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# **Dynamic Analysis of Damaged Carbon Nanotubes Micro-Beam Structures**

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Abstract. In this article, the intact and damaged multi-walled carbon nanotubes (MWCNTs) models were performed in micro-beam structures to detect the damaged parts. To that end, numerical analysis using ABAQUS software was implemented to achieve free vibration including the calculating of natural frequencies and normalized mode shapes. Vibration-based damage detection techniques were proposed to assess and localize the damaged parts. Within this study, the damage was presented in terms of reducing the local stiffness at 10%, 20%, and 30% E<sub>c</sub> at each location. Then to accomplish the detection task, the irregularity of the higher derivative index was calculated. According to the computed results, the peak of the irregularity index precisely shows the effect of the defect, although it was unseen in the mode shape. Keywords. Dynamic analysis, Carbon nanotubes, Micro-beams, Damage detection.

#### **1. Introduction**

Composite materials are a highly demanded material and widely used in various mechanical equipment such as aerospace structures, wind turbine blades, and maritime carriers where superior stiffness or strength to low weight ratio, integrated with their unique tailoring merit, and good chemical resistance [1-6]. In the past few decades, there has been an enormous expansion in research efforts that considered the polymer nanocomposites owing to the development of new and multifunctional materials for potential applications [5], [7]. Particularly, since the carbon nanotubes (CNTs) has been discovered in the early1990s [7], CNTs became one of the extremely important parameters which play a promising role for material scientists and numerous engineers due to their extraordinary functional, superior mechanical properties, better structural, and wide range utilization in different fields [5], [8-11]. With their outstanding mechanical properties, CNTs have also been treated as a typical reinforcement in polymer composites [12]. The CNTs may be distributed as variant grading patterns through a particular direction to enhance the mechanical features and to reinforce the structures of the composite [5], [13], [14]. The enhancing influence level fundamentally relies on the following aspects: the alignment, the dispersion of CNTs, the interfacial adhesion, and the aspect ratio [15]. On the other hand, compared to metallic sensors, the main feature of utilizing polymeric microbeams lies in their inexpensive and easy fabrication. Moreover, due to the low Young's modulus of pure polymeric micro-beams, the pure polymeric micro-beams may discover low natural frequencies, consequently narrower sensing ranges [16]. Several researchers have reported that the mechanical characteristics of polymeric matrices may be increased drastically by adding solely a few weight percent (wt.%) [16], [17]. Furthermore, there are many papers that have established the fact that the



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scales of length present in strain gradient theory and nonlocal elasticity describes two totally different physical properties of structures and materials at the Nano-scale [7]. In the stages of design, it is crucial when using a few percentages of CNTs along with polymeric matrices to achieve secure high natural frequencies and reasonable ranges of high deflection [16]. Recently, several researchers have considered the micro-cantilever polymer beams to investigate the influence of micro-systems on enhancing the natural frequencies [18]. In spite of the fact that there are numerous investigations that have used the differential theory of nonlocal continuum of Eringen's to forecast the dispersive relationship between wave number and frequency for the propagation of the wave in CNTs, the ability of these studies to identify size-dependent stiffness is narrow [7]. Vibration has been studied by several researchers using Eringen's nonlocal model [19], [20]. For a nonlocal Timoshenko beam, the equation of motion was derived to explore the wave propagation properties in CNTs. Shell models were employed to examine in detail the effect of nonlocal on the wave properties [21]. Simulations and experimental studies demonstrated that CNTs hold outstanding characteristic over carbon fibers [3]. However, simulations have been powerful tools to estimate nanomaterials' physical properties [7], [22], [23]. In addition, simulations have grown widespread in the investigations of strengthening mechanisms in CNT-polymer due to difficulties in studying the CNT-polymer

