

PAPER • OPEN ACCESS

Modal Analysis of Fixed - Free Beam Considering Different Geometric Parameters and Materials

To cite this article: Enass H Flaieh *et al* 2021 *IOP Conf. Ser.: Mater. Sci. Eng.* **1094** 012118

View the [article online](#) for updates and enhancements.

You may also like

- [Molecular dynamics investigation of carbon nanotube resonance](#)
I-Ling Chang
- [Modeling and analysis of a coupled novel nonlinear magneto-electro-aeroelastic lumped model for a flutter based energy harvesting system](#)
Mehran Zaheri Abdehvand, Seyed Alireza Seyed Roknizadeh and Hamid Mohammad Sedighi
- [Towards the LISA backlink: experiment design for comparing optical phase reference distribution systems](#)
Katharina-Sophie Isleif, Lea Bischof, Stefan Ast et al.



ECS
The
Electrochemical
Society
Advancing solid state &
electrochemical science & technology

DISCOVER
how sustainability
intersects with
electrochemistry & solid
state science research

Modal Analysis of Fixed - Free Beam Considering Different Geometric Parameters and Materials

Enass H Flaieh, Abbas Aziz Dwech and Mohammed Refaat Mosheer

Mechanical Department, University of Technology, Baghdad, Iraq

E-mail: 20057@uotechnology.edu.iq

Abstract. The early studying of natural frequencies and associated mode shapes for different geometric parameters and different boundary conditions is considered an integral approach that has received great attention in industrial applications to prevent catastrophic failure in machines. The effect of different diameters (Solid and Hollow) on the transverse bending and torsional natural frequencies on a uniform steel beam with a circular cross-section is studied. The effect of different materials and different beam lengths on the fundamental transverse bending and torsional natural frequencies are introduced for Fixed - Free supported beam. Theoretical analysis calculation and the Finite Element Methods using ANSYS Workbench 17 software results are introduced. Theoretical and numerical methods give approximately the same results. Figures of effects of the beam length, material types, and different inner diameters on the transverse bending, and torsional natural frequencies of the uniform beam are performed.

Keywords. Modal analysis, Free beam, Geometric parameters, ANSYS Workbench 17, Finite element.

1. Introduction

Free torsional and transverse vibration analysis of beams with different geometric characteristics and different boundary conditions is important in mechanical engineering. Rotating shafts are extensively implemented for power transmission in different industries. Most machinery may encounter torsional and transverse vibrations in their rotary elements. Such vibrations could be caused by random exciting torque, disturbance of electricity, or interaction of different parts of the system like shafts and bearings. The most common type of vibration which occurs in rotary systems is torsional vibration of elements due to the resonance phenomenon. However, many studies have been introduced in this field. Alansari, L.S et al. [1] calculated the natural frequencies of a hollow sectional cantilever beam with internal steps with circular and square cross section area. The study introduced a numerical analysis using the classical Rayleigh method, modified Rayleigh method, and Finite Element Method using ANSYS-Workbench (17.2). The equivalent cross-section area of the hollow stepped beam was calculated in two methods, the classical method, and the modified method. Seven values of width for a large part were used with different length vales of small part for square and circular cross-section areas. It was concluded that when the length of small parts increases, the natural frequencies increases to reach their maximum values at a specific length then decrease.



The natural frequencies increase when the width of small parts increases for the same width of the larger part. Also, the natural frequency of the circular CSA is less than the natural frequencies for square CSA for the same dimensions. Jammal et al. [2] introduced a numerical solution for transverse vibration of the simply-supported beam with symmetric overhang. The natural frequencies of different beam lengths with constant cross-section area were calculated. An approximation solution and numerical solutions for frequency equation with small overhang were calculated, using beam flexural stiffness calculation. The free vibration of square cross-section aluminum beam was investigated by Avcar [3] analytically and numerically under four different boundary conditions: clamped-clamped (C-C), clamped-free (C-F), clamped-simply (C-S), and simply supported beam (S-S). Analytical solution was carried out using Euler-Bernoulli beam theory and Newton – Raphson method. Effects of geometric characteristics, boundary conditions on natural frequencies were obtained for the first three modes. Numerical results were obtained from the finite-element method (FEM) using ANSYS software to confirm the reality of the analytical solution. It was found that the beam with (C-C) boundary condition has the highest natural frequencies, and lowest natural frequencies under (C-F).

