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Probabilistic Load Flow Analysis for Power System Containing Wind Farms Based on Cumulant Method

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Abstract. In this paper, a probabilistic load flow computation method based on the cumulants and Edgeworth expansion theory is put forward, and is used in the analysis of system containing wind farms composed of doubly-fed induction generator. Firstly a 2-parameter Weibull function is adopted to fit the random distribution of wind speed, and then the probabilistic model for wind power generator can be established based on active power output characteristic of wind power generator. Based on the linearized AC load flow model, analysis for IEEE 30-node system with wind farm is carried out. Finally the method is compared with Monte-Carlo simulation method, and the effects of wind farms integrated to the operation of power system is analysed in detail.

1. Introduction

Wind energy is clean and renewable. And the rational development of wind energy resources plays a very important role in reducing exhaustible resource consumption and protecting the environment. Wind power, one of the most mature renewable technologies, has developed rapidly in recent years. The output of the wind turbine generator set is also random, uncertain and uncontrollable as the wind energy is naturally volatile, intermittent and uncontrollable. For the power grid, grid-connected wind turbine generator sets are equivalent to an unstable disturbance source. And the grid connection will not only cause the problems of voltage instability and power quality, but also change the load flow distribution of the entire system, therefore posing a great challenge to the system operation and control. Accordingly, the impact of wind power integration on the system load flow and voltage is in urgent need of research, and an effective tool to quantify the impact is the load flow calculation of the power system including wind farms.

The load flow calculation of the power system in the matter of wind farms is now mostly based on deterministic methods [1-2], regardless of the random changes in the wind farm output. But the probabilistic load flow may involve various random factors during the system operation, like the impact of the load fluctuation and random failure shutdown of generation & transmission components on the steady-state operation of the system. Essentially, the probabilistic load flow aims at obtaining the stochastic distribution of state variables under the action of stochastic variables, so it is feasible to apply it to study the impact of stochastic volatility of the wind power on the system operation. For the probabilistic load flow, a great number of studies have been done at home and abroad [3-5], but there are few studies in which the probabilistic load flow is applied to analyse the power system with wind



farms. Literature [6] considers the impact of the wind farm terrain and wake effects on the wind speed. The Monte Carlo Simulation Method is applied to analyse the probabilistic load flow of the power system in the wind farm, and the results are accurate as calculation is simple, but more calculations are needed as multiple stochastic simulations are required. In Literature [7], the method of probabilistic load flow is applied to study the impact of the stochastic volatility of the wind power and solar power on the operation of the distribution network, which more comprehensively describes the system operation, but the accuracy is not ideal as the model used is too simple. In Literature [8], the Cumulant Method and the Gram-Charlier Series Method are applied to analyse the probabilistic load flow of the power system containing wind farms. If the ordinary asynchronous wind turbine generator set is directly deemed as a PQ node in the calculation, the calculation results are not accurate. Literature [9] involves the effects of correlation between the reactive power and active power of the ordinary asynchronous wind turbine generator set and the terminal voltage, but this method is not applicable to the analysis of the power system in the wind farm containing mainstream wind turbine generator sets - doubly-fed wind turbine generator sets.

In this paper, the method combining the semi-invariants method and the Edgeworth series method is used to analyse the probabilistic load flow to comprehensively consider uncertainty factors like the random failure shutdown of conventional generator sets, the random changes in load, and the stochastic volatility of the wind farm output and also to calculate the probability distribution of the node voltage and branch load flow of the power system in the wind farm, which provides a reference for analysing the impact of the wind power integration on the system.

