PAPER • OPEN ACCESS

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To cite this article: Guanghui Sun *et al* 2021 *IOP Conf. Ser.: Earth Environ. Sci.* **632** 042074

View the article online for updates and enhancements.

A Mixed-Integer Linear Programming for Nodal Clearing Price in Day Ahead Market Considering Security Constraints

Guanghui Sun[*], Xiaowei Wang, Libo Yang, Peng Lu, Bin Ma, Hao Lv, Long Yuan

State Grid Hebei Electric Power CO., LTD, Shijiazhuang, 050021, Hebei, China

*Corresponding author email: guanghuisun@sgcc.com.cn

Abstract. A nodal clearing price model in day ahead market considering security constraints is established. First, the intra-region and inter-region transaction modes are explained, and the order types are shown. The clearing model takes the total social welfare as the goal, considers the security constraints of the network, and specifies the market clearing price expressed by the Lagrangian multiplier. Then the mixed linear programming algorithm is used to solve the model in this paper, and a verification method based on the N-1 criterion is performed. Finally, a numerical case is performed to calculate the clearing price of the network, which illustrates the effectiveness of the proposed method and model.

Keywords: Clearing price; electricity transaction order; mixed-integer linear programming; security constraints.

1. Introduction

With the advancement of electricity marketization, the types of electricity transactions in China are gradually enriched. In the process of users participating in electricity market, demand response project is a common way. For price-based demand response, users can adjust their electricity consumption behavior according to price policy; On the other hand, the power generation side needs to adjust its quotation according to the energy consumption of consumers, and finally get the market clearing price. At present, the research on the mechanism of market clearing price is not deep enough.

In view of the existing research, literature [1] gives an idea on the construction of power market in China Southern Power Grid. Literature [2] puts forward a power market equilibrium model considering demand response. Literature [3] designed the competition mechanism of electricity market based on fair distribution of value. None of the above documents have analyzed the relationship between the power transaction order type and the market clearing price.

In this paper, referring to the EU power trading block order mode, firstly, the intra-regional and inter-regional trading modes are analyzed, and the order types are explained. Then, a clearing model of node electricity price in the prior market is established, aiming at maximizing the total social welfare and considering the network security constraints, and the market clearing electricity price expressed by Lagrange multiplier is given. Then, the mixed integer linear programming algorithm is used to solve the model in this paper, and the verification method according to N-1 criterion is given. Finally, an

example is analyzed, and the clearing price of the network is calculated, which shows the effectiveness of the proposed method and model.

2. European Electricity Trading and Electricity Market

2.1. European power trading orders
This paper mainly considers the order types of European power transactions [4].

Order type 1: the demand is quoted for supply step by step in a single time period order, and the demand is bid. A single price is controlled for each submitted electricity quantity.

Order Type 2: Regular block orders Regular block orders include a fixed price ceiling and a fixed transaction volume. Regular orders are mainly for the time range set by users or the time range set by exchanges. This kind of block order is an order that is completely closed or cancelled immediately. Its acceptance and rejection depend on the average market clearing price.

Order type 3: Overview block orders have been widely used by EU power exchanges at present. The main feature of regular order composition is that the transaction volume changes with the uploading time of block orders. That is to say, the delivery date of the order is discontinuous. In addition, the minimum acceptance ratio of profile orders can generally be less than 1.

2.2. Overview of European electricity market
Electricity market is also a kind of market form. According to the different degree of market competition, there are mainly perfect competition market, monopoly competition market, oligopoly market and complete monopoly market. See table 1 below for the characteristics of each market model.

Tab.1 Classification of electricity markets

Market type	Number of vendors	Degree of product difference	Degree of price control
perfect competition	A lot of	Completely undifferentiated	No
Monopoly competition	A lot of	Have difference	There are some
Oligopoly	Several	Not necessarily	Quite a degree
complete monopoly	One	Only	To a great extent

Microeconomic theory points out that a perfectly competitive market can be regarded as a model to minimize costs or maximize profits. Therefore, for a perfectly competitive electricity market, it is the best form to describe it with an optimization model. For complete monopoly, it can be simulated as the profit maximization problem of monopolist. In this model, electricity price comes from demand function. In imperfect competition, that is, monopoly competition, it is necessary to solve the profit maximization problem of each participant at the same time. In addition, oligopoly model will change with time [5].