The natural frequencies decreased with increasing beam length. Also, the natural frequencies increased with the increasing cross-sectional area. Al-Saffar [4] studied the natural frequency of aluminum of cantilever stepped beam experimentally and theoretically by modeling the experimental data using artificial neural network (ANN) for different values of large and small diameters and for different lengths of the larger diameter step. The results showed that the natural frequencies increased with the increasing of larger diameter length of the stepped shaft. The ANN technique had a high performance to predict the experimental results. Augustyn et al. [5] investigated the existence of torsional modes in low-frequency range for a fixed-free beam with a rectangular cross-section with different ratios between width and height and different lengths. Analytical model was introduced and verified using the finite element method. For some cross-sections with an especially short length, the existence of torsional modes in the low frequency range was observed. Ece et al. [6] studied the vibration of the isotropic beam with variable cross-section. The governing equation was reduced to ordinary differential equations in special coordinates for exponentially varying cross-section area width. An analytical solution of vibration of beams was conducted with three different boundary conditions. Natural frequencies and mode shapes were determined for each set of conditions of boundaries.

The results showed that non-uniformity influenced the natural frequencies and mode shapes. The natural frequencies increased for widening beams and decreased for narrowing beams. Al-Ansari [7] also presented an investigation of the natural frequencies of the stepped beam using modified Rayleigh's method to calculate the equivalent moment of inertia of the stepped beam to verify a new method. The proposed analytical method and finite element method using Ansys software were implemented on four types of beams: circular beam, square beam, rectangular beam with stepping in width only, and rectangular beam with stepping in height only. A good agreement was found between the results of static deflection calculated using Ansys and those found by modified Rayleigh's method. It was concluded that modified Rayleigh's method can be used for calculating the static deflection and natural frequencies for a stepped beam with a number of steps larger than two and a non-prismatic beam. Da Silva [8] studied the flexible beams carrying attachment and non-classical boundary conditions in engineering structures. The analysis of vibrating beams with ends elastically restrained against rotation a translation or with ends carrying concentrated masses or rotational inertial. (unintelligible)

In this study, the calculations based on mathematical models and Finite Element Methods using ANSYS Workbench 17.2 software were presented in this paper to study the fundamental transverse and torsional natural frequencies of cantilever solid and hollow beams with the uniform circular cross-sectional area to study the effect of shaft length, different materials, and different inner diameter of the hollow beam.

2. Theoretical analysis

2.1. Transverse natural frequency

According to Euler-Bernoulli beam theory, the equation of motion can be obtained for homogeneous material properties and uniform cross -section area as follows [9]:

$$c^2 \frac{\partial^4 w}{\partial x^4}(x, t) + \frac{\partial^2 w}{\partial t^2}(x, t) = 0 \quad 0 \leq x \leq L \quad (1)$$

Where L , is the beam's length as shown in figure (1), I is the area moment of inertia, ρ mass density with uniform cross - section area, A , w is the transverse displacement, and t is the time, at distance x . The constant parameter c depends on the material properties.

$$c = \sqrt{\frac{EI}{\rho A}} \quad (2)$$

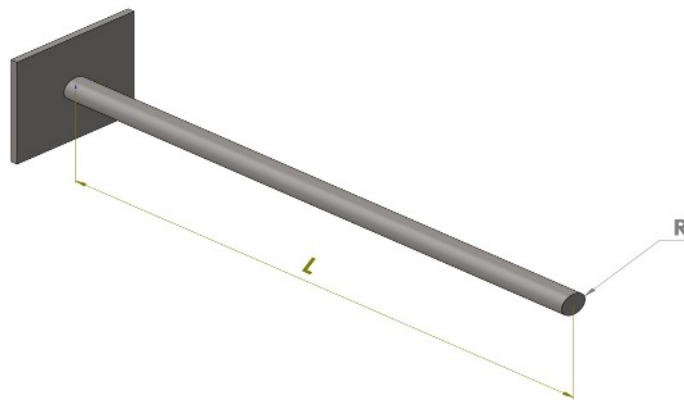


Figure 1. Geometry of the beam.

