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# **Effect of Assumed Boundary Conditions in Numerical Model** of Road Pavement-Mining Subsoil System on Criterial Values **Used in Design Using Mechanistic Methods**

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Abstract. Underground mining brings benefits in the form of the extracted mineral. The negative effects of mining exploration are deformations of the rock mass, which also cause deformations on the ground surface. There are continuous deformations, discontinuous deformations and mining-induced tremors. Recommendations regarding the protection of the structure of cubature building against the negative effect of mining operations are discussed in detail, for example, in the recommendation published by the Building Research Institute (ITB) in Warsaw. In the case of road structures, the situation is different. Firstly, there are no general rules that would provide clear guidelines for the procedure for designing road pavement in mining areas, similarly to cubature buildings. Secondly, in the computer programs used for the individual design of road pavement, it is not possible to assign additional actions, including mining impact. Therefore, in order to analyze the behavior of the pavement-mining subsoil system, an advanced numerical analyze should be carried out. In this case, the subsoil thickness, the boundary conditions and the constitutive relationships of the materials of the road pavement layers and subsoil should be determined. This paper presents an attempt to select kinematic boundary conditions for the FEM model of the road pavement-mining subsoil system, analogically to the model of the building-mining subsoil system. The paper is aimed at assessment of the influence of kinematic boundary conditions selection on the criterial values that are taken into account during the design process of road pavement using mechanistic methods. For this purpose, three cases were considered: (i) horizontal mining strain ( $\varepsilon_{design}$ ), (ii) curvature of surface (K<sub>design</sub>), (iii) combined impact of these actions. In these cases, each time vehicle wheel load was assumed. Based on the analyzes, the computational horizontal strain of the mining area  $\varepsilon_{comp}$  is decisive when assessing the criterial values taken into account in the design process of road pavement structures.

### 1. Introduction

There are significant benefits from mining in the form of minerals, e.g. coal. However, in addition to the benefits, negative effects are also observed, see Figure 1.

A surface subsidence curve is formed due the impact of continuous displacement [4], [5]. It is described by a number of different indicators, e.g.: subsidence (w), terrain inclination (T), horizontal ground strain ( $\varepsilon$ ), horizontal displacement (u), ground curvature (K) or radius of curvature (R=1/K), see Figure 2.

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Figure 1. Effects of the mining impact on the surface area, where: w – subsidence, T – terrain inclination, K – ground curvature, u – horizontal displacement,  $\varepsilon$  – horizontal ground strain (developed based on Kawulok [1], Kwiatek[2], Judycki [3])



Figure 2. The course of deformation indicators in the area of the edge of a large exploitation field according to the Knothe theory [4], [6] where: H- the depth of exploitation,  $\beta$ - the angle of the main influences range,  $w_{max}$  - maximum subsidence, w - subsidence,  $\varepsilon_{max}^+$  - maximum loosening strain,  $R_{min}^+$  - minimum loosening radius of curvature,  $\varepsilon_{max}^-$  maximum compressive strain,  $R_{min}^-$  - minimum compressive radius of curvature [1]

All mining impact (continuous or discontinuous deformations in mining area, changing water levels and mining-induced tremors) the appearance of damage in all types of building structure subject to these impacts [1]. In the case of road pavement, this may cause a problem in their use or even

serious difficulties in ensuring continuous communication in important areas of road transport (Catalogue 2013 [3]), see gray area in Figure 1. The subsidence causes changes in the longitudinal profile [7] (see Figure 3) and the cross-section of the road and influences the creation of additional load in the pavement layers and subsoil . These damages may cause counter-falls and thus stagnation of water on the pavement surface, which endangers traffic safety (e.g. through aquaplaning) [8]. Except subsidence, effect of horizontal (compressive or loosening) strain on the pavement layers materials and subsoil is very important. While horizontal (compressive or loosening) strain may change parameters of pavement layers materials and subsoil [3], [9]-[11]. The compressive strain causes the uplift of terrain, including pavement or stones [12] (Figure 4) and deformation of the associated infrastructure and equipment of road elements [13], [14] (Figure 5). The loosening strain may reduce soil stiffness [15] and layers made from aggregate not stabilized with hydraulic binders [8] and cause transverse cracks of pavement structure [7] (Figure 6).

