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Improved Bi-Direction Evolutionary Structural Optimization Method and Its Application

Jialei Zhu, Xiaodong Liang and Shuai Shao*, Yongdong Bi, Runze Miao

Northwest Institute of Nuclear Technology, xi'an 710024, China

*Corresponding author e-mail: 17736126435@163.com

Abstract. In the process of lightweight design of structures, the topology optimization method does not depend on the experience of designers, so it can design the optimal products more flexibly and creatively, which has become a hot issue in structural design. However, the current topological optimization results usually have problems such as difficult processing and high cost, which limit their wide application in engineering design. Therefore, this paper is based on the two-way progressive structure optimization method (BESO), considering the existence of jagged edges in the optimization results. In the finite element analysis of the fixed grid method, the boundary element is judged by the sensitivity of the element node, and the boundary element is localized divide and realize the partial deletion of the unit, so as to achieve a smooth boundary of the optimization result. The sensitivity of element nodes during structural flexibility optimization is deduced in detail, its basic principles and specific steps are explained, and finally the feasibility of this method is verified by an example.

1. Introduction

In the 1990s, Yimin Xie [1], [2] first introduced Evolutionary Structural Optimization (ESO). ESO is based on the idea that by gradually removing inefficient elements from a structure, the resulting structure will evolve towards an optimum. With the study of ESO theory, Querin et al. [3] proposed the Bidirectional Structural Optimization (BESO), which improves the global optimality of the ESO method by deleting the elements with low stresses and adding the elements in the locations with high stresses. Garcia et al. [4] introduced the fixed grid method into the ESO method to avoid re-meshing and thus improving the efficiency of optimization. In view of the shortcomings of the traditional ESO method that the boundary of the optimized structure shall be sawtooth in shape, Chunjiang Du et al. [1] put forward the reverse engineering method, which can effectively solve the manufacturing and processing problem of the optimization results without increasing the calculation. With regard to the study on the sawtooth boundary of a structure, this paper [6] proposes a CAD model reconstruction method from topological optimization results based on feature and constraint so as to make the boundary of topological optimization results smooth. By doing so, the optimization results can be accurately used in the subsequent design and manufacture as quickly as possible. At present, improving the manufacturability and machinability of optimized structures is a hot topic in structure optimization.

In order to improve the smoothness of optimized structures, Kim et al. [7] proposed to introduce the fixed gird method into the ESO method. On this basis, this paper introduces fixed grid method into

BESO method for finite element analysis so as to realize the smoothness of optimized structures. Also, the sensitivity formula of element nodes is deduced. The boundary element is judged by the node sensitivity, for which the local mesh division is applied. As such, the boundary smoothness is realized. In the end, this paper lists out some examples by which the rationality and effectiveness of the proposed method are verified.

2. BESO element sensitivity

In the optimization of a structure, the principle of deleting materials in BESO method is based on the contribution rate of materials to the structure. That is, the materials with low contribution rate shall be gradually deleted in the optimization process so that the remaining materials can be optimized. Therefore, the element sensitivity shall be derived in the optimization of structural flexibility.

In the structural rigidity model, the objective function is the structural flexibility:

$$C = f^T u \tag{1}$$

In the finite element analysis, the structural equilibrium equation is:

$$Ku = f \tag{2}$$

Where, K is the stiffness matrix of the structure; f and u are the structural force load vector and the nodal displacement vector respectively.

According to the interpolation model of the material in the variable density method [8] and in the BESO method, the elastic modulus of the material and the stiffness matrix of the structure are respectively defined as:

$$\begin{aligned} E(x_i) &= E_0 x_i^p \\ K &= \sum x_i^p K_i^0 \end{aligned} \tag{3}$$

Where, E_0 is the elastic modulus of the solid material; p is the penalty factor; K_i^0 is the stiffness matrix of the solid element i in the structure.