The natural frequencies can be calculated using the equation[9]:

$$\omega_n = (\beta)^2 \sqrt{\frac{EI}{\rho A}} = (\beta l)^2 \sqrt{\frac{EI}{\rho A l^4}} \quad (3)$$

And, the natural frequency f_n (Hz) of the beam is determined as follows:

$$f_n = \frac{\omega}{2\pi} \quad (4)$$

βl_n , represents the natural frequency parameter for nth mode for fixed -free boundary conditions beam for hollow circular cross-section area beam, Table 1:

$$f_n = \frac{(\beta L)^2}{2\pi \times L^2} \times \sqrt{\frac{E \times (D^4 - d^4)}{16 \times \rho \times (D^2 - d^2)}} \quad (5)$$

Where; D , and d are the outer and inner diameters of beam.

Table 1. Dimensionless mode shape parameter.

Mode number, n	Natural frequency parameter βl_n ,
1	$\beta l_1 = 1.875104$,
2	$\beta l_2 = 4.694091$
3	$\beta l_3 = 7.854757$

2.2. Calculating the fundamental torsional natural frequencies of beams have different diameters (solid and hollow beams).

The equation of motion for the torsional vibration of the beam is [9]:

$$I_0 \frac{\partial^2 \theta}{\partial t^2} = \frac{\partial}{\partial x} \left(G I_p \frac{\partial \theta}{\partial x} \right) + m_t(x, t) \quad (6)$$

Where; $I_0 = \rho I_p$ is the mass moment of inertia of the beam per unit length, for uniform cross section beam. G =shear modulus, I_p =polar moment of inertia of cross section of the beam, θ =angular displacement of the cross- section area, t = time, $m_t(x, t)$, m_t = external torque acting on the beam per unit length at distance x , along the beam axis.

Where;

$$c = \sqrt{\frac{G I_p}{I_0}} \quad (7)$$

For uniform cross section area beam

$$c = \sqrt{\frac{G}{\rho}} \quad (8)$$

The natural frequencies of vibration are given by the roots;

$$\frac{\omega l}{c} = \frac{\pi}{2} + n\pi, \quad n = 0, 1, 2, \dots \quad (9)$$

Simplifying equation (9)

$$\omega_n = \pi c \left(\frac{2n+1}{2l} \right) \quad n = 0, 1, 2, \dots \quad (10)$$

The corresponding normal mode shapes

$$\varphi_n(x) = F_n \sin \frac{\omega_n x}{c}, \quad n = 0, 1, 2, \dots \quad (11)$$

F_n , constant for n mode. To calculate the beam torsional natural frequency of fixed – free ends boundary condition the following equation can be used [9]

$$\omega_n = \left(\frac{2n+1}{2} \pi \right) \times \frac{c}{L} = \left(\frac{2n+1}{2L} \pi \right) \times \sqrt{\frac{G \times J}{I_0}} \quad (12)$$

For a uniform cross section area beam $I_0 = \rho J$ and, the natural frequency, [9]

$$f_n = \left(\frac{2n+1}{4 \times L} \right) \times \sqrt{\frac{G}{\rho}} \quad (13)$$

3. Numerical results and discussion

3.1. Fundamental transverse and torsional natural frequencies for different inner diameter hollow beams

A comparative study was performed for a Fixed -Free beam with outer diameter 40mm and length 850 mm structural steel to validate the analytical results which were compared with the FEM – based ANSYS Workbench17 results. Three transverse natural frequencies and a fundamental torsional natural frequency for fixed-free boundary conditions of different inner hollow beams. Figures (2-5) shows obviously that the transverse natural frequencies f_n (Hz) increase with increasing the inner diameter of the hollow shaft for the first three modes, where the numerical results were consistent for both methods.