2. Stochastic system model

2.1. Stochastic model of wind farm output

2.1.1. Stochastic model of wind speed

For the distribution of the wind speed, many studies have been done at home and abroad, in which many models have been proposed to simulate the probability distribution of the wind speed, like Weibull distribution, Rayleigh distribution and lognormal distribution, etc. One of them, the Weibull distribution is known as the most applicable probability distribution which describes the wind speed statistics. The two-parameter Weibull function is adopted in this paper to do the approximate fitting of the probability distribution of the wind speed. And the probability density function is:

$$f(v) = \frac{k}{\lambda} \left(\frac{v}{\lambda}\right)^{k-1} \exp\left[-\left(\frac{v}{\lambda}\right)^k\right] \quad (1)$$

where v indicates the wind speed, k indicates the shape parameter of the Weibull distribution, and λ indicates the scale parameter of the Weibull distribution. The parameters may be approximately calculated from the average wind velocity \bar{v} and standard deviation σ based on the local meteorological data statistics of the wind farm:

$$k = \left(\frac{\sigma}{\bar{v}}\right)^{-1.086} \quad (2)$$

$$\lambda = \frac{\bar{v}}{\Gamma(1+1/\alpha)} \quad (3)$$

where $\Gamma(\cdot)$ is a gamma function.

2.1.2. Probabilistic model of output of doubly-fed wind turbine generator set

There are various forms of wind turbine, and these wind turbines may be divided into horizontal-axis wind turbines and vertical-axis wind turbines, according to the position of the wind wheel rotation axis in space. The mainstream wind turbine generator set - doubly-fed wind turbine generator set now uses horizontal-axis wind turbines, and the relationship between the active power output and the wind speed basically conforms to the typical conversion curve of fan power shown in Figure 1. Wherein, P_r indicates the rated active output of the wind turbine generator set, v_i indicates the cut-in wind speed, v_r indicates the rated wind speed, and v_o indicates the cut-out wind speed.

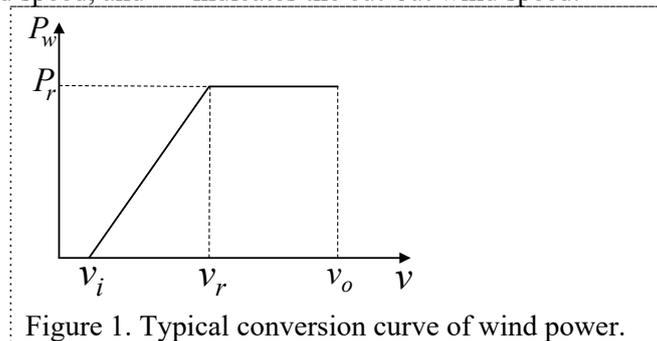


Figure 1. Typical conversion curve of wind power.

As seen from Figure 1, the expression of the active output P_w of the wind farm is:

$$P_w = \begin{cases} 0 & v \leq v_i, v \geq v_o \\ k_1 v + k_2 & v_i < v \leq v_r \\ P_r & v_r < v \leq v_o \end{cases} \quad (4)$$

wherein: $k_1 = P_r / (v_r - v_i)$, $k_2 = -P_r \cdot v_i / (v_r - v_i)$.

The stochastic distribution of active outputs of the wind turbine generator set may be determined by combining only the wind speed distribution and the output characteristics of the wind turbine generator set. The probability distribution of the active power output of the wind turbine generator set is discontinuous at the zero output and the rated output point in the wind farm. And as shown in Figure 4, the probability distribution of the output in the wind farm is discretized to simply calculation and improve calculation efficiency. The probability of the zero output of the wind farm is:

$$\begin{aligned} P(0) &= \int_0^{v_i} f(v) dv + \int_{v_o}^{+\infty} f(v) dv \\ &= 1 - \exp\left[-\left(\frac{v_i}{\lambda}\right)^k\right] + \exp\left[-\left(\frac{v_o}{\lambda}\right)^k\right] \end{aligned} \quad (5)$$

The probability of the rated output of wind farm is:

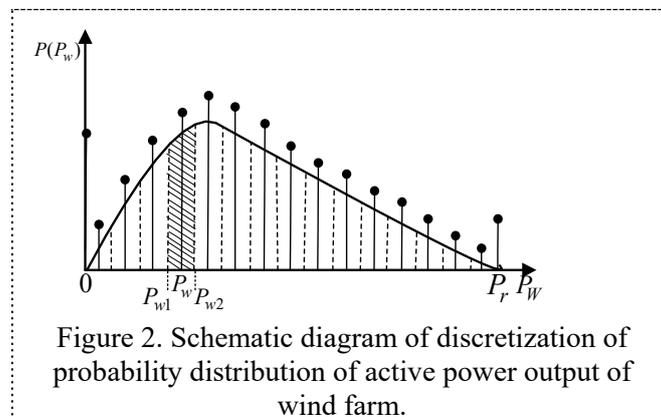
$$\begin{aligned} P(P_r) &= \int_{v_i}^{v_o} f(v) dv \\ &= \exp\left[-\left(\frac{v_i}{\lambda}\right)^k\right] - \exp\left[-\left(\frac{v_o}{\lambda}\right)^k\right] \end{aligned} \quad (6)$$

The continuous probability distribution is satisfied when the active power output of the wind farm is within the range of $(0, P_r)$:

$$\begin{aligned}
 P(p_w) &= P(P_w < p_w) \\
 &= \int_0^{v_i} f(v) dv + \int_{v_i}^{(p_w - k_2)/k_1} f(v) dv
 \end{aligned} \tag{7}$$

Discretization:

$$\begin{aligned}
 P(P_w) &= P(P_{w1} < P_w < P_{w2}) \\
 &= F(P_{w2}) - F(P_{w1}) \\
 &= \exp\left[-\left(\frac{P_{w1} - k_2}{\lambda \cdot k_1}\right)^k\right] - \exp\left[-\left(\frac{P_{w2} - k_2}{\lambda \cdot k_1}\right)^k\right]
 \end{aligned} \tag{8}$$



The doubly-fed wind turbine generator set adopts the doubly-fed induction generator, its reactive power consumption may be compensated by the controller, and the power factor can be controlled to a constant value. As a result, there is:

$$Q_w = P_w \cdot \tan \varphi \tag{9}$$

where $\tan \varphi$ indicates the power factor of the wind turbine generator set. Hence, the active power absorbed by the wind turbine generator set is a linear function of its active output, and the output of the entire wind farm only corresponds to one random variable, namely the active output of the wind farm.

2.1.3. Stochastic model of load

The grid load is also uncertain, like random changes and prediction errors, etc. The normal distribution is generally used to represent the random uncertainty of the load. Suppose that the active power and the reactive power of the load are independent random variables, and the probability density functions are respectively:

$$f(P_l) = \frac{1}{\sqrt{2\pi}\sigma_{P_l}} \cdot \exp\left[-\frac{(P_l - \bar{P}_l)^2}{2\sigma_{P_l}^2}\right] \tag{10}$$

$$f(Q_l) = \frac{1}{\sqrt{2\pi}\sigma_{Q_l}} \cdot \exp\left[-\frac{(Q_l - \bar{Q}_l)^2}{2\sigma_{Q_l}^2}\right] \tag{11}$$

where \bar{P}_l , $\sigma_{P_l}^2$, \bar{Q}_l and $\sigma_{Q_l}^2$ represent the expected value and the variance of the active power, and the expected value and the variance of the reactive power respectively.

2.1.4. Stochastic model of the conventional generator set

The conventional generator set generally has two states: normal operation and failure shutdown, so binomial distribution is often used to describe the randomness of the generator. Stochastic model of the generator set is:

$$f(X=x) = \begin{cases} p & (x = S_N) \\ 1-p & (x = 0) \end{cases} \quad (12)$$

where p represents the availability probability of the generator, and S_N represents the rated capacity of the generator.