Discontinuous deformation fractures the subsoil area. Their effects on the pavement and road subsoil were presented by Kotyrba and Kowalski [16], Grygierek 2018 [8], Adelsohn i in. 2020 [17], Grygierek and the others [18] - [21], Ju and Xu [22].





Figure 3. Subsidence of road profile. Source: [7]

**Figure 4.** The uplift of paving stones and deformation of the kerbline. Source: [12]



Figure 5. Deformation of energy-consuming barriers. Source: [13]



Figure 6. Transverse crack of road pavement. Source: [7]

The rules for taking into account mining impacts when designing cubature structures or assessing their technical condition are presented in the Instructions of the Building Research Institute [23], [24], [25] as well as in the monograph [1]. In the case of road structures, the situation is different. Firstly,

there are no general rules that would provide clear guidelines for the procedure for designing road pavement in mining areas, similarly to cubature buildings. Secondly, in the computer programs used for the individual design of road pavement, it is not possible to assign additional actions, including mining impact. Therefore, in order to analyze the behavior of the pavement-mining subsoil system, an advanced numerical analyze should be carried out. In this case, the subsoil thickness, the boundary conditions and the constitutive relationships of the materials of the road pavement layers and subsoil should be determined. This paper presents an attempt to select kinematic boundary conditions for the FEM model of the road pavement-mining subsoil system, analogically to the model of the buildingmining subsoil system. The paper is aimed at assessment of the influence of kinematic boundary conditions selection on the criterial values that are taken into account during the design process of road pavement using mechanistic methods. For this purpose, three cases were considered:

- (i) horizontal mining strain ( $\varepsilon_{design}$ ),

- (ii) curvature of surface ( $K_{design}$ ),

- (iii) combined impact of these actions. In these cases, each time vehicle wheel load was assumed.

Based on the analyzes, the computational horizontal strain of the mining area  $\varepsilon_{comp}$  is decisive when assessing the criterial values taken into account in the design process of road pavement structures.

## 2. Designing road pavement structures in mining areas

The guidelines used for designing road pavement structures [26] recommend that the design of structures in mining areas be applied individually. In this purpose, the numerical analyses can be used. Examples of these numerical analysis were presented by Fedorowicz and Fedorowicz [25], Grygierek et al. [8],[9],[18],[19], Kawalec et al. [21].

In the general approach, the task involving the building-subsoil system subjected to mining impact can be solved using the following methods [25]:

A – conventionally using the methods of building structure mechanics, using substitute static diagrams and parametric description of the subsoil;

B – using 2D or 3D FEM analysis and parametric description of the subsoil;

C – using FEM analysis (2D or 3D) of the building-subsoil system.

For contact-related problems concerning the pavement-subsoil system, so far two main trends used in the description of these systems have been applied, which are included in the classical approach (A):

- slab model lying on Winkler substrate [28] (Figure 7);

- model of a layered slab lying on elastic half-space [29] (Figure 8, 9).

Unfortunately, the above-mentioned models hardly simplify the currently recognised description of subsoil behaviour [30]-[35]. It should be emphasised that considerable progress has been made in the constitutive modelling of the subsoil in the last fifty years [33], [36], [37], [39], [40]. This is probably related to the needs of large construction projects, the development of implementation capabilities of large endeavours, as well as the development of engineering and computational technology related to numerical modelling of the advances structures. Moreover, the analysis of the pavement structure as isotropic elastic layers lying on the half-space does not allow additional impacts, including the mining impacts.

The modelling inconsistencies with reality relate to the adoption of linear elasticity relationships to describe the behaviour of the pavement layers and subsoil, as well as to assume of the subsoil area cooperated with the pavement structure. These issues were the subject of the authors' earlier considerations.