Figure (6) shows the first three mode shapes for a hollow beam using Ansys Workbench (17.2) software. Finite element analysis was used to calculate the natural frequencies using (Ansys workbench 17.2)

software as shown in figure (7). Table (2) summarizes the fundamental torsional natural frequency using both methods.

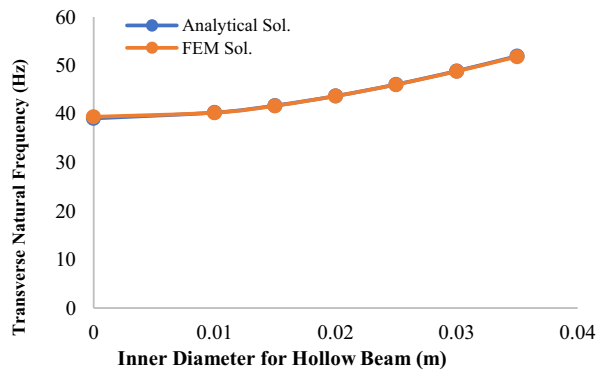


Figure 2. Comparison between the analytical and FEM results of the transverse natural frequencies ($n=1$).

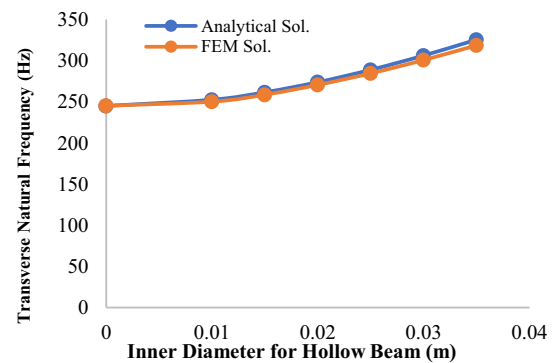


Figure 3. Comparison between the analytical and FEM results of the transverse natural frequencies ($n=2$).

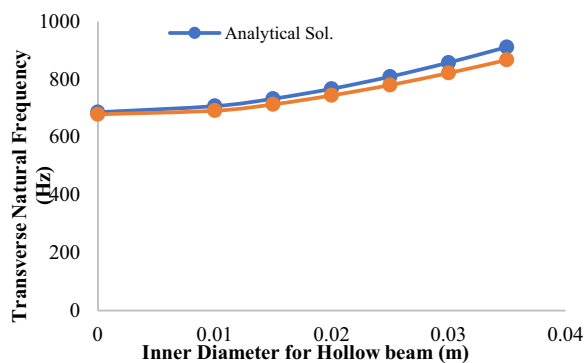


Figure 4. Comparison between the analytical and FEM results of the transverse natural frequencies ($n=3$).

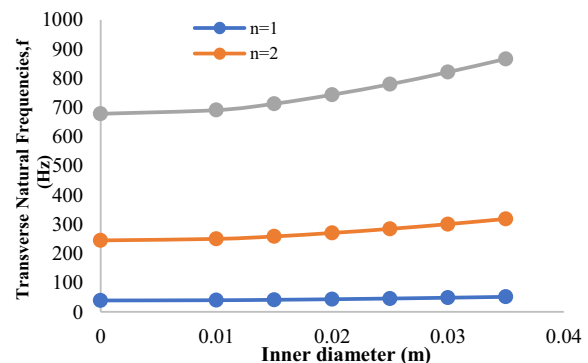


Figure 5. Change of transvers frequencies due to the change of inner diameter.

