cdot r}{E_c I_c + E_s I_s} r \tag{10}$$

Differentiating relation (4), the slip strain can also be expressed in the following form:

$$\varepsilon_{slip} = \frac{ds}{dx} = -\frac{i_c}{K}\frac{d^2 N_c}{dx^2} \tag{11}$$

Using equations (10) and (11), the second order differential equation of steel-concrete composite beams with partial interaction can be derived:

$$\frac{d^2 N_c}{dx^2} - \frac{K}{i_c}\frac{(EI)^\infty}{\overline{\overline{EA}}(EI)^0} N_c = -\frac{K}{i_c}\frac{r}{(EI)^0} M \tag{12}$$

in which:

$$\begin{aligned} \frac{1}{\overline{\overline{EA}}} &= \frac{E_c A_c + E_s A_s}{E_c A_c E_s A_s}; \\ (EI)^0 &= E_c I_c + E_s I_s; \\ (EI)^\infty &= (EI)^0 + r^2 \overline{\overline{EA}}; \end{aligned} \tag{13}$$

$(EI)^0$ and $(EI)^\infty$ are the elastic flexural rigidities of the composite beam cross section without interaction and with full interaction, respectively. Furthermore, by introducing the following notations:

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$$k = \frac{K}{i_c}; \qquad \alpha^2 = k\frac{(EI)^\infty}{\overline{EA}(EI)^0}; \tag{14}$$

the governing second order differential equation becomes:

$$\frac{d^2 N_c(x)}{dx^2} - \alpha^2 N_c(x) = -\alpha^2 \frac{r \cdot \left(\overline{EA}\right)}{(EI)^\infty} M(x) \tag{15}$$

The main unknown in equation (15) is the axial force in the concrete slab and it depends on the cross-sectional properties, the level of interaction (through the non-dimensional parameter α) and on the bending moment value. Consequently, closed-form solutions can be analytically provided for different loading scenarious.

Once the solution of the differential equation is determined, it may be used to evaluate the longitudinal slip by Eq. (4) and the transverse displacement by double integration of the following second order differential equation:

$$\frac{d^2 w}{dx^2} = -\frac{M}{(EI)^\infty} + \frac{r}{k}\frac{\overline{EA}}{(EI)^\infty}\frac{d^2 N_c}{dx^2} \tag{16}$$

which is obtained by combining relations (7), (9) and (15). Noticing that the first term of the sum in Eq. (16) represents the transverse displacement of the steel-concrete beam with full interaction, w_{fi}, the solution of the previous differential equation may be written in the following form:

$$w_{pi} = w_{fi} + \frac{r}{k}\frac{\overline{EA}}{(EI)^\infty} N_c \tag{17}$$

where w_{pi} is the elastic transverse displacement of the steel-concrete element under partial interaction. It is worth mentioning that Eq. (17) rigorously includes the partial interaction effects on the transverse displacement of the element by considering the actual shear stiffness of the connection, k.

3. Closed-form solution of the differential equation

In this section, we provide the solution of the governing second order differential equation of steel-concrete composite beams with partial interaction for two different loading schemes. The solution of the homogeneous equation, $N_{c,0}$, may be expressed as follows:

$$N_{c,0} = C_1 \cdot ch(\alpha x) + C_2 \cdot sh(\alpha x) \tag{18}$$

where C_1 and C_2 are integration constants and are derived by imposing proper Dirichlet or Neumann boundary conditions depending on the static scheme of the beam, as presented in the following subsections. The particular solution $N_{c,p}$ depends on the external bending moment variation and, implicitly, on the loading conditions.

3.1 Simply supported composite beam with a transverse force at midspan

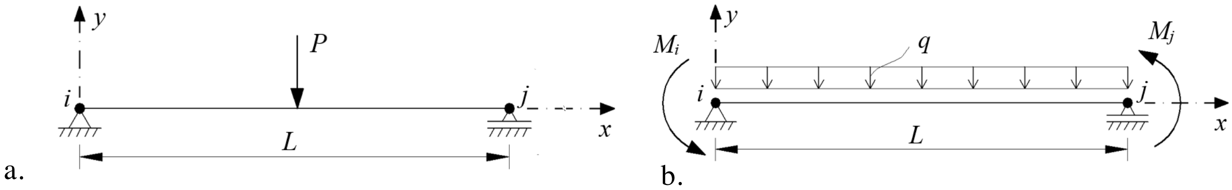

Figure 2. a. Simply supported beam subjected to concentrated load; **b.** Simply supported beam subjected to end bending moments and uniform distributed load.