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Figure 7. Werstergaard's model diagram. Source: [34]



Figure 8. Double-course system diagram. Source: [34]



The paper presents numerical analyses used to solve the task involving the pavement structuremining subsoil system, assuming approach C, i.e. FEM analyses for both subsystems. Such an approach seems appropriate to solve the problems presented earlier. When considering the possibility of a reliable assessment of the road pavement structure (without mining impacts) operation in the FEM model, the strong coupling of the pavement structure's response to the behaviour of subsoil should be included [32].

#### 3. Numerical analyses

The numerical analysis of the pavement-mining subsoil system was performed using a 2D model (flat strain state). Due to symmetry, only one half of the model was analysed. The analysis was carried out using ZSoil.PC ver. 16.03. The analysed the structure of the road pavement with the layout and parameters are shown in Figure 10. A rectangular subsoil area with overall dimensions of 7.50 x 5.00 m was assumed. The dimensions of the subsoil area were assumed according to recommendations in Fedorowicz [36], Fedorowicz [37] and Kadela [32], [38]. The finite elements with dimensions in the highest-density area of 0.05 x 0.04 m (the ratio of the dimensions of the sides of finite elements in other grid subareas does not exceed 1:3) was assumed.

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Figure 10. Model of the road pavement and mining substrate system. Dimensions in meters [m]. Source: [12]

For the purpose of the analyses, the following assumptions were adopted:

- there is full interlayer bonding;

- pavement load is 100 kN per axle, which results in vehicle wheel load (50 kN) uniformly transferred to the surface of the wheel-shaped trace (contact pressure q = 850 kPa); the value of the substitute distributed load q(2D) = 180 kPa was assumed according to Kadela [32] so that the resulting values of strains in the state (2D) are sufficiently close to the values determined in the axial symmetry state for the actual load q = 850 kPa.

- a elastic model (e) was used, as accepted in mechanical analyses;

- there is no friction between the pavement structure and the surface of the subsoil;

- mining impacts were determined in the form of vertical and horizontal displacements applied to the lower edge of the subsoil area and to the outer vertical edge of the same subsoil area (Figure 11).

The combined action of the vehicle wheel load and the impact of mining-related area deformation were assumed. Wheel load was introduced in two computational steps with an increment of 0.5; after the last step, mining impacts were introduced in ten stages with an increment of 0.1. The impact of mining deformation of the area, represented by kinematic input (vertical and horizontal displacements were introduced into the model nodes respectively), was analysed for values of horizontal  $\varepsilon$  and the curvature radius R=I/K corresponding to categories of mining areas 0 to IV. Characteristic and design values of the parameters ( $\varepsilon$  and R) as well as partial values of safety factors are included in Table 1. The computational analysis was performed for three cases as follows: I – impact of vehicle wheel load and  $\varepsilon_{design}$  impact, II – impact of vehicle wheel load and  $R_{design}$  impact, III – impact of a vehicle wheel and the joint impact of  $\varepsilon_{design}$  and  $R_{design}$ .

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Figure 11. Loads and impacts adopted in the model analysis: a)  $\varepsilon_{design}$  impact; b)  $R_{design}$  impact. Source: [12].

 Table 1. Parameters of mining deformation of the area assumed in the computational analysis [25]

Category	$\varepsilon$ [mm/m]	γdesign [-]	Edesign [mm/m]	<i>R</i> [km]	γdesign [-]	R <sub>design</sub> [km]
0	0.30		0.39	40.00		23.53
Ι	1.50		1.95	20.00		11.77
II	3.00	1.3	3.90	12.00	1.7	7.06
III	6.00		7.80	6.00		3.53
IV	9.00		11.70	4.00		2.35

### 4. Results and discussions

Figures 12 to 15 presents the selected results of the numerical analyses, obtained in the last computational step. In this study the pavement deflection  $u_y$  and the following criterion values, taken into account when designing the pavement using mechanical methods were considered. The deflection  $u_y$  is determined as vertical displacement of the node in the symmetry axis of model, directly under the vehicle wheel load – Figures with index "a". While criterion values:

– horizontal strain in the bottom of asphalt courses  $\varepsilon_x$ ,

- vertical strain in the top surface of the subsoil  $\varepsilon_{\nu}$ ,

determined in the elements located on the symmetry axis (see Figures with index "b" and "c" respectively). The background is composed of the values determined without taking into account the mining impacts (continuous line in grey).