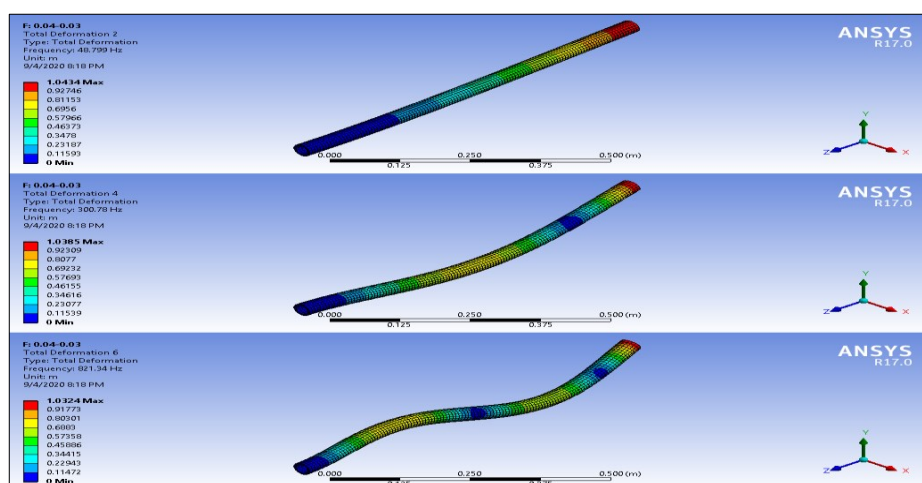


Figure 6. the first modes build using ANSYS Workbench (17.2) for hollow beam, di=0.03m, steel).

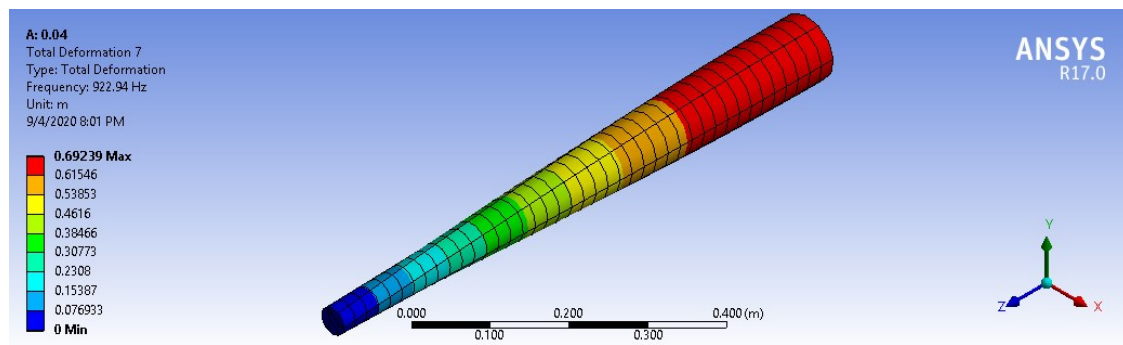


Figure 7. Solid beam torsional mode shape.

Table 2. Theoretically obtained torsional natural frequencies (solid beam).

Mode	Anslys (Hz)	Theoretical (Hz)
1	922.94	920.691

As shown in figure (7), for the first torsional mode shape of beam, the angle of twisting increases along the beam length away from the fixed end, and it increases with increasing beam length towards the free end.

3.2. Calculating the first three bending natural frequencies and fundamental torsional natural frequencies for the beam have different lengths

The first tree bending natural frequencies and the first torsional frequency of fixed-free boundary condition beams have been introduced for five different lengths for structural steel.

As shown in figures (8-13), the transverse natural frequencies and the fundamental torsional natural frequency for the first three modes decrease with increasing beam lengths.

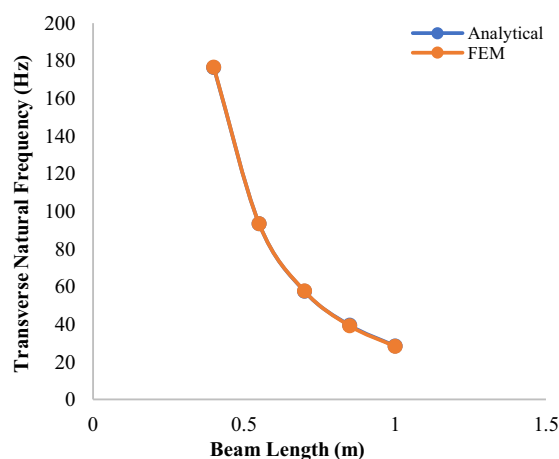


Figure 8. Comparison between the analytical and FEM results of the transverse natural frequencies (n=1)