Consider the element shown in Figure 2a. The external bending moment M in terms of the distance x of the cross section from the left support can be expressed as:

$$M(x) = \begin{cases} \frac{P}{2}x \to x \leq \frac{L}{2} \\ -\frac{P}{2}x + \frac{PL}{2} \to x \geq \frac{L}{2} \end{cases} \tag{19}$$

For this case the particular and, consequently, the general solution depends on the position along the element:

$$\begin{aligned} N_c'(x) &= C_1' \cdot ch(\alpha x) + C_2' \cdot sh(\alpha x) + \frac{r \cdot (\overline{EA})}{(EI)^\infty}\frac{P}{2}x \to x \leq \frac{L}{2} \\ N_c''(x) &= C_1'' \cdot ch(\alpha x) + C_2'' \cdot sh(\alpha x) + \frac{r \cdot (\overline{EA})}{(EI)^\infty}\left[-\frac{P}{2}x + \frac{PL}{2}\right] \to x \geq \frac{L}{2} \end{aligned} \tag{20}$$

It is worth mentioning that the particular solutions are introduced as first order polynomials in accordance with the bending moment expression. To determine the integration constants, the following Dirichlet boundary conditions are imposed in terms of axial force in the concrete slab:

$$\begin{aligned} N_c|_{x=0} &= 0; \\ N_c|_{x=L} &= 0; \end{aligned} \tag{21}$$

Since four constants must be determined, two compatibility conditions are further imposed at the midspan cross-section:

$$\begin{aligned} N_c'\big|_{x=\frac{L}{2}} &= N_c''\big|_{x=\frac{L}{2}}; \\ \frac{dN_c'}{dx}\bigg|_{x=\frac{L}{2}} &= \frac{dN_c''}{dx}\bigg|_{x=\frac{L}{2}}; \end{aligned} \tag{22}$$

By solving the system with four equations given by conditions (21) and (22), the following expressions for the integration constants are obtained:

$$\begin{aligned} C_1' &= \frac{1}{\alpha}\frac{r \cdot (\overline{EA})}{(EI)^\infty}P\frac{ch^2\left(\frac{\alpha L}{2}\right) - ch\left(\frac{\alpha L}{2}\right)}{sh(\alpha L)}; \\ C_2' &= -\frac{1}{\alpha}\frac{r \cdot (\overline{EA})}{(EI)^\infty}\frac{P}{2}; \\ C_1'' &= \frac{1}{\alpha}\frac{r \cdot (\overline{EA})}{(EI)^\infty}\frac{P}{2}\left(\frac{ch(\alpha L) + 1}{sh(\alpha L)} - \frac{ch(\alpha L)}{sh\left(\frac{\alpha L}{2}\right)}\right); \\ C_2'' &= -\frac{1}{\alpha}\frac{r \cdot (\overline{EA})}{(EI)^\infty}\frac{P}{2}\left(1 - \frac{sh(\alpha L)}{sh\left(\frac{\alpha L}{2}\right)}\right); \end{aligned} \tag{23}$$

Further, the general solution of the equation may be written as:

$$\begin{aligned} N_c'(x) &= -\frac{P}{\alpha}\frac{r \cdot (\overline{EA})}{(EI)^\infty}sh\left(\frac{\alpha L}{2}\right)\frac{sh(\alpha x)}{sh(\alpha L)} + \frac{r \cdot (\overline{EA})}{(EI)^\infty}\frac{P}{2}x, \qquad x \leq \frac{L}{2}; \\ N_c''(x) &= -\frac{P}{\alpha}\frac{r \cdot (\overline{EA})}{(EI)^\infty}sh\left(\frac{\alpha L}{2}\right)\frac{sh(\alpha L - \alpha x)}{sh(\alpha L)} + \frac{r \cdot (\overline{EA})}{(EI)^\infty}\left(-\frac{P}{2}x + \frac{PL}{2}\right), \qquad x \geq \frac{L}{2}; \end{aligned} \tag{24}$$

Additionally, by introducing the axial force in the concrete slab under full interaction conditions, derived in [1, 2]:

$$N_{cf}(x) = M(x)\frac{r(\overline{EA})}{(EI)^\infty}; \tag{25}$$

the solutions of the governing differential equation can be further written in the following form:

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$$N_c'(x) = N_{cf}'(x)\left[1 - \frac{P}{\alpha \cdot M(x)}\frac{sh(\alpha x)}{sh(\alpha L)} sh\left(\frac{\alpha L}{2}\right)\right], \quad x \leq \frac{L}{2};$$
$$N_c''(x) = N_{cf}''(x)\left[1 - \frac{P}{\alpha \cdot M(x)}\frac{sh(\alpha L - \alpha x)}{sh(\alpha L)} sh\left(\alpha\frac{L}{2}\right)\right], \quad x \geq \frac{L}{2}; \tag{26}$$

or, in condensed form:

$$N_c(x) = N_{cf}(x) \cdot f(\alpha L); \tag{27}$$

In this way, the axial force in the concrete slab under partial interaction is computed as a fraction of the axial force in the concrete slab under full interaction. The reduction factor $f(\alpha L)$ considers the degree of composite action and the cross-sectional properties. It is important to emphasize that for composite beams with full interaction $f(\alpha L) = 1$ and consequently $N_c(x) = N_{cf}(x)$. Contrarily, for no interaction case, $f(\alpha L) = 0$, therefore $N_c(x) = 0$.

3.2 Simply supported composite beam subjected to end bending moments and uniform distributed load

Similarly, as presented in the previous subsection, the expression of the axial force in the concrete slab under partial interaction is derived for the element shown in Figure 2b. The bending moment equation is given by the following expression:

$$M(x) = -\frac{q}{2}x^2 + \left(\frac{M_i + M_j}{L} + \frac{qL}{2}\right)x - M_i \tag{28}$$

In this loading scenario, the particular solution of the governing second order differential equation is a second order polynomial and the general solution takes the following form:

$$N_c(x) = C_1 \cdot ch(\alpha x) + C_2 \cdot sh(\alpha x) + \frac{r \cdot \overline{(EA)}}{(EI)^\infty}\left(\frac{M_i + M_j}{L}x - M_i - \frac{q}{2}x^2 + \frac{qL}{2}x - \frac{q}{\alpha^2}\right) \tag{29}$$

The integration constants can be determined by imposing as in [5] zero slip conditions at beam ends:

$$s|_{x=0} = 0;$$
$$s|_{x=L} = 0; \tag{30}$$

which, in accordance with equation (4), are equivalent to the following Neumann boundary conditions:

$$\left.\frac{dN_c}{dx}\right|_{x=0} = 0;$$
$$\left.\frac{dN_c}{dx}\right|_{x=L} = 0; \tag{31}$$

With C_1 and C_2 determined enforcing conditions (31), the axial force in the concrete component is completely defined by the following expression:

$$\begin{aligned} N_c(x) = {} & \frac{1}{\alpha}\frac{r \cdot \overline{(EA)}}{(EI)^\infty}\left[\frac{M_i + M_j}{L}\frac{ch(\alpha L) - 1}{sh(\alpha L)} + \frac{qL}{2}\frac{ch(\alpha L) + 1}{sh(\alpha L)}\right] \cdot ch(\alpha x) - \\ & -\frac{1}{\alpha}\frac{r \cdot \overline{(EA)}}{(EI)^\infty}\left(\frac{M_i + M_j}{L} + \frac{qL}{2}\right) \cdot sh(\alpha x) + \\ & +\frac{r \cdot \overline{(EA)}}{(EI)^\infty}\left(\frac{M_i + M_j}{L}x - M_i - \frac{q}{2}x^2 + \frac{qL}{2}x - \frac{q}{\alpha^2}\right); \end{aligned} \tag{32}$$

or:

$$\begin{aligned} N_c(x) = N_{cf}(x)\Bigg[1 - \frac{q}{\alpha^2 \cdot M(x)} + \frac{1}{\alpha \cdot M(x)}\Bigg(\frac{M_i + M_j}{L}\frac{ch(\alpha L - \alpha x) - ch(\alpha x)}{sh(\alpha L)} + \\ + \frac{qL}{2}\frac{ch(\alpha L - \alpha x) + ch(\alpha x)}{sh(\alpha L)}\Bigg)\Bigg]; \end{aligned} \tag{33}$$

where, $N_{cf}(x)$ is the axial force in the concrete slab under full interaction conditions.