Introduce the Lagrange parameter vector λ and add the addition term $\lambda^T(f\text{-}Ku)$ to the formula (1), and then the derivative of objective function to the design variable can be expressed as:

$$\frac{dC}{dx_i} = \frac{df^T}{dx_i}u + f^T\frac{du}{dx_i} + \frac{d\lambda^T}{dx_i}(f - Ku) + \lambda^T(\frac{df}{dx_i} - \frac{dK}{dx_i}u - K\frac{du}{dx_i}) \tag{4}$$

Considering that deleting elements has no effect on the load f, and $df/dx_i = 0$, the above equation can be simplified as:

$$\frac{dC}{dx_i} = \left(f^T - \lambda^T K\right)\frac{du}{dx_i} - \lambda^T\frac{dK}{dx_i}u \tag{5}$$

For (f-Ku)=0, the parameter λ can be selected freely. It is proposed in the reference [9] that in order to eliminate the unknown term du/dx_i, the selection of λ is based on the following expression:

$$f^T - \lambda^T K = 0 \tag{6}$$

Based on the static equilibrium equation, the Lagrangian vector λ can be obtained as:

$$\lambda = u \tag{7}$$

By substituting the equation (7) into the equation (5), the following can be obtained:

$$\frac{dC}{dx_i} = -u^T \frac{dK}{dx_i} u \tag{8}$$

By substituting the equation (3) into the equation (8), the derivative of the design variable x_i of the element i can be obtained as:

$$\frac{\partial C}{\partial x_i} = -p{x_i}^{p-1} u_i^T K_i^0 u_i \tag{9}$$

The element sensitivity represents the degree to which the element affects the objective function and is defined as:

$$\alpha_i = -\frac{1}{p}\frac{\partial C}{\partial x_i} = \begin{cases} u_i^T k_i u_i & x_i = 1 \\ {x_{\min}}^{p-1} u_i^T k_i u_i & x_i = x_{\min} \end{cases} \tag{10}$$

For “inefficient or non-efficient” elements, the sensitivity depends on the size of the penalty factor p. When p approaches infinity, the equation (10) can be simplified as:

$$\alpha_i = \begin{cases} u_i^T k_i u_i & x_i = 1 \\ 0 & x_i = x_{\min} \end{cases} \tag{11}$$

It can be seen from the above formula that the sensitivity of solid elements and empty elements is equal to the strain energy and zero value of the elements respectively. The design variable of empty elements is $x_{\min}$. The corresponding element stiffness and elastic modulus are also minimum, thereby realizing the deletion of the elements.

3. BESO method based on fixed grid

Applying fixed grid method into finite element analysis in the ESO method can improve the optimization efficiency. As shown in Figure 1, according to the location of elements in the design domain, the elements can be divided into three types. That is, inner element (I), outer element (O), boundary element (NIO). As we can see from Figure 1, NIO element affects the smoothness of the structure. Therefore, this paper considers inserting a new material model into the NIO element and allowing part of the materials in the elements to be deleted.

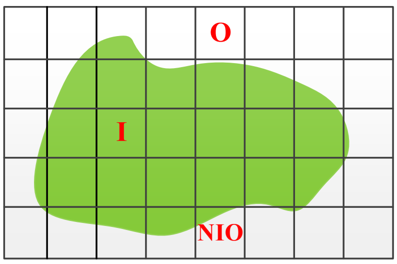


Figure 1. Fixed grid model

3.1. *Design model and finite element model*

As shown in Figure 2, when the design model (yellow area) is projected to a fixed grid, the design model can be divided into three types of elements. That is, empty element (element 2, V_f=0), solid element (element 1, V_f=1) and boundary element (element 3, 0<V_f<1).