3. Calculation method of the probabilistic load flow of the power system in the wind farm

3.1. Calculation model of probabilistic load flow

The calculation model of the load flow of the power system in the wind farm, as described in this paper, is based on a linear equation of the load flow. The random factors considered here include the uncertainties of the output of the wind turbine generator set, the output of the conventional generator set and the load power, and the powers injected on all nodes is regarded as independent random variables. The conventional equation of the power system is summarized as:

$$\begin{cases} W = f(X) \\ Z = g(X) \end{cases} \quad (13)$$

where W represents the injected power on each node, X represents the node voltage, and Z represents the load flow of each branch. Given that the randomness of the injected power on each mode, the Formula (13) is linearized at the reference operating point is:

$$\begin{cases} \Delta X = J_0^{-1} \Delta W \\ \Delta Z = G_0 \Delta X = G_0 J_0^{-1} \Delta W \end{cases} \quad (14)$$

where J_0 represents the Jacobian matrix used in the last iteration of the load flow solution; $G_0 = \frac{\partial g(X)}{\partial X} \Big|_{X=X_0}$, X_0 represents the expected value of the state variable at the reference operating point.

The Formula (14) is essentially the linear conversion state variable of each random variable and can also be solved analytically through convolution operation. This paper combines the semi-invariants method and the Edgeworth Series Method to replace the convolution calculation to reduce calculation.

3.2. Semi-invariant [10] and Edgeworth Series

The semi-invariants are additive, that is, each order of semi-invariant of the convolution of the independent random variables is equal to the sum of each order of semi-invariant of each random variable. Each order of semi-invariant and its origin moment of the random variable are correlated as follows:

$$\begin{cases} K_1 = M_1 \\ K_{r+1} = M_{r+1} - \sum_{i=1}^r C_r^i M_i K_{r-i+1} \quad r = 1, 2, 3, \dots \end{cases} \quad (15)$$

where K_r and M_r represent the semi-invariant and its origin moment of the random variable respectively, and r indicates the order. Hence, each order of semi-invariant can be calculated by knowing only the origin moment.

According to the Edgeworth Series Method, the distribution function of the random variable is expressed as a series composed of each order of the derivative of a normal random variable. The expression is:

$$f(x) = \sum_{k=0}^{\infty} A_k H_k(x) \phi(x) \quad (16)$$

wherein,

$$A_k = \frac{1}{k!} \int_{-\infty}^{\infty} H_k(x) f(x) dx \quad (17)$$

where $f(x)$ indicates the probability distribution function of the random variable, $H_k(x)$ indicates the k -order polynomial of Hermite, $\phi(x)$ indicates the probability density function of the standard normal distribution, and A_k indicates the linear combination of each order of semi-invariant.

3.3. Calculation steps

The calculation steps of the probabilistic load flow of power system in the wind farm are shown as follows:

- a) Input the data required for conventional load flow calculation, calculate the wind speed distribution parameters, and give the distribution parameters of known random variables;
- b) Adopt the conventional Newton-Raphson method to solve the reference operating state, and calculate J_0 and G_0 ;
- c) Obtain each step interval of each given random variable, and obtain the semi-invariant of each given random variable according to the relationship between the moment and the semi-invariant, and then get the semi-invariant of the power injected on each node;
- d) According to the linear conversion as shown in Formula (13), obtain the semi-invariants of node voltages ΔX and branch load flows ΔZ (the accuracy can be satisfied by taking only the first 11 orders);
- e) According to the semi-invariants of node voltages ΔX and branch load flows ΔZ , determine the probability distribution of each variable to be solved by the Edgeworth Series expansion.