4. Exact stiffness matrix

In this section a flexibility-based approach is used to derive the force-displacement relations of a 2-noded steel-concrete beam element with partial interaction. The effects of interlayer slip are explicitly captured by considering in the formulation the solution of the governing differential equation derived in the previous sections.

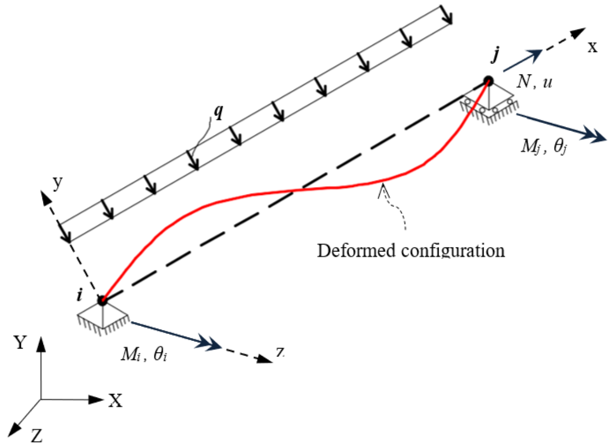

Figure 3. Beam-Column element with rigid-body modes removed.

For exemplification, consider the beam-column element shown in Figure 3, represented in the natural coordinates (i.e. a simply supported beam) subjected to end bending moments and uniform distributed load. The element flexibility matrix which relates the end displacements $\boldsymbol{u}_r$ to end actions, $\boldsymbol{s}_r$ can be derived by applying the Maxwell-Mohr rule for computation of generalized displacements. Specifically, the end rotation can be evaluated using the following expressions:

$$\theta_i = \int_0^L \frac{M(x)}{(EI)_{el}} \frac{\partial M(x)}{\partial M_i} dx$$
$$\theta_j = \int_0^L \frac{M(x)}{(EI)_{el}} \frac{\partial M(x)}{\partial M_j} dx \tag{34}$$

where $M(x)$ is the bending moment at the current location x along the member length and $(EI)_{el}$ is the cross-sectional stiffness of the beam under partial interaction conditions and can be computed using the following equation as derived in [1]:

$$(EI)_{el} = \frac{(EI)^0}{1 - r\dfrac{N_c(x)}{M(x)}} \tag{35}$$

Eq. (34) can be further expressed as:

$$\theta_i = \int_0^L \frac{M(x) - r \cdot N_c(x)}{(EI)^0} \left(\frac{x}{L} - 1\right) dx$$
$$\theta_j = \int_0^L \frac{M(x) - r \cdot N_c(x)}{(EI)^0} \frac{x}{L} dx \tag{36}$$

Supposing that the element is subjected to uniform distributed loads (Figure 3), the bending moment can be expressed as functions of nodal element forces as described by Eq (28) and the axial force in the concrete slab under partial interaction, $N_c(x)$ is given by Eq. (32) and can be included in relations (36), thus the relationship between nodal displacements and nodal efforts becomes:

$$\begin{bmatrix} \theta_i \\ \theta_j \end{bmatrix} = \begin{bmatrix} \int_0^L \frac{(\xi-1)[\xi-1-r\cdot\beta(A+\xi-1)]}{(EI)^0}dx & \int_0^L \frac{(\xi-1)[\xi-r\cdot\beta(A+\xi)]}{(EI)^0}dx \\ \int_0^L \frac{\xi[\xi-1-r\cdot\beta(A+\xi-1)]}{(EI)^0}dx & \int_0^L \frac{\xi[\xi-r\cdot\beta(A+\xi)]}{(EI)^0}dx \end{bmatrix} \begin{bmatrix} M_i \\ M_j \end{bmatrix} + \boldsymbol{\delta}_r \tag{37}$$

where $\boldsymbol{\delta}_r$ are the nodal displacements resulting from loads acting along the member length and can be expressed as:

$$\boldsymbol{\delta}_r = \begin{bmatrix} \int_0^L \frac{(\xi-1)\left[(1-r\beta)\left(\frac{qL}{2}x-\frac{q}{2}x^2\right)-r\beta B+r\beta\frac{q}{\alpha^2}\right]}{(EI)^0} \\ \int_0^L \frac{\xi\left[(1-r\beta)\left(\frac{qL}{2}x-\frac{q}{2}x^2\right)-r\beta B+r\beta\frac{q}{\alpha^2}\right]}{(EI)^0} \end{bmatrix} \tag{38}$$