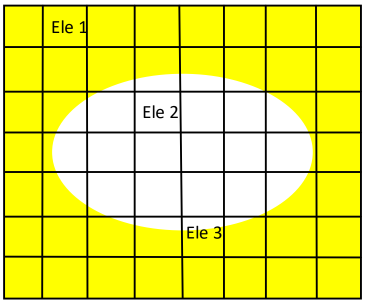


Figure 2. Projection relationship between design model and fixed grid

In the finite element analysis, the density and elastic modulus of elements can be expressed as:

$$\begin{aligned} \rho_i(V_i^f) &= (1-V_i^f)\rho_{\min} + V_i^f \rho_0 \\ E_i(V_i^f) &= (1-V_i^f)E_{\min} + V_i^f E_0 \end{aligned} \tag{12}$$

Where, V_i^f is the volume fraction of the element i; ρ_i is the density of the element i; E_i is the Young's modulus of the element i; $\rho_{\min}$ and ρ_0 are the density of the empty element and solid element respectively; $E_{\min}$ and E_0 are the Young's modulus of the empty element and solid element respectively.

3.2. *Volume constrained topology optimization model*

3.2.1. Stiffness optimization model. The structural optimization is to find the optimal distribution of the material under certain constraints, such as the maximum stiffness. In this paper, the problem of flexibility minimization under the constraint of volume is considered, and the corresponding mathematical model is as follows:

$$\begin{aligned} &\mathit{find}\ x=[x_1,x_2,...,x_n] \\ &\min\ \ C = F^T U \\ &s.t\ \ \ V \le V^* \\ &\qquad\ \ x_i = \{0,1\} \end{aligned} \tag{13}$$

Where, C is the flexibility of the structure; V is the volume after optimization; V^* is the volume constraint of the structure; F and U are the displacement vector and the load array of the structure respectively; x_i is the design variable of any node i in the design domain.

The volume of the structure can be expressed as:

$$V = \sum_{i=1}^{N} V_i^f V_i \quad 0 \le V_i^f \le 1 \tag{14}$$

Where, V_i is the volume of the element i; N is the total number of elements in the finite element model.

3.2.2. Element and node sensitivity. As we can see from the equation (10), the sensitivity of the element can be expressed as:

$$\alpha_i = \begin{cases} u_i^T k_i^0 u_i & x_i = 1 \\ x_{\min}{}^{p-1} u_i^T k_i^0 u_i & x_i = x_{\min} \end{cases} \tag{15}$$

By substituting the density of the element in the equation (12) to the equation (15), the sensitivity of the element i can be expressed as:

$$\alpha_i = \left[x_{\min}^{p-1} \left(1 - V_i^f\right) + V_i^f \right] u_i^T k_i^0 u_i \tag{16}$$

As for the sensitivity of nodes, the method proposed in the reference [9] is adopted. Take the sensitivity of the elements around the nodes as the sensitivity of the nodes.

$$\alpha_j^n = \frac{\sum_{i=1}^{M} V_i \alpha_i}{\sum_{i=1}^{M} V_i} \tag{17}$$

Where, α_j^n is the sensitivity of the node j; M is the number of elements related to the node j.

3.2.3. Update and convergence criterion of design variable. When the design model is projected onto the finite element model, there will be three different types of elements, as shown in Figure 3, namely solid element, empty element, and boundary element. Therefore, the volume fraction V_i^f of each element in the finite element model is expressed as:

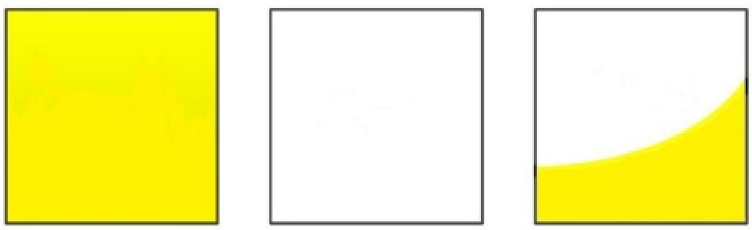


Figure 3. Types of elements

$$V_i^f = \begin{cases} 1 & \min(\alpha_i^j) > 0 \\ 0 & \max(\alpha_i^j) < 0 \\ N_{lp} / N_{ap} & else \end{cases} \tag{18}$$

Where, $a_i^j (j = 1,2,3,4)$ is the sensitivity of node j of the element i. If the node sensitivity of the element is greater than 0, the element shall be considered as a solid element. If the node sensitivity of the element is less than 0, the element shall be considered as an empty element. Otherwise, it shall be considered as a boundary element. For the boundary element ($0 < V_i^f < 1$), the local grid shall be differentiated. N_{ap} is the total number of nodes in the local grid, and N_{lp} is the number of nodes whose sensitivity is greater than 0.