For all the analysed cases, horizontal deformation of the area  $\varepsilon_{design}$  has the decisive influence on the values of deformation  $u_y$  and the criterion values ( $\varepsilon_x$ , and  $\varepsilon_y$ ). The influence of the  $R_{design}$  radius value is negligible.

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**Figure 12.** Results of numerical analyses of the pavement-mining subsoil system for flexible pavement of type A1 (t = 1.0): a)  $u_y$ ; b)  $\varepsilon_x$ ; c)  $\varepsilon_y$  [12]



**Figure 13.** Results of numerical analyses of the pavement-mining subsoil system for flexible pavement of type A2 (t = 1.0): a)  $u_y$ ; b)  $\varepsilon_x$ ; c)  $\varepsilon_y$ 

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**Figure 14.** Results of numerical analyses of the pavement-mining subsoil for flexible pavement of type B (t = 1.0): a)  $u_y$ ; b)  $\varepsilon_x$ ; c)  $\varepsilon_y$ 



**Figure 15.** The results of numerical analyses of the pavement-mining subsoil for rigid pavement with the layers system and its parameters as in Fig. 6 (t = 1.0): a)  $u_y$ ; b)  $\sigma_x$ ; c)  $\varepsilon_y$ 

Figure 16 shows the effect of system rigidity on the obtained values. It can be observed that, along with the rigidity of the pavement structure, the criterion values taken into account in designing are lower for the same category of the mining area.



**Figure 16.** Results of numerical analyses of the pavement-mining subsoil for flexible pavement of type A1 (t = 1.0): a)  $u_y$ ; b)  $\varepsilon_x$ ; c)  $\varepsilon_y$ 

# 5. Conclusions

This paper presents an attempt to select kinematic boundary conditions for the FEM model of the pavement-mining subsoil system, referring to the method of taking into account mining impacts in the models of the building-mining subsoil system and their impact on the criterion values taken into account in designing road pavements. For this purpose, the impact of vehicle wheel loads and three cases were considered: separate impacts of horizontal mining strain ( $\varepsilon_{design}$ ) and the area curvature radius ( $R_{design}$ ) as well as the combined impact of these parameters. The following conclusions can be drawn on the basis of the results of the carried out numerical analyses:

- 1) For the road pavements structure exposed to mining impacts, the kinematic boundary conditions can be assumed; the procedure for determining displacements on the edges of the computational model of this system is the same as for the building-subsoil system [37].
- 2) When assessing the criterion values and deflections taken into account in the assessment of the technical condition and designing pavement structures (for the cases analysed in this paper), horizontal deformation of the mining area  $\varepsilon_{design}$  is decisive, whereas the impact of the values of the  $R_{design}$  radius is negligible.
- 3) There is an effect of system rigidity on the analysed values, namely with the rigidity of the pavement structure the criterion values and deflections taken into account in design engineering are lower for the same category of the mining area.

In order to full assessment regarding the operation of road surfaces in mining areas, further analyses are required; it is also necessary to use advanced constitutive models to describe the behaviour of pavement layers materials and subsoil. This will be the subject of further research.