experimentally [24]. In the structures where thin plates are used like aircraft and spacecraft, vibration problems are more significant [25]. In addition, the mechanical and electrical properties of the CNT/polymer composites are deteriorated due to the localized damage to CNTs. Therefore, many attempts have been proposed for developing processes that are convenient to apply with low cost and minimal damage to the CNT structure [8]. The main purpose of this research was to have a good understanding of the nonlinear vibration behavior of multi walled carbon Nano-tubes (MWCNTs') damaged beams. On one hand, it is therefore very substantial to have a good comprehension of how the MWCNT's weight percent and damage locations influence the mode shapes and natural frequencies. Therefore, different damages location and volume fraction of MWCNTs are discussed in this study. A detailed numerical study is conducted to examine the influences of imperfection mode, amplitude, and location on the nonlinear vibration behavior of MWCNTs beams. A generic imperfection is employed to model possible imperfection shape. ABAQUS software is used to model and analyze the undamaged/damaged MWCNTs beams. Furthermore, the third-order derivative is used to derive the governing equations which are then compared with the numerical results to obtain the linear and nonlinear frequencies of imperfect MWNTCs beams. Little has been expressed about the impact of MWCNT's weight percent and damage locations on the natural frequencies, or their impact on the mode shapes. To the best of the researchers' knowledge, nothing has been created in the modeling of damaged MWCNT's cantilever beam.

### 2. Methodology of predictive modeling for binary classification

Highlighting damaged areas in mechanical structures is vital to avoid early failure. One of the recommended approaches as an active method to detect the damaged areas is vibration-based damage detection techniques. According to the research in this area, the modification and improving of common indexes used in this field are important to satisfy the localization of damaged elements. In the current research, the calculated data mentioned in numerical analysis is used to assess the reduction in local stiffens caused by expected damage. Meanwhile, the highest derivative index represented in Eq.1 [26] was computed by applying the forward difference displacement on the normalized mode shape in each case.

$$\vartheta_{ji}^{\prime\prime\prime} = \frac{\left(\emptyset_{(j+3)i} - 3\emptyset_{(j+2)i} + 3\emptyset_{(j+1)i} - \emptyset_{(ji)}\right)}{(\Delta x^3)} \tag{1}$$

Where  $\vartheta_{ji}^{\prime\prime\prime}$  third derivative, i = mode shape number; j = node number;  $\Delta x$  = the distance between nodes, and  $\emptyset_{ji}$  is the mode shape. As reported in the literature review, the simple damage is not easy to be detected. To this aim, the Irregularity Index R<sup>2</sup>, Eq.2 has been suggested to modify the damage detection task. This idea is an extension for the work investigated by [27] where the calculation and filtration of the irregularities in the calculated dynamic response (mode shapes) are useful to highlight

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the damaged parts in composite structures. The proposed indicator in this research represents the square of the difference between the damaged and intact higher derivative indexes, where  $(\emptyset_{ji}^{\prime\prime\prime})_{H}$  is the intact index, and  $(\emptyset_{ji}^{\prime\prime\prime})_{D}$  is the damaged value.

Irregularity Index (R<sup>2</sup>) = 
$$\left(\left(\phi_{ji}^{\prime\prime\prime}\right)_{\rm D} - \left(\phi_{ji}^{\prime\prime\prime}\right)_{\rm H}\right)^2$$
 (2)

Where the main idea underlying this step is to separate the fluctuation in dynamic response due to the damage effect. Then the square vale can be used to identify the damaged parts precisely. To the best of the researchers' knowledge, this index has not been previously used to assess the damaged regions in nanotubes composite structures.

# 3. Numerical details

### 3.1. Calculating the effect of multi-walled carbon nanotubes on natural frequency

To ensure the accuracy of modelling micro-beams in ABAQUS, natural frequencies with different boundary conditions were accomplished. The mechanical properties of pure polymer micro-beams and multi-walled carbon Nano tubes /epoxy 1% wt was reported by [24] which are implemented in the current model. To this end, 3D deformable shell element was used to model the micro-beam in the current research as described in Figures 1 and 2. Table 1 presents the comparison of natural frequencies which were computed in this research with the magnitudes displayed by the exact solution [28] and numerical analysis investigated by [24]. According to Table 1, the numerical modelling of micro-beam using the shell element (current research) shows an excellent convergence with the exact solution even with calculating the higher mode under free vibration. FE validate models gave quantitatively accurate and qualitatively descriptive results when compared to the published research. Thus, the proposed numerical model is reliable to be used in modelling the damaged areas as illustrated in the following sections.



Figure 1. Schematic of pure polymer micro-beams model used in the current study.



**Figure 2.** Schematic of MWCNTs micro-beams model used in the current study, where the MWCNTs were distributed uniformly along the length of beam.