In the structural optimization, when the design volume V_{l-1} of the step l-1 is provided, the iteration volume V_l of the next step can be expressed as follows：

$$V_l = \begin{cases} \max(V_{l-1}(1-ER), V^*) & V_{l-1} \geq V^* \\ \min(V_{l-1}(1+ER), V^*) & V_{l-1} < V^* \end{cases} \tag{19}$$

Where, ER is the evolutionary rate of volume.

In the finite element analysis, the convergence criterion of BESO method shall be adopted until the volume constraints and convergence conditions are satisfied. Therefore, the convergence criterion can be expressed as:

$$\frac{\left|\sum_{i=1}^{N}\left(C_{k-i+1} - C_{k-N-i+1}\right)\right|}{\sum_{i=1}^{N} C_{k-i+1}} \leq \tau \tag{20}$$

Where, C is the flexibility of the structure, k is the current number of iterations, τ is the allowable convergence error, and N is a positive integer (usually take N=5).

3.3. Algorithm steps

The optimization steps of the updated BESO method can be summarized as follows:

(1) Specify the design domain, define the load and boundary conditions, use the finite element grid to discretize the design domain, and initialize the elements in the design domain;

(2) Through finite element analysis, calculate the sensitivity of the element, and calculate the node sensitivity by the equation (18).

(3) Confirm the volume fraction V_i^f of the element in the design domain through the equation (18) and then confirm the current volume of the structure through the equation (14).

(4) Repeatedly execute the step (2)-(3) until the volume constraint (19) and convergence criterion (20) are satisfied.

4. Numerical examples

The following two classical examples are used to calculate the minimum flexibility of the structure with the aim of verifying the feasibility and efficiency of the algorithm.

Example 1: The traditional optimal Michell structure is shown in Figure 4. Figure 5 is the initial design domain of the structure. Load F=1000N. Elastic modulus of the material is 3GPa. Poisson's ratio is 0.3, and volume constraint is 0.4.

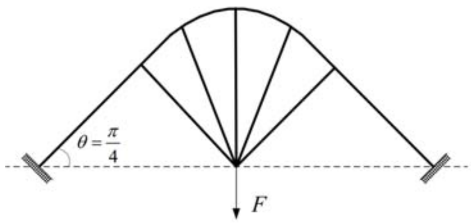


Figure 4. Michell structure

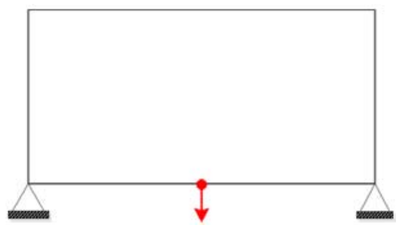


Figure 5. Initial design domain of the structure

Figure 6. Optimization result of modified BESO

Figure 7. Optimization result of traditional BESO

As we can see from Figure 6 and Figure 7, by using the modified BESO method, the boundary of the structure does not have the sawtooth compared with that by using the traditional BESO method, and the structure is highly smooth. The structure is similar to the "Michell" type structure with a close to 45 degrees of support similar to the umbrella structure.

Example 2: As shown in Figure 8, the cantilever structure is subjected to concentrated load F=1kN at the midpoint of the right end. Elastic modulus E=100GPa. The Poisson's ratio is υ=0.3. The left side is the fixed end. A 80*50 four-node rectangular discrete design domain is adopted. The optimization objective is to minimize the flexibility of the structure with a volume fraction of 0.5.