4. Case study

A wind farm is added in the IEEE 30-node system as an example. There are 10 sets of 1.5MW doubly-fed wind turbine generator set in the wind farm. And the output voltage of the wind turbine generator set is 690V and is boosted to 10kV bus through the box-type transformer of the wind turbine. After that, the voltage is boosted to 110kV through the boosting transformer in the wind farm, and then the 110kV transmission line is connected to the Node 26 of the IEEE 30-node system. The impedance parameter of the transmission line is $(12.83+25.63j) \Omega$. For the wind turbine generator set, the cut-in wind speed is 3m/s, the rated wind speed is 13m/s, and the cut-out wind speed is 25m/s. In this paper, the power factor of the wind turbine generator set is -0.98 in the calculation, and the minus sign indicates the reactive power absorbed by the wind turbine generator set. The wind speed data are taken from the annual wind speed measurement data of one wind farm in Shandong, whereby the Weibull parameters can be determined: $k=2.3$, $\lambda=7.65$. The load of each node of the system conforms to the normal distribution, the expected value is the given value of the load, and the root mean square value takes 5% of the expected value. The failure ratio of the conventional generator set is 0.02.

4.1. Comparison between the Semi-invariants Method and the Monte Carlo Simulation Method

The Monte Carlo Simulation Method is used as a standard in this paper to verify the accuracy of the algorithm, and the calculation results from the method described in this paper are compared with the calculation results of the Monte Carlo Simulation Method. The number of Monte Carlo simulations is

3000 and is programmed based on Visual C++6.0. The corresponding expected values and variances of the node voltages are shown in Table 1.

Table 1. The expected value and variance of the node voltage.

Node	Monte Carlo Simulation Method		Semi-invariant Method	
	Voltage expectations	Mean square error	Voltage expectations	Mean square error
1	1.06	0	1.06	0
2	1.045	0	1.045	0
3	1.0238	0.00031	1.0239	0.00032
4	1.0152	0.00039	1.0153	0.00037
5	1.01	0	1.01	0
6	1.0115	0.00032	1.0116	0.00035
7	1.0031	0.00043	1.0031	0.00044
8	1.01	0	1.01	0
9	1.0406	0.00066	1.0408	0.00058
10	1.0229	0.00167	1.0234	0.00161
11	1.082	0	1.082	0
12	1.0513	0.00088	1.0515	0.0009
13	1.071	0	1.071	0
14	1.0339	0.00119	1.0343	0.00131
15	1.0277	0.00114	1.0281	0.00129
16	1.0319	0.00118	1.0322	0.00123
17	1.0205	0.00159	1.0209	0.00167
18	1.0137	0.00140	1.0141	0.00152
19	1.0086	0.00156	1.009	0.00149
20	1.0114	0.00158	1.0118	0.0132
21	1.0106	0.00198	1.0112	0.00213
22	1.0113	0.00199	1.0119	0.00185
23	1.0129	0.00154	1.0133	0.00171
24	1.0016	0.00208	1.0018	0.00192
25	1.0094	0.00269	1.0101	0.00265
26	1.0079	0.00414	1.0093	0.00441
27	1.0180	0.00397	1.0187	0.00373
28	1.0067	0.00058	1.0062	0.00059
29	0.9989	0.00419	0.9998	0.00397
30	0.9865	0.00423	0.9873	0.00395
31	1.0143	0.01603	1.0151	0.01537

According to the data in Table I, the maximum difference between the expected values of the voltage amplitudes on each node is calculated to be 0.0014 and also exists on the access point in the wind farm - Node 26; The maximum difference of the standard deviation of the peak voltage of each node is 0.00066 and also exists in the bus on the high-voltage side of the boosting transformer in the wind farm - Node 31. Thus, compared with the Monte Carlo simulation results, the calculation results from the method in this paper also have higher accuracy.

4.2. Analysis of the probability distribution of the system node voltage and the branch load flow

the Node 26 and the Branch 25-26 that are closer to the wind farm and the Node 6 and the Branch 7-6 that are further away from the wind farm are taken to compare and analyze the probability distribution, and the analysis results are shown in Figure.3- Figure.6.