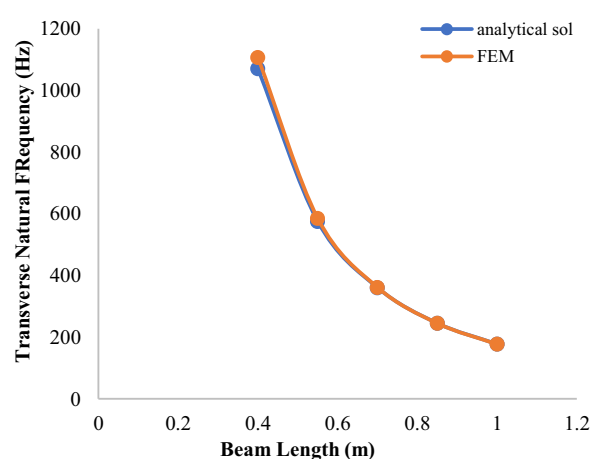


Figure 9. Comparison between the analytical and FEM results of the transverse natural frequencies (n=2)

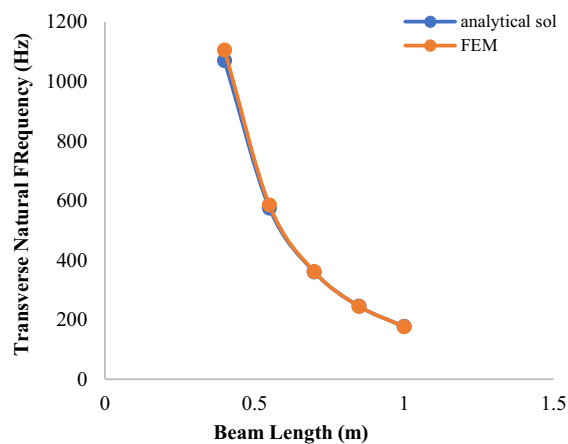


Figure 10. Comparison between the analytical and FEM results of the transverse natural frequencies ($n=3$)

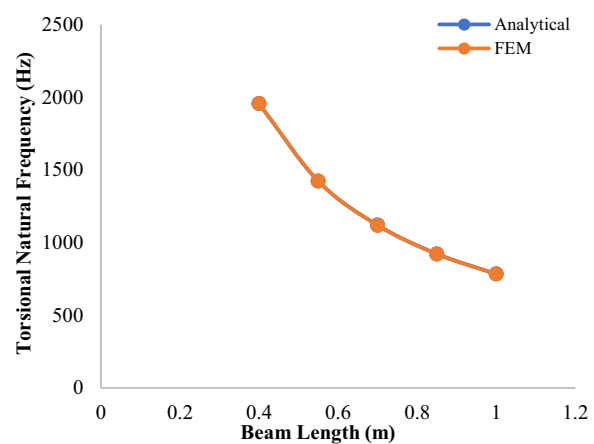


Figure 11. Comparison between the analytical and FEM results of the torsional natural frequencies ($n=1$)

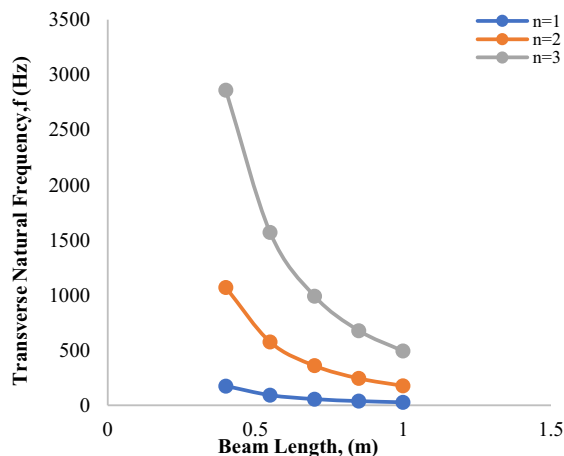


Figure 12. Effect of changing beam lengths on transverse natural frequencies.

