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Harmonic Linearization Method for Computing the Motions of a Specific Type of Three-Joint Arms

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Abstract. The paper presents a three-joint arm that has two translational fifth-class kinematic pairs and a rotational one. The first two kinematic pairs are axially orthogonal, while the second and the third one are axially parallel. Such robots find use in machinery handling for arc welding. Systems of three nonlinear second-order differential equations were obtained to describe the motions of this arm. As it was necessary to assess the moment of load emerging on the control-drive shaft, the external generalized forces were reduced to a corresponding generalized coordinate. Harmonic linearization of nonlinearities was applied to analyze the arm system. This method can be used to fine the core properties of the system (frequency, amplitude, and phase of the oscillations; dependence of the form of nonlinearities or on the parameters of the linear part; etc.) without studying the transient.Ratios were obtained to find the parameters of natural oscillational motion occurring in such arms. These rations can be used to design and set up such an arm. The finding is that where the system of equations describing the motion of such an arm (including the drive motion equations) contains no prominent nonlinearities, these rations are a system of two algebraic frequency equations: a sixth-degree one and a fifth-degree one.

1. Introduction

Systems of differential equations used to describe the mechanical dynamics of three-joint, three-DoF arms are mostly nonlinear. Besides, the motion transmission mechanisms and the degree-of-freedom drives contain a number of significant nonlinearities like dead-zones, freeplay, saturation, etc. Harmonic linearization of nonlinearities is advisable for early analysis of such systems when the research has to structure and configure the system given its existing nonlinearities. What makes this method advisable is that it can be used to fine the core properties of the system (frequency, amplitude, and phase of the oscillations; dependence of the form of nonlinearities or on the parameters of the linear part; etc.) without studying the transient of the system per se.

2. Materials and Methods

Consider the problem of finding the conditions, under which periodic oscillational motion may occur and is steady; the three-joint arm in question has two translational fifth-class kinematic pairs and a rotational one [1]. Assume that the arm is controlled by three independent electric drives located in the joints. Let the mathematical model of the drive be the following equation:

$$M_{\text{DV}}^{i} = d_{\text{i}} k_{\text{y}}^{i} (T_{\text{y}}^{i} p + 1)(q_{\text{i}} + q_{\text{i}}^{*}) - h_{\text{i}} p \tag{1}$$

where $d_{i,} h_i -$ are the design parameters of the i^{th} drive, $k_y^i -$ is a coefficient inversely proportional to the gain of the i^{th} motor, $q_i^* -$ is the programmed value of the generalized coordinate of the i^{th} degree of freedom, p is the operator of differentiation[2]. Assume that a moment of resistance forces generated by the viscous friction forces in the mechanical transmission is applied to the output axis of the second drive.

$$M_{\text{tp}}^{2} = -K_{\text{V.TR}}^{2} \square \dot{q}_2 \tag{2}$$

Assume that viscous friction forces are applied to the output shafts of drives #1 and #3:

$$F_{\text{TR}}^{j} = -K_{\text{V.TR}}^{j} \square \dot{q}_{\text{j}}, \; j = 1.3 \tag{3}$$

Based on these assumptions, the motion equations for the arm in question can be written together with the control-drive equations as follows:

$$(I_{\text{p}1} n_1^2 + a_1)\ddot{q}_1 + a_5 \ddot{q}_2 + a_6 (\dot{q}_2)^2 = d_1 n_1 k_{\text{y}}^1 (T_{\text{y}}^1 \dot{q}_1^* - T_{\text{y}}^1 \dot{q}_1 + q_1^* - q_1) - h_1 n_1 \dot{q}_1 - K_{\text{V.TR}}^1 \square \dot{q}_1, \tag{4}$$

$$\left(I_{p2} n_2^2 + a_2\right)\ddot{q}_2 + a_5 \ddot{q}_1 = d_2 n_2 k_y^2 \left(T_y^1 \dot{q}_2^* - T_y^1 \dot{q}_{12} + q_2^* - q_2\right) - h_2 n_2 q_2 - K_{\text{V.TR}}^2 q_2$$

$$+ a_7 [p_2 + 2(p_3 + p_\Gamma)] \tag{5}$$

$$(I_{p2} n_3^2 + a_3)\ddot{q}_3 = d_3 n_3 k_y^3 (T_y^3 \dot{q}_3^* - T_y^3 \dot{q}_3 + q_3^* - q_3) - h_3 n_3 \dot{q}_3 - K_{\text{V.TR}}^3 \square \dot{q}_3. \tag{6}$$