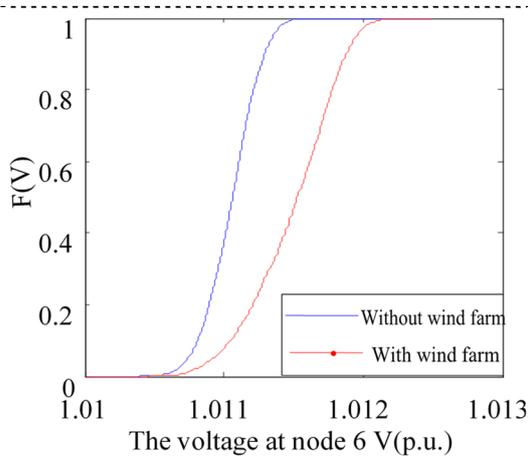


Figure 3. The cumulative distribution curve of the voltage at node 6.

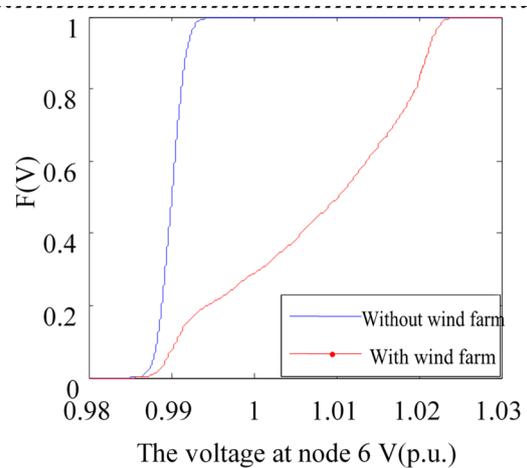


Figure 4. The cumulative distribution curve of the voltage at node 26.

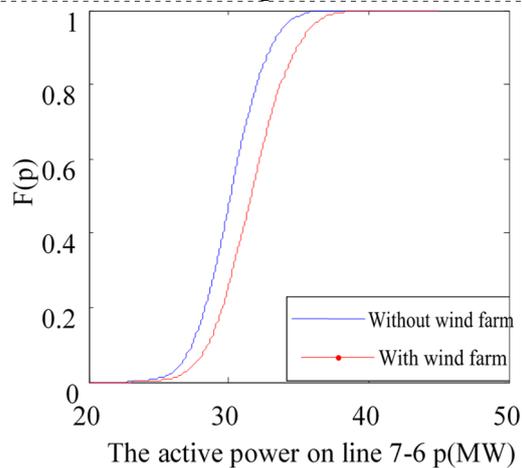


Figure 5. The cumulative distribution curve of the active power transmitted by the transmission line 7-6.

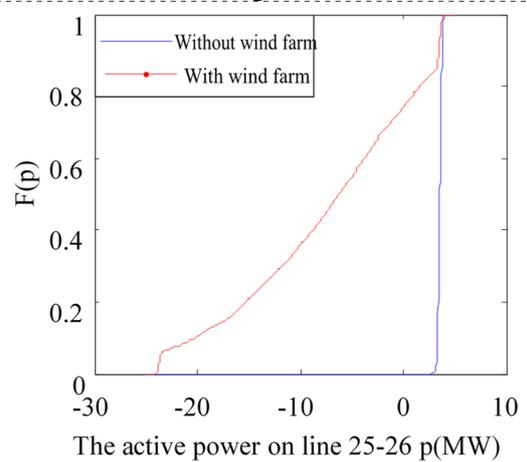


Figure 6. The cumulative distribution curve of the active power transmitted by the transmission line 25-26.

According to the above compensation, the node voltage and the branch load flow of the system are impacted to various extent after the transmission line is connected to wind farm, and the closer to the access point of the wind farm is, the more severe the impact is.

5. Conclusion

In this paper, the calculation method gives an overall consideration of uncertainty factors like the random failure shutdown of conventional generator sets, the random changes in load, and the stochastic volatility of the wind farm output to effectively evaluate the load flow state of the power system in the wind farm. This paper combines the semi-invariants and the Edgeworth series to reduce the calculation of the probabilistic load flow convolution. The example analysis shows that the calculation accuracy is very impressive.

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