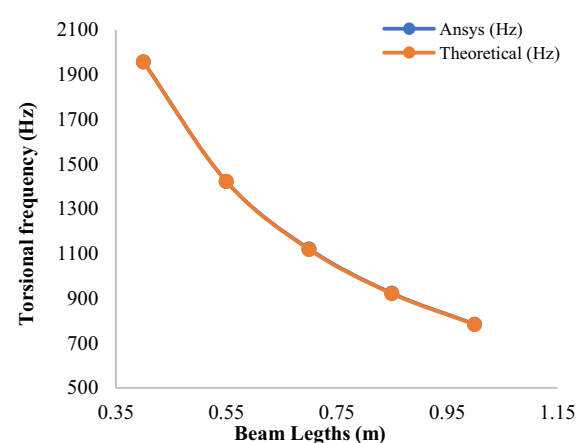


Figure 13. Effect of changing beam lengths on torsional natural frequencies.

3.3. Calculating the fundamental torsional natural frequencies for different materials

In this section, the first three bending and first torsional natural frequencies of three different materials (copper, aluminum, and steel) were calculated for the fixed-free boundary condition for a solid beam of the length of 850mm, and beam diameter of 40mm. The material properties used for theoretical analysis are shown in table (3).

Figures (14-17) illustrate the beam transverse natural frequencies for the first three modes for steel, copper, and aluminum materials. Transverse natural frequencies for steel and aluminum materials were higher than these of copper material for the studied beam.

Table 3. The beam material properties.

Material	Density, ρ (Kg/m^3)	Young's modulus, E (GPa)	Shear's modulus, G (GPa)
Structural steel	7850	200	76.923
Aluminum	2770	71	26.69
Copper	8300	110	41.04

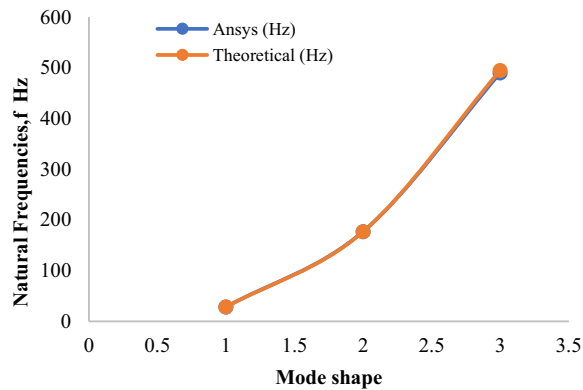


Figure 14. Copper beam natural frequency.

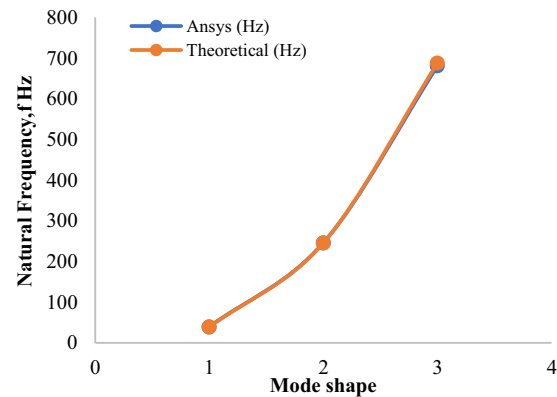


Figure 15. Aluminum beam natural frequency.

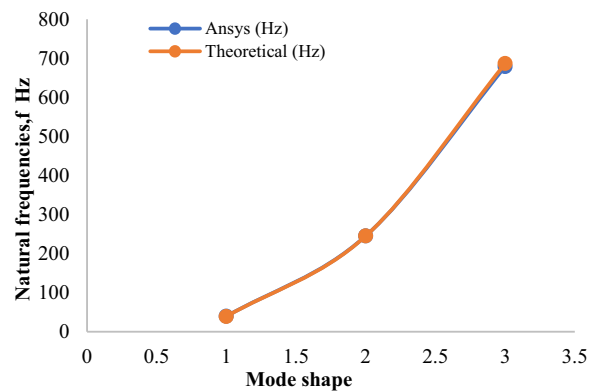


Figure 16. Steel beam natural frequency.