Coefficients for this system can be found by the formulas:

$$a_1 = m_1 + m_2 + m_3 \tag{7}$$

$$a_2 = I_{z2} + I_{z3} + m_2 \left(\frac{l_2}{2} \right)^2 + m_3 l_2^2, a_3 = m_3, \tag{8}$$

$$a_4 = m_2 l_2 \cos q_2 + 2 m_3 l_2, a_5 = \frac{1}{2} a_4, \tag{9}$$

$$a_6 = -\frac{1}{2}(m_2 + 2m_3) l_2 \sin q_2, \tag{10}$$

$$a_7 = -\frac{l_2}{2} \cos q_2. \tag{11}$$

q_j are the generalized coordinates that configure the gripper and the arm of the robot; $m_j, l_j, p_j, I_{xi}, I_{yi}, I_{zi}$ are the mass, the weight, and the axial moments of inertia of the link i, respectively, p_Γ is the weight of the payload in the gripper[3].

The system can be written in the operator form as follows:

$$Q_1(p) q_1 = s_1 F_{11}(q_2, p^2 q_2) - s_1 F_{12}(q_2, p q_2) + d_1 n_1 k_y^1 (T_y^1 p + 1)(q_1^*), \tag{12}$$

$$Q_2(p) q_2 = s_2 F_{22}(q_2, p^2 q_1) + d_2 n_2 k_y^2 (T_y^2 p + 1)(q_2^*), \tag{13}$$

$$Q_3(p) q_3 = d_3 n_3 k_y^3 (T_y^3 p + 1)(q_3^*). \tag{14}$$

Below is the notation used for these operator equations:

$$s_1 = -\frac{l_2}{2n_1}(m_2 + 2m_3), s_2 = -\frac{l_2}{2n_2}(m_2 + 2m_3), F_{11}(q_2, p^2 q_2) = p^2 q_2 \cos q_2, \tag{15}$$

$$F_{12}(q_2, pq_2) = (pq_2)^2 \sin q_2, F_{22}\left(q_2, p^2 q_1\right) = p^2 q_1 \cos q_2 + \frac{p_2 + 2\left(p_3 + p_\text{г}\right)}{m_2 + 2m_3}\cos q_2, \tag{16}$$

$$Q_i(p) = \frac{I_{\text{p}i} n_i^2 + a_i}{n_i} + (k_2^i + \frac{k_{\text{B.TP}}^i}{n_i})p + k_1^i, \tag{17}$$

$$k_2^i = d_i k_y^i T_y^i + h_i, k_1^i = d_i k_y^i, i = 1, 2, 3. \tag{18}$$

Figure 1 shows the structure of the equation system (13).