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Boundary	Mode	Natural frequencies in Hz						
conditions	no.	Exact [28]	PP <sup>a</sup> [24]	PP <sup>a</sup> [Current]	UD <sup>b</sup> of CUT[24]	UD <sup>b</sup> of CUT [Current]		
C-F	1	658.45	664.1	658.37	718.95	713.67		
	2	4126.3	4158.6	4123.8	4507.8	4470.2		
	3	6584.3	6545.9	6548.9	7095.7	7043		
	4	11554	11645	11538	12623	12507		
	5	-	12257	12115	13290	13198		
C-G	1	1047.4	1059	1047.3	1147.9	1135.2		
	2	5660.2	5725.4	5656.2	6206.1	6131.3		
	3	13977	14139	13955	15326	15127		
	4	-	24903	24806	27001	26980		
	5	25990	26304	25915	28512	28095		
C-C	1	4189.7	4254.3	4186.8	4611.4	4538.5		
	2	11549	11715	11531	12698	12499		
	3	22641	22957	22577	24884	24473		
	4	-	25207	25110	27330	27150		
	5	37426	37949	37262	41134	40392		

Table 1.	Calculating natural	frequencies	of micro-beams	s at different	boundary	conditions
					/	

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a: Pure polymer; b: Uniform distribution.

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#### 3.2. Finite element modelling

In this work, a FE model presents the MWCNT's cantilever beams. The mechanical properties and dimensions of the MWCNT's beams investigated by [29] were implemented in the current model to be used in detecting the damaged parts. ABAQUS (Version 6.14-5, Dassault Systemes Simulia Corp.) was used to create, solve and validate a 3D FE model of the MWCNT's cantilever beams to quantify the damage locations and local stiffness reduction effect. MATLAB was used to create a 3D finite element model of the MWCNTs cantilever beams, which are damaged and/or intact. Two FE models of MWCNTs cantilever beams were created in this research. One model includes modelling the damaged part in MWCNTs cantilever beam as illustrated in Figure 3. On the other hand, the intact model was presented in Figure 2. The dimensions of cantilever beam were: length ( $l = 4000 \ \mu m$ ), width ( $w = 400 \ \mu m$ ), and thickness ( $t = 40 \ \mu m$ ). Cantilever beams was created by using reduced integration Axisymmetric shell elements (S4R) (1000 elements) to avoid shear and membrane locking. The MWCNT's cantilever beam was assumed homogenous and isotropic elastic. Encastre boundary condition was applied to one end of the cantilever beams. The MWCNT's cantilever did not involve complex contact conditions and large deformations; therefore, an implicit solver was used. Composite Nano tubes material properties were held constant while damage location varied. An implicit solver was used.



Figure 3. Schematic diagram of the physical model for damaged micro-beams, d is the length of damaged part = 100  $\mu m$ .

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# 4. Results and discussion

Obviously, nondestructive techniques are among the most commonly used approaches in evaluating the integrity of structure. Thus, vibration-based damage detection technique was utilized to achieve the evaluation of defects/damage in the current study. A 3D FE model for 1.5% wt MWCNT's cantilever beam was implemented and assessing the effect of damage locations and the reduction percentage of local stiffness. Within this study the damaged locations were modelled at 1000, 2000 and 3000  $\mu m$  to the fixed end as shown in Fig. 3. Also, in all models the length of damaged part (d) is 100  $\mu$ m. In composite Nano-tubes the local stiffness of the damaged part was reduced at 10%, 20% and 30% from the original value  $(E_c)$  at each damage location. The mode shape of intact and damaged was considered, as depicted in Figures 4 and 5, respectively. In both cases, the numerical analysis was used to compute the normalized second mode shape for intact beam and damage (10% stiffness reduction) at 1000 µm to the fixed end of the beam. As depicted in these Figs., the alter in the mode shapes is not considerable. Therefore, damage localization cannot be achieved depends on the mode shape itself. To increase the ability of damage detection, the higher derivative index was calculated as shown clearly in Figure 6. Despite the ability of the higher derivative index to define the location of the damage, it still somehow fluctuated. This means that more fluctuation is expected with the noise effect accompanied with the experimental data. Subsequently, the irregularity index of higher-order derivative was used to enhance the detection of the damage locations. Figures 7-15 illustrate the relation between of the irregularity index of the higher derivative versus the beam length with stiffness reduction 10%, 20%, and 30%, at locations 1000, 2000, and 3000 µm from the fixed end, respectively. In addition, the Figures showed that the damaged locations can be specified according to the jump in the values of the irregularity index of the higher derivative which occurs exactly between the beginning and the end of internal damage. The irregularity index of the higher derivative showed that the efficiency in the identification of locating damages through the results that were more stable, as illustrated in Figs. 7-15. So, the irregularity index can be used in more investigations as an indicator to reveal the location of damages in material structures.