in which:

$$\xi = \frac{x}{L}; \qquad \beta = \frac{r\cdot(\overline{EA})}{(EI)^\infty};$$
$$A = \frac{1}{\alpha\cdot L}\frac{ch(\alpha L-\alpha x)-ch(\alpha x)}{sh(\alpha L)}; \qquad B = \frac{1}{\alpha}\frac{qL}{2}\frac{ch(\alpha L-\alpha x)+ch(\alpha x)}{sh(\alpha L)}; \tag{39}$$

Eq (37) may be written in condensed form as:

$$\boldsymbol{u}_r = \boldsymbol{f}_r\cdot\boldsymbol{s}_r+\boldsymbol{\delta}_r \tag{40}$$

where $\boldsymbol{f}_r$ is the flexibility matrix of the steel-concrete composite element with partial interaction. The exact stiffness matrix of the composite element may be further computed by inverting the flexibility matrix and it can be emphasized that the partial composite action may be included at the element level by means of a series of correction factors applied to the standard full-interaction stiffness matrix coefficients.

5. Numerical example

In this section, a numerical example is given to validate the analytical procedure described above and to highlight particular features concerning the behaviour of composite beams with partial interaction. The beam E1 experimentally tested by Chapman & Balakrishan [6] is considered. The geometric configuration of the beam is presented in Figure 4 and the main material and shear connection properties can be found in [2]. Based on the number of shear connectors and the spacing between them, the shear connection stiffness, k, has been determined using Eq. (14).

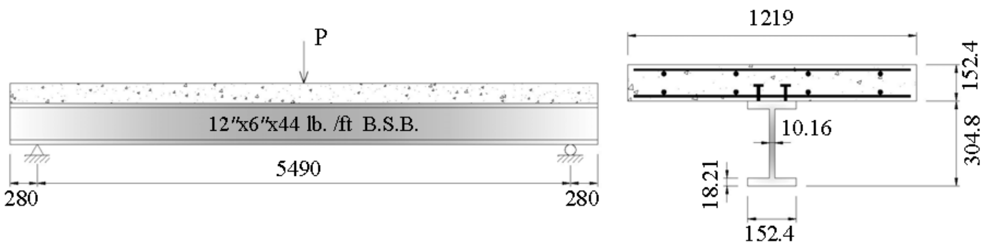

Figure 4. Beam E1 layout

In Figure 5a comparative load mid-span deflection curves are shown. As it can be seen, the elastic behaviour of the composite beam predicted with the proposed analytical model is in very close agreement with those published by Queiroz et al. [3] in which the behaviour is explicitly modelled by using advanced three-dimensional finite element software. Compared to the experimental results, both numerical [3] and analytical procedures slightly underestimate the initial stiffness of the composite element.

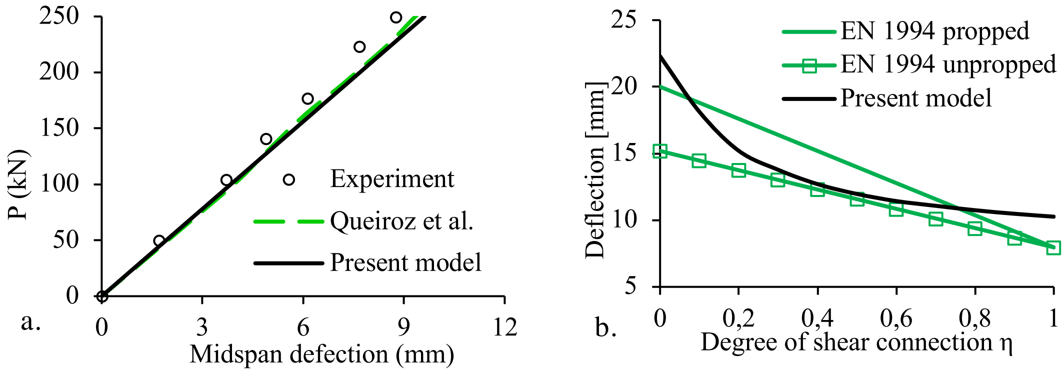

Figure 5. a. Load-deflection curves; **b**. Deflection-shear connection degree curves.