- [1] M. Kawulok, "Mining damages in building" [Szkody górnicze w budownictwie], *Publishing House of Building Research Institute (Wydawnictwo ITB)*, Warsaw, 2015.
- [2] J. Kwiatek (editor): "Protection of construction objects in mining areas" [Ochrona objektów budowlanych na terenach górniczych], *Mining Institute Publishing (Wydawnictwo GiG)*, 2007.
- [3] J. Judycki (editor): "Catalogue of reinforcements of and repairs of flexible and semi-rigid pavements KPRNPP-2013" [Katalog wzmocnień i remontów nawierzchni podatnych i półsztywnych KPRNPP-2013], Publishing House of Gdańsk University of Technology (Wydawnictwo Politechniki Gdańskiej), Gdańsk, 2013.
- [4] S. Knothe: "Forecasting the influence of mining exploitation" [Prognozowanie wpływów eksploatacji górniczej], *Silesia Publishing House (Wydawnictwo Śląsk)*, Katowice 1984.
- [5] R. Hejmanowski, A. Kwinta: "Measurement of horizontal displacements in european coalfields". 10th FIG International Symposium on Deformation Measurements, Orange, California/USA, pp. 31-39, 2001.
- [6] R. Hejmanowski, A. A. Malinowska, A. Kwinta, P. Ulmaniec: "Modeling of land subsidence caused by salt cavern convergence applying Knothe's theory". Works of the Strata Mechanics Institute (Prace Instytutu Mechaniki Górotworu PAN), Vol. 20, No. 2, pp. 87-94, 2018.
- [7] Statutory study NZK-64/2018, rev. 2 entitled "Evaluation of behaviour of slab-type course structures in mining areas – Final report", Building Research Institute [Praca statutowa NZK-64/2018, zad. 2 pt. Ocena zachowania się płytowych konstrukcji warstwowych na terenach górniczych Sprawozdanie końcowe], Building Research Institute (Instytut Techniki Budowlanej, 2018.
- [8] M. Grygierek: "Problems of road operation in mining areas" [Problematyka funkcjonowania dróg na terenach górniczych], *Highways (Autostrady)*, No. 1-2/2018, pp. 24-29, 2018.
- [9] M. Grygierek: "Effect of mining subgrade deformation on deflections of road pavement" [Wpływ deformacji podłoża górniczego na ugięcia nawierzchni drogowej], Scientific Journals of the Silesian University of Technology. Series: Construction (Zeszyty Naukowe Politechniki Śląskiej. Seria: Budownictwo), Book 112, Vol. 1763, pp. 83-90, 2007.
- [10] M. Grygierek: "Impact of mining subgrade loosening on the bearing capacity of road pavement in field tests" [Wpływ górniczych rozluźnień podłoża na nośność nawierzchni drogowej w świetle badań terenowych], Land, air and water roads (Drogi lądowe, powietrzne, wodne), Vol. 10/2008, pp. 35-39, 2008.
- [11] J. Kawalec, M. Grygierek, E. Koda, P. Osiński: "Lessons Learned on Geosynthetics Applications in Road Structures in Silesia Mining Region in Poland", *Applied Sciences*, Vol. 9, 1122, doi: 10.3390/app9061122, 2019.
- [12] B. Parkasiewicz: "Attempt of numerical evaluation of criterion values in the road pavement and mining subgrade system model". Joint publication, edited by Krzysztof Gromysz and Karolina Knapik. Monograph [Próba numerycznej oceny wielkości kryterialnych w modelu układu nawierzchnia drogowa-podłoże górnicze. Praca zbiorowa pod redakcją Krzysztofa Gromysza i Karoliny Knapik. Monografia] Publishing House of Silesian University of Technology (Wydawnictwo Politechniki Śląskiej), pp. 237-244, 2018.
- [13] B. Parkasiewicz, "Monitoring of building elements displacements in conditions of mining exploitation conducting at the example of viaduct observation in Ruda Śląska" [Monitoring przemieszczeń elementów budowlanych w warunkach prowadzenia eksploatacji górniczej na przykładzie obserwacji wiaduktu w Rudzie Śląskiej], *Mining Review (Przegląd Górniczy)*, Vol. 8, pp. 83-91, 2012.
- [14] B. Parkasiewicz, M. Kadela, P. Bętkowski, R. Sieńko, Ł. Bednarski:"Application of Structure Monitoring Systems to the Assessment of the Behaviour of Bridges in Mining Areas". *IOP Conf. Series: Materials Science and Engineering*, Vol. 245, 032018, doi: 10.1088/1757-899X/245/3/032018, 2017.
- [15] J. Fedorowicz, L. Fedorowicz, M. Kadela: "Description Of Limit States In The Subsurface