Since 1990s, many countries in the world have started to reform the electricity market one after another, and are exploring and establishing the power industry system and market-oriented operation mechanism which are suitable for their own national conditions. Europe, as one of the more open areas of electricity marketization, has already started the process of marketization in major countries. Take the electricity markets in Britain and Northern Europe as examples. Britain is the first country to carry out market-oriented reform. In the early days, Britain adopted the trading mode of electricity bank. Since 2001, Britain has adopted a new trading mechanism instead of electricity bank. Britain's power market is separated from each other in three production links: power generation, transmission and distribution. There is a power joint venture market composed of multiple power generation companies, independent power grid companies and multiple distribution companies, and its market trading platform mechanism is more efficient. Nordic electricity market is a transnational electricity market, and most electricity transactions in Nordic electricity market are conducted through bilateral contracts.

In the market-oriented power system, the formulation of electricity price mainly includes agreement pricing method and competition formation pricing method. Competition to form electricity price means that buyers and sellers of electric energy form a balanced price through supply and demand competition and price competition and fair trade in the spot market of electric power to determine the market price. Due to the oligopoly characteristics of electricity market and the problem of information asymmetry, the competition mechanism is different under different operation modes of electricity market. The main pricing methods are as follows: 1. System marginal unified pricing mechanism. 2, the market unified clearing price. 3. Quotation and payment of electricity price mechanism. This paper mainly considers the market unified clearing price mechanism. Its principle is: the power generation market is based on the auction system of power economy, with independent power producers providing their own quotations, and the power broker system realizes the unified clearing price of the power market according to the quotations of various power producers and users, and then determines the power generation plan and the electricity price settlement method in the form of contract.

3. Electricity Price Clearing Model

3.1. Objective function

With the goal of maximizing the total social welfare, the objective function of the optimization model is as follows:

$$
f = \max \sum_{d \in D} \sum_{b \in B} \sum_{t \in T} \left[P_{db}^t Q_{db}^t x_{db}^t \right] - \sum_{s \in S} \sum_{b \in B} \sum_{t \in T} \left[P_{sb}^t Q_{db}^t x_{db}^t \right]
$$
$$
+ \sum_{bo \in BO} \sum_{t \in T} \left[P_{bo} Q_{bo}^t x_{bo} \right]
$$

(1)

s.t.

$$
p_n^t + \sum_{d \in D_n} \sum_{b \in B} \left[Q_{db}^t x_{db}^t \right] - \sum_{s \in S_n} \sum_{b \in B} \left[Q_{sb}^t x_{sb}^t \right]
$$
$$
+ \sum_{bo \in BO_n} \left[Q_{bo}^t x_{bo} \right] = 0
$$

(2)

$$
\sum_{n \in N} p_n^t = 0
$$

(3)

$$
F_{lc}^{\min} \leq \sum_{n \in N} \left(PTDF_{\ln}^c p_n^t \right) \leq F_{lc}^{\max}
$$

(4)

$$
x_{sb}^t \leq 1
$$

(5)

$$
x_{db}^t \leq 1
$$

(6)

$$
R_{bo}^{\min} u_{bo} \leq x_{bo} \leq u_{bo}
$$

(7)

The above objective function is to maximize social welfare, and the set of decision variables is as follows:

$$V = \left\{ u_{bo}, x_{db}^t, x_{sb}^t, x_{bo}, p_n^t \right\} \tag{8}$$

In which u_{bo} is a binary variable.

The total social welfare of single-period orders and block orders is the difference between the total load utilization and the total quoted cost. It should be noted that in the block order quotation, the value of $P_{bo}Q_{bo}^t$ is negative, which assumes that the market clearing problem is a social welfare maximization model and Q_{bo}^t is less than zero. Among them, P_{db}^t is the bidding price of b-stage demand of demand d in t period; Q_{db}^t is the bidding amount of b-stage demand of demand d in t period; x_{db}^t is the B-stage demand acceptance ratio of demand d in t period; P_{sb}^t is the b-stage supply quotation of t-period supply s; Q_{db}^t is the b-stage supply of sin t period; x_{bo} is the acceptance ratio of block order bo; $R_{bo}^{\min}$ is the minimum acceptance ratio of block order bo; $PTDF_{\ln}^c$ is the power transfer distribution factor of l branch of node n in emergency state c; p_n^t is the net injection power of node n in t period; $F_{lc}^{\max}$ is the upper limit of power flow of branch l in emergency c.

Constraint (2) represents the net injection of node n in t period. Equation (3) represents the power balance constraint of the system per hour. Equation (4) represents the power flow balance constraint and the emergency state constraint c of the transmission line within each hour of the system. Equations (5) and (6) represent the upper limit of order clearing in simple time period. Equation (7) ensures that the clearance status of profile block orders should be 0 or between the minimum acceptance ratio and 1. If the regular block orders $R_{bo}^{\min} = 1$, equation (7) becomes full execution or immediate cancellation of constraint.