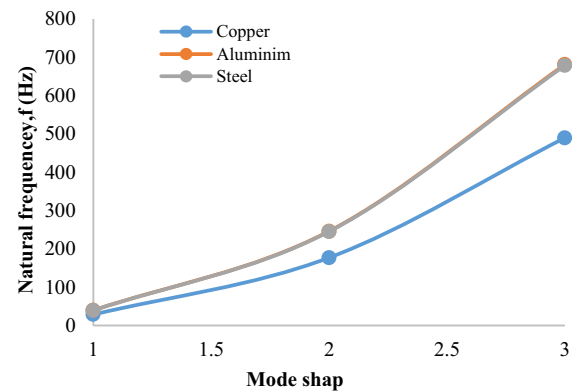


Figure 17. Relation between frequency and material for three mode shapes.

4. Conclusions

The transverse natural frequencies are increased when decreasing the inner circular beam diameter. A change of the inner diameter of the hollow circular beam does not affect the torsional natural frequency of the cross-section symmetric area even when it is hollow. The transverse natural frequencies of the copper beam are less than the frequencies of aluminum and steel for the same mode shape. For the circular cross-section area, the transverse natural frequencies are identical in the perpendicular y-z plane and x-z plane. This finding is the same for all uniform cross-sectional area where the sectional areas have the same area moment of inertia in perpendicular planes to the beam axis. The torsional natural frequencies are directly proportional to the modulus of rigidity and inversely proportional with the density of the beam material. The transverse and torsional natural frequencies are inversely proportional to the beam length. In conclusion, the bending natural frequency is directly proportional to the ratio of Young's modulus to the density of the material, and the torsional natural frequency is directly proportional to the ratio of modulus of rigidity to the density of the material.

5. References

- [1] L S Al-Ansari 2013 *Calculating Static Deflection and Natural Frequency of Stepped Cantilever Beam Using Modified Rayleigh Method* (Int. J. Mech. Prod. Eng. Res. Dev.)

- vol 3 no 4 pp 107–118
- [2] Z Shakeeb Al-Sarraf and S Mohammed Jamel Ali 2009 *Study the Transverse Vibration of a Beam With Different Length* (AL-Rafdain Eng. J.) vol 17 no 1 pp 83–91
 - [3] M Avcar 2014 *Free Vibration Analysis of Beams Considering Different Geometric Characteristics and Boundary Conditions* (Int. J. Mech. Appl.) vol 4 no 3 pp 94–100
 - [4] A A Al-Saffar, A A Diwan, L S Al-Ansari and A Alkhatat 2020 *Experimental and Artificial Neural Network Modeling of Natural Frequency of Stepped Cantilever Shaft* (J. Mech. Eng. Res. Dev.) vol 43 no 4 pp 299–309
 - [5] E Augustyn and M S Kozień 2015 *Possibility of Existence of Torsional Vibrations of Beams in Low Frequency Range* (Czas. Tech., vol 2015, no Mechanika Zeszyt) vol 3-M no 21 pp 3–11
 - [6] M C Ece, M Aydogdu and V Taskin 2007 *Vibration of a Variable Cross-Section Beam* (Mech. Res. Commun.) vol 34 no 1 pp 78–84
 - [7] S Indexed, L S Alansari, H Z Zainy, A A Yaseen and M Aljanabi 2019 *Calculating the Natural Frequency* vol 10 no 01 pp 898–914
 - [8] João Fernandes da Silva, Lucas Allende Dias do Nascimento and Simone Dos Santos Hoefel 2016 *Free Vibration Analysis of Euler-Bernoulli Beams Under Non-Classical Boundary Conditions* (An. do IX Congr. Nac. Eng. Mecânica) no 1990
 - [9] S S Rao 2007 *Vibration of Continuous Systems*