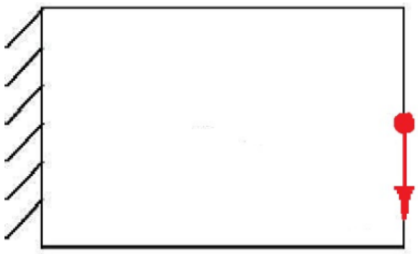
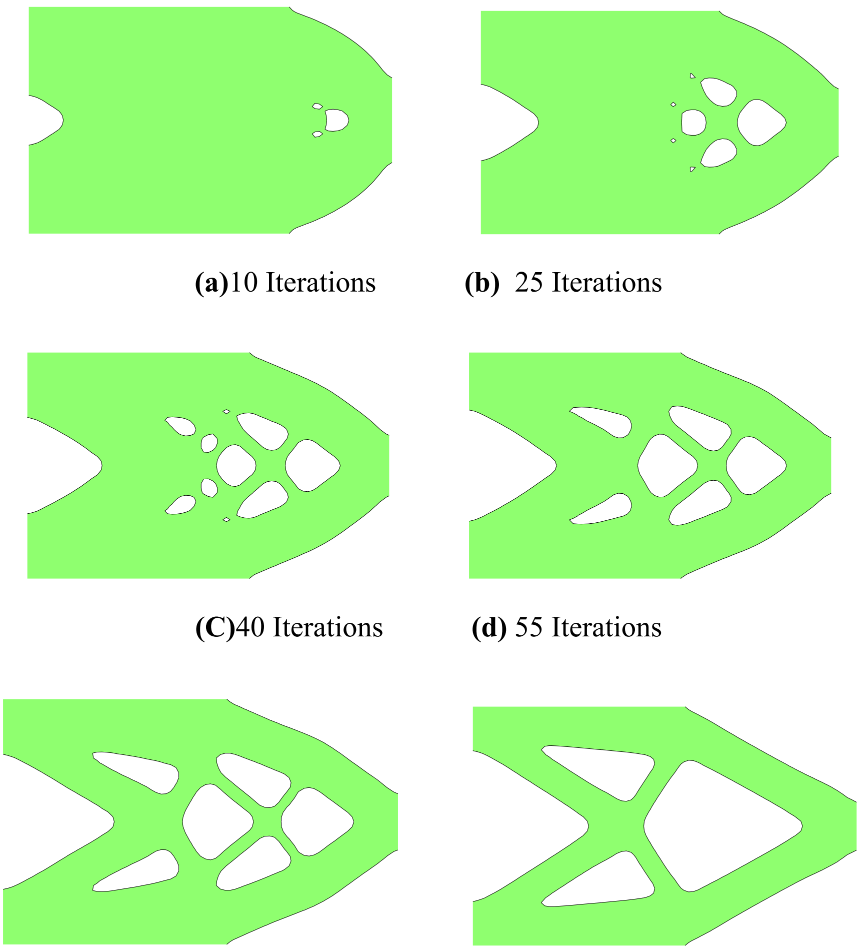


Figure 8. Cantilever structure



(a)10 Iterations **(b)** 25 Iterations

(C)40 Iterations **(d)** 55 Iterations

(e) 60 Iterations **(f)** 75 Iterations

Figure 9. Structural optimization process diagram

As we can see from Figure 9, during the optimization process of the cantilever, with the removal of materials in the structure, the structure boundary always has a good smoothness, and the structure is optimized after 75 iterations.

5. Conclusion

Based on the problem of sawtooth boundary appeared in structural optimization through the traditional BESO method, this paper proposes to introduce the fixed grid method into the BESO method and derive the node sensitivity of elements. Through the node sensitivity of elements, the boundary elements in the structure can be determined. Then, the local grid division can be used for boundary elements to ensure the smoothness of optimized results. As such, the feasibility is verified.

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