3.2. Electricity price clearing model
The following formula is needed to calculate the node clearing price MCP [6] in t period:

$$MCP_n^t = \lambda_t - \sum_{l \in L} \sum_{c \in C} \left[PTDF_{\ln}^c \left(\mu_{lct}^+ - \mu_{lct}^- \right) \right] \tag{9}$$

Where λ_t is the Lagrange multiplier of power balance constraint of scheduling period t; μ_{lct}^+ and μ_{lct}^- are lagrange multipliers of the upper and lower limits of transmission line power flow in emergency c. This constraint shows that the final market clearing price depends not only on the basic situation, but also on the emergency situation.

4. Model Solving Algorithm
In this paper, an iterative algorithm is proposed to solve the proposed model. The steps are as follows:

4.1. Parameter setting
The $PTDF_{\ln}$ of transmission line l in emergency c is calculated according to reference [7]. For each PTDF matrix, it is necessary to calculate the parameters in emergency state, that is, to remove the power cut line in emergency state.

4.2. Carry out internal iteration, and the specific steps include:
1) Optimize without considering constraint (4);

2) The formula for calculating the power flow F_{lc}^t of the transmission line l for each scheduling period t and the considered progress state c is as follows:

$$F_{lc}^t = \sum_{n\in N}\left(PTDF_{\ln}^c\, \overline{p}_n^t\right) \qquad (10)$$

Among them, $\overline{p}_n^t$ is the optimal parameter of the node net position, which is determined by the previous step;

3) Identify overloaded transmission lines. If there is no overload transmission line, proceed to step 3; Otherwise, the constraint (4) is included in the optimization problem, that is, the thermal stability value of each transmission line is 90% of its rated value.

Continue the iteration of step 1), and consider all the constraints in step 3) until there is no overload line.

4.3. Identify the order type

This step carries out external iteration, which specifically includes:

1) The above iterative algorithm is used to solve the feasible solution, and (8) is used to solve the market clearing price of node n in scheduling period t;

2) All the block orders can be obtained through the total social welfare obtained from the previous transaction. The welfare W_{bo} calculation formula for each order is as follows:

$$W_{bo} = \sum_{t\in T}\left\{\left(P_{bo} - \sum_{n\in N}\left(A_n^{bo} MCP_n^t\right)\right)Q_{bo}^t \overline{x}_{bo}\right\} \qquad (11)$$

$\overline{x}_{bo}$ is the optimal value of the variable x_{bo}, which is obtained by the last iteration of the internal iteration. A_n^{bo} is an association matrix, which indicates the association between block order bo and node n. If the social welfare of the block order is negative, it means that the price clearing condition is not satisfied, that is, the clearing status is 0, and it is removed from the order.

3) If there are contradictory orders, repeat step 1), and perform internal iteration in each external iteration, otherwise, the algorithm is terminated.
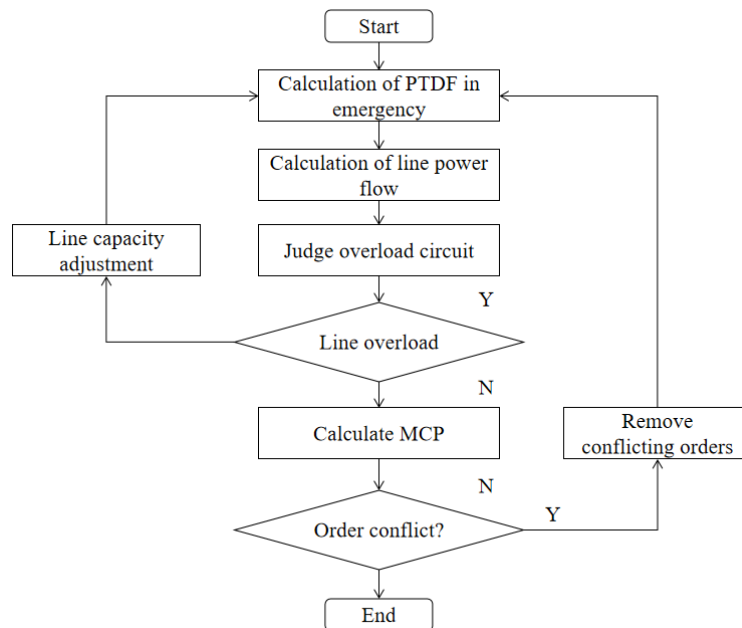

Fig.1 Flow chart of iteration

5. Example Analysis

5.1. Hypothesis and variable description

In this paper, the simulation system is used to analyze an example. Order types of related nodes are given in the following table. All single-bucket orders and block orders are randomly generated.