Generally, Figures 7-15 clarify that the peak values of the irregularity index of the higher derivative was lowest in the middle of the beam. This was due to the change of the beam at the middle from tension to compression or vice versa. Whereas, the magnitudes of the irregularity index of the higher derivative were the highest at the free end of the beams because the damage far away from the fixed ends. At the same damage location (1000  $\mu$ m), the magnitude of the irregularity index peak of the higher derivative was proportional inversely to the stiffness reduction percentage, Figures 7-9. Meanwhile, increasing the stiffness reduction percentage with the other two damage locations caused an increase in the magnitude of peak of the irregularity index of the higher derivative, Figures 10-15.



Figure 4. Normalized second mode shape for intact cantilever micro-beam.

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Figure 5. Normalized second mode shape for damaged cantilever micro-beam, damage was modeled at 100  $\mu$ m with 10% local stiffness reduction, damage part was not detected.



**Figure 6.** Higher derivative index for damaged cantilever micro-beam, damage was modeled at 100  $\mu$ m with 10% local stiffness reduction damaged part was detected with some fluctuation at damage location.



Figure 7. The irregularity index of higher order derivative, damage was modeled at 1000  $\mu$ m from the fixed end and 10% reduction in local stiffness, damage was precisely detected.

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**Figure 8.** The irregularity index of higher order derivative, damage was modeled at 1000  $\mu$ m from the fixed end and 20% reduction in local stiffness, damage was noticed.



Figure 9. The irregularity index of higher order derivative, damage was modeled at 1000  $\mu$ m from the fixed end and 30% reduction in local stiffness, damage was precisely detected.



Figure 10. The irregularity index of higher order derivative, damage was modeled at 2000  $\mu$ m from the fixed end and 10% reduction in local stiffness, index peak was correlated to the damage location.

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Figure 11. The irregularity index of higher order derivative, damage was modeled at 2000  $\mu$ m from the fixed end and 20% reduction in local stiffness, index peak was correlated to the damage location.



Figure 12. The irregularity index of higher order derivative, damage was modeled at 2000  $\mu$ m from the fixed end and 30% reduction in local stiffness, index peak was correlated to the damage location.



**Figure 13.** The irregularity index of higher order derivative, damage was modeled at 3000  $\mu$ m from the fixed end and 10% reduction in local stiffness, index peak clearly shows the damage location.

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Figure 14. The irregularity index of higher order derivative, damage was modeled at 3000  $\mu$ m from the fixed end and 20% reduction in local stiffness, index peak clearly shows the damage location.



Figure 15. The irregularity index of higher order derivative, damage was modeled at 3000  $\mu$ m from the fixed end and 30% reduction in local stiffness, index peak clearly shows the damage location.

#### 5. Conclusions

The damage detection procedures of multi-walled carbon nanotubes of microbeam structures were investigated in this research. Vibration-based damaged detection techniques were used in this analysis to calculate the mode shapes of intact and damaged beams. The higher derivative index was calculated to improve the damage assessment and overcome the expected fluctuation in dynamic responses. In addition, the irregularity of the higher derivative index was investigated. In all models, the irregularity of the higher derivative index was efficient even with a low reduction in local stiffness at different locations with respect to the fixed end. The calculated results show that the peak of the irregularity index approximate range were 7e15, 5e14, and 1.7e16 at three damage locations, respectively. The FE results showed that the peak of the irregularity index depends on the damage part location. Model and theoretical results were presented using Nanotubes composite structures. The numerical results are qualitatively correct and consistent with the exact solution.

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Symbol	Meaning	Unit
θ'''	Third dervative index	-
Ø <sub>ji</sub>	Mode shape	-
Δx	Increment in x	m
$\left( \phi_{ji}^{\prime\prime\prime} \right)_{\rm D}$	Third dervative of damaged mode shape	-
$\left( \emptyset_{ji}^{\prime\prime\prime\prime} \right)_{H}^{D}$	Third dervative of healthy mode shape	-
R	Irregularity Index	-
1	Beam length	μm
W	Beam width	μm
t	Beam thichness	μm
Ec	Young modulus of composite nanotubes	GPa
d	Damage length	μm
$l_1$	Damage location	μm

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