Layer Of Loosened Subsoil In View Of Critical State Soil Mechanics", Materials, 2021 - in press

- [16] A. Kotyrba, A. Kowalski: "Linear discontinuous deformation of A4 highway within mining area 'Halemba'", Mineral Resource Management (Gospodarka Surowcami Mineralnymi), Vol. 25, z. 3, pp. 303-317, 2009.
- [17] E. Adelsohn, A. Iannacchione, R. Winn: "Investigations on longwall mining subsidence impacts on Pennsylvania highway alingnments", International Journal of Mining Science and Technology, Vol. 30, pp. 85-92, 2020.
- [18] M. Grygierek, J. Kawalec: "A4 motorway operation in the area of linear discontinuous surface deformations". Geotechnical and geophysical site characterization 5. Proceedings of the Fifth International Conference on Geotechnical and Geophysical Site Characterization (ISSMGE TC-102 - ISC'5), Vol. 2, pp. 1337-1342, 2016.
- [19] M. Grygierek, K. Sternik: "Identification of Pavement Model Parameters in the Area of Discontinous Surface Deformation Based on FWD Tests". International Journal of Civil Engineering, Vol. 8, 2020
- [20] M. Grygierek, M. Zieba: "Damage to Road Pavements in the Area of Linear Discontinuous Deformations on the Surface Caused by Deep Mining". IOP Conference Series: Earth and Environmental Science, Vol. 362, 012151, doi: 10.1088/1755-1315/362/1/012151, 2019.
- J. Kawalec, M. Grygierek, K. Chlipalski: "Communication right-of-way structures in mining [21]areas" [Komunikacyjne obiekty liniowe na terenach górniczych], Highways (Autostrady), Vol. 3/2015, pp. 35-42, 2015.
- [22] J. Ju, J. Xu: "Surface stepped subsidence related to top-coal caving longwall mining of extremely thick coal seam under shallow cover", International Journal of Sciences Rock Mechanics & Mining Sciences, vol. 78 (2015), pp. 27-35, 2015.
- [23] A. Cholewicki, M. Kawulok, Z. Lipski, J. Szulc: "Rules for determining loads and verifying limit states of buildings located in mining areas with reference to Eurocodes" [Zasady ustalania obciążeń i sprawdzania stanów granicznych budynków lokalizowanych na terenach górniczych w nawiązaniu do Eurokodów], Publishing House of Building Research Institute (Wydawnictwo ITB), Warsaw, 2012.
- [24] Instruction of the Building Research Institute No. 391/2003: Design of buildings subject to impact of mining tremors. . [Instrukcja ITB nr 391/2003: Projektowanie budynków podlegających wpływom wstrząsów górniczych], Publishing House of Building Research Institute (Wydawnictwo ITB), Warsaw, 2003.
- [25] Instruction of the Building Research Institute No. 416/2006: Designing buildings in mining areas. [Instrukcja ITB nr 416/2006. Projektowanie budynków na terenach górniczych], Publishing House of Building Research Institute (Wydawnictwo ITB), Warsaw, 2006.
- [26] J. Judycki (editor): "Catalogue of typical flexible and semi-rigid pavement structures" [Katalog typowych konstrukcji nawierzchni podatnych i półsztywnych], Publishing House of Gdańsk University of Technology (Wydawnictwo Politechniki Gdańskiej), Gdańsk, 2012.
- L. Fedorowicz, J. Fedorowicz: "Numerical modelling of in situ phenomena occurring in subgra-[27] des in mining deformation areas" [Numeryczne modelowanie zjawisk in situ występujących w podłożach gruntowych w obszarach deformacji górniczych], Land, air and water roads (Drogi lądowe, powietrzne, wodne), Vol. 11/2009, pp. 62-67, 2009.
- [28] A. Khajeansari, G.H. Baradaran, J. Yvonnet: "An explicit solution for bending of nanowires lying on Winkler-Pasternak elastic substrate medium based on the Euler-Bernoulli beam of theory", International Journal Engineering Vol. Science, 52, doi: 10.1016/j.ijengsci.2011.11.004, pp. 115-128, 2012.
- [29] O. Xu, J.A. Prozzi: "A time-domain finite element method for dynamic viscoelastic solution of layered-half-space responses under loading pulses", Computers and Structures, Vol. 160, doi: 10.1016/j.compstruc.2015.07.005, pp. 20-39, 2015.
- [30] M. Gryczmański, R. Uliniarz: "A simple critical state model with small strain nonlinearity for