Figure 5b depicts the mid-span deflections evaluated using Eq. (17) for shear connection levels ranging from 0 (no interaction) to 1 (full shear connections) and those calculated according to Eurocode-4 [7] provisions for propped and unpropped construction. In the proposed approach, for each shear connection level the associated stiffness has been determined based on the appropriate number of shear connectors and the distance between them. The load level was set to 250 kN to assure the elastic behaviour of the beam. It can be noted that the Eurocode 4 provisions asses in an approximate manner the deflections of composite beams with partial shear connection. In this study, the deviations between the exact and code-based deflections are ranging from 3.5% to 22.4% and from 3.2% to 31.9% for propped and unpropped construction, respectively.

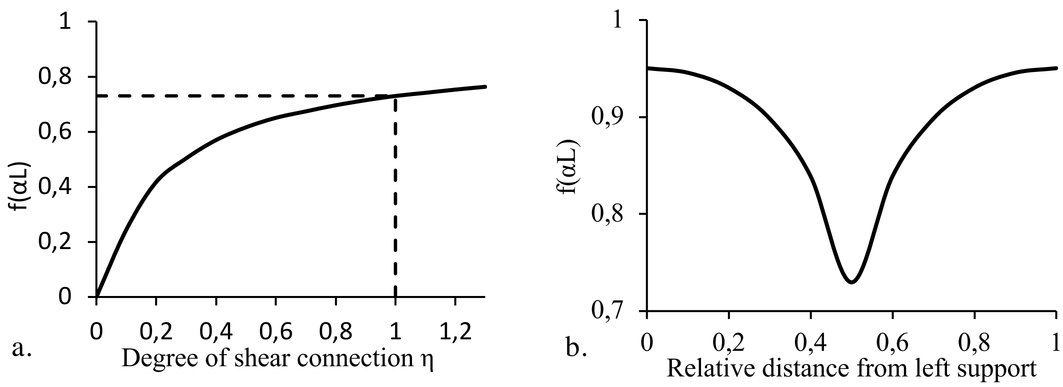

Figure 6. a. f(αL)-degree of shear connection curve; **b**. f(αL) variation along the beam length

The following study presents the variation of the $f(\alpha L)$ function, which reduces the axial force in the concrete slab under full interaction conditions to obtain the axial force under partial interaction conditions, with respect to the shear connection degree (the element length is assumed to be constant). Using Eq. (26), the value of $f(\alpha L)$ function was computed at mid-span (extreme bending moment) for different shear connection ratios (Figure 6a). It can be observed that by increasing the shear connection degree η and, consequently the connection stiffness k, the value of the $f(\alpha L)$ function approaches the theoretical value of 1(one) which correspond to full interaction case (no relative slip). It is worth noting that for full shear connection ($\eta=1$) the associated $f(\alpha L)$ value is less than 1 (one), which indicates that the degree of shear connection (related to the strength-based property) is different from that of interaction (related to the stiffness-based property). The variation of the $f(\alpha L)$ function along the beam length under full shear conditions ($\eta=1$) is shown in Figure 6b. It can be observed that the values of $f(\alpha L)$ are ranging between 0.7 and 1.0, hence a constant value can be used to compute the approximate deflection of steel-concrete composite beams as accepted by most relevant standards.

6. Conclusions

This paper presented an exact analytical method for the elastic analysis of steel-concrete composite beams under partial interaction conditions. Adopting the basic assumptions of Newmark's model, the governing second order differential equation has been derived and closed-form solutions have been provided for two loading scenarious. The study emphasized that the axial force in the concrete slab under partial interaction can be evaluated as a fraction of the axial force in the concrete slab under full interaction through a reduction factor $f(\alpha L)$ which explicitly considers the shear stiffness of the connection and the cross-sectional properties. This conclusion may be used to formulate approximate methods of quantifying the effect of the partial composite action in linear and non-linear analysis of composite beams. Moreover, the solutions have been used to derive the force-displacement relations of a 2-noded beam-column element with 6DOF under partial interaction in a flexibility-based approach. The stiffness matrix was derived, and it is emphasized that the partial composite action may be included at the element level by means of a series of correction factors applied to the standard full-interaction stiffness matrix coefficients. The analytical model can be further developed to consider non-uniform shear connector distribution by simply dividing the beam according with variable spacing and treating each segment as beams with uniform distribution.

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