Tab.2 Nodes data

Node	Line	Number of block orders	Node	Line	Number of block orders
16	21	5	11	11	10
36	40	19	6	6	8
59	82	42	83	120	106
265	412	178	39	56	59
27	30	11	10	11	18
39	52	44	591	841	500

5.2. Simulation analysis

1. Regardless of the N-1 criterion

Because the N-1 principle is not considered, the value of C is equal to zero. In the basic case of this paper, the iterative algorithm needs three times to converge.

Among them, each external iteration needs three internal iterations. In each internal iteration, the total social welfare is continuously reduced. This is mainly due to the constraint of transmission lines in each iteration, which is considered in the optimization model. The following figure shows the total social welfare of the third external iteration. As shown in the figure, the total social welfare is declining from the first to the third internal iteration, which is due to the fact that the constraints of transmission lines are not considered in the first iteration.
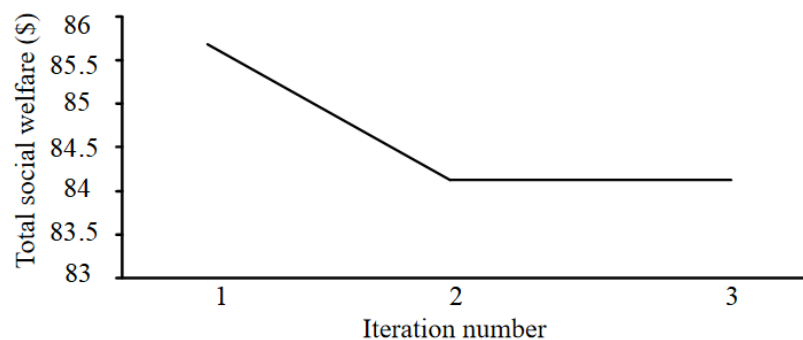

Fig.2 Total social welfare in the final iteration

In the second iteration, transmission line constraints are considered, and the number of transmission line constraints is more than that in the third iteration. The following table shows the number of normal operation and overload lines during iteration. It can be seen that the line constraints of normal operation are taken into account in each iteration. This kind of constraint is necessary, mainly to avoid the oscillation in the internal iterative algorithm, especially when the normal line is put into operation and cut off in the continuous iterative algorithm, which is mainly caused by the congestion in other parts of the network. On the other hand, the total number of overloaded lines will

decrease in each internal iteration and external iteration algorithm. When the overload line has been reduced to zero in the current iterative process, the internal iterative algorithm is terminated.

Tab.3 Total number of normal and overloaded lines

External iteration	Transmission line status	Internal iteration		
		1	2	3
1	Normal	246	325	326
	Overload	165	52	0
2	Normal	234	317	315
	Overload	168	45	0
3	Normal	234	302	312
	Overload	168	45	0

The following table shows the initial clearing results of some orders. BO409 is cleared in the first external iteration, which is also in line with the biggest goal of total social welfare. The clearing of orders reduces the number of overloaded lines, thus optimizing social welfare. The MCP of order BO422 is equal to the weighted average price, so the total benefit in the transaction is 0.

Tab.4 Clearing of certain block orders

Order	The minimum acceptance ratio (p.u.)	Offer ($/MWh)	Weighted average market clearing price(€/MWh)	x_{bo} (p.u.)	Welfare ($)
BO408	1.000	46.930	9.352	1.000	-28,844.652
BO490	0.370	57.710	57.892	0.360	-7.525
BO496	0.920	50.180	70.173	0.910	-7,125.469
BO144	0.784	21.450	37.594	1.000	7,457.081
BO422	0.163	36.526	36.520	0.586	0.000

2. Consider the N-1 principle

Consider the n-1 principle for simulation. The following table gives the results of social welfare and solution time. As shown in the table, the total social welfare obtained by iterative algorithm is lower than that without considering the n-1 principle. This is due to the additional transmission line constraints, which fully shows the possibility of power failure in each interconnected area in consideration of the optimization problem. The additional constraints mentioned above mainly affect the total solution time, which is determined by the convergence time of the iterative algorithm.

Tab.5 Results under N-1 criterion

	Solving time	Total social welfare
Iterative algorithm	15.03min	82327654.2$

6. Summary

In this paper, aiming at maximizing the total social welfare, the corresponding electricity market model is established. In this paper, an iterative algorithm is proposed to simulate the order clearing and take into account the safety constraints of transmission lines, and the corresponding node model and the marginal price model of electric energy producers and consumers in each node are established. The simulation example shows that the algorithm proposed in this paper can be applied to the power system with N-1 principle, and can fully reflect the constraints of safe and stable operation of the line. The results can tell the electricity market to carry out the next transaction.

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