IOP Conf. Series: Materials Science and Engineering 1203 (2021) 032021 doi:10.1088/1757-899X/1203/3/032021

overconsolidated soils", Foundations of Civil and Environmental Engineering, Vol. 12/2008, pp. 49-60, 2008.

- [31] W. Świdziński: "Mechanisms of compaction and liquefaction of loose soils" [Mechanizmy zagęszczenia i upłynniania gruntów sypkich], *Publishing House of Institute of Hydroengineering of the Polish Academy of Science (Wydawnictwo IBW PAN)*, Gdańsk, 2006.
- [32] M. Kadela: "Road pavement and subgrade systems in numerical models and in-situ testing" [Układy nawierzchnia drogowa – podłoże gruntowe w modelach numerycznych i badaniach in situ], *Publishing House of Silesian University of Technology (Wydawnictwo Politechniki Śląskiej*, Gliwice, 2016.
- [33] M. Kadela, L. Fedorowicz: "Model obliczeniowy układu konstrukcja warstwowa-podłoże gruntowe zgodnie z EC7", *Acta Scientiarum Polonorum*. *Architectura*, Vol. 12, pp. 17-25, 2013.
- [34] A. Szydło: "Road pavements made of cement concrete. Theory. Dimensioning. Execution" [Nawierzchnie drogowe z betonu cementowego. Teoria. Wymiarowanie. Realizacja]. *Publishing House of Polski Cement Sp. z o. o.* (Wydawnictwo Polski Cement Sp. z o. o.), Kraków, 2004.
- [35] M. Cudny, A. Truty: "Refinement of the Hardening Soil model within the small strain range". *Acta Geotechnica*, Vol. 15, doi: 10.1007/s11440-020-00945-5, pp. 2031-2051, 2020.
- [36] L. Fedorowicz: "Contact-related building and subgrade problems. Part I. Criteria for modelling and analysis of basic building and subgrade problems" [Zagadnienia kontaktowe budowla – podłoże gruntowe. Część I. Kryteria modelowania i analiz podstawowych zagadnień konstrukcja budowlana – podłoże gruntowe], Publishing House of Silesian University of Technology (Wydawnictwo Politechniki Śląskiej), Gliwice, 2006.
- [37] J. Fedorowicz: "Contact-related building and subsoil problems. Part II. Criteria for creating and evaluating computational models of the building and mining subgrade problems" [Zagadnienia kontaktowe budowla – podłoże górnicze. Część II. Kryteria tworzenia i oceny modeli obliczeniowych układów konstrukcja budowlana – podłoże górnicze], Publishing House of Silesian University of Technology (Wydawnictwo Politechniki Śląskiej), Gliwice, 2008.
- [38] M. Kadela: "Model of multiple-layer pavement structure-subsoil system", *Bulletin of the Polish Academy of Sciences*. *Technical Sciences*, Vol. 64, no. 4, doi: 10.1515/bpasts-2016-0084, pp. 751-762, 2016.
- [39] L. Fedorowicz, L. M. Kadela: "Recreation of Small Strains Phenomenon under Pavement Structure and Consequences of Failure to Address It", *IOP Conference Series: Materials Science and Engineering*, Vol. 245, no. 2, 022005, doi: 10.1088/1757-899X/245/2/022005, 2017.
- [40] M. Holičková, M. Drusa, J. Vlček: "Testing of Subsoil Support in Physical Model of Piled Embankment", *Transportation Research Procedia*, Vol. 40, doi: 10.1016/j.trpro.2019.07.100, pp. 711-717, 2019.