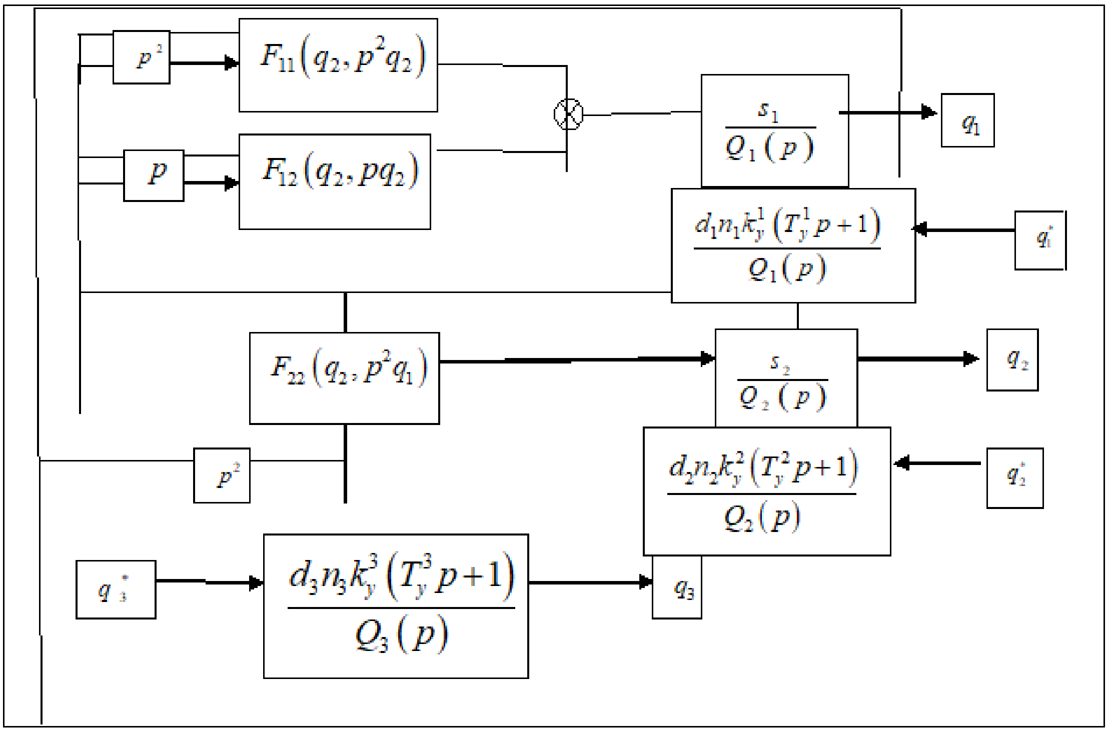

Figure 1. The structure of the equation system (13).

Consider the arm movements unaffected by any external disturbances or control actions [4]. In this regard, assume that q_i^*, (i=1,2,3) equal zero, and $F_{22}\left(q_2, p^2 q_1\right) = p^2 q_1 \cos$. Let all the intermediate parts of the system between nonlinearities have a generalized filter property. Pursuant to the harmonic linearization method, assume that for variables under the signs of nonlinear functions, the period solution is close enough to a sinusoid, i.e.

$$q_2 = A_{q_2} \sin \Omega t, q_1 = A_{q_1} \sin(\Omega t - \theta) \tag{19}$$

Here, $A_{q_1}, A_{q_2}, \Omega, \Theta$ stands for the amplitudes of oscillations, the frequency and phase shift of the oscillations of the variables $q_1.q_2$ under the signs of nonlinear functions. Perform harmonic linearization of the nonlinearity

$$F_{12}\left(q_2, pq_2\right) = \left(pq_2\right)^2 \sin q_2 = F_{12}\left(A_{q_2}\sin\Omega t, A_{q_2}\Omega\cos\Omega t\right) = (A_{q_2}\Omega)^2\cos^2\Omega t\ \sin A_{q_2}\sin\Omega t\,. \tag{20}$$

Expand this function into a Fourier series to obtain a harmonically representation of the nonlinear function $F_{12}\left(q_2, pq_2\right)$:

$$F_{12}(q_2, pq_2) = \tfrac{1}{2}\left\{J_1(A_{(q_2)}) + J_3(A_{(q_2)})\right\}(A_{(q_2)})^2\Omega^2 q_2 = q_{12}(A_{(q_2)}, \Omega)q_2. \tag{21}$$

Here, $J_1\left(A_{q_2}\right).J_3(A_{q_2})$ stand for fist- and third-kind of Bessel functions, respectively, and

$$q_{12}(A_{(q_2)}, \Omega) = \frac{1}{2}\left\{\left[J_1(A_{(q_2)}) + J_3(A_{(q_2)})\right]\right\}(A_{(q_2)})^2\Omega^2. \tag{22}$$

Similarly obtain a harmonically linearized expression for the function $F_{11}\left(q_2, p^2 q_2\right)$:

$$F_{11}(q_2, p^2 q_2) = \frac{a_{11}(q_2\Omega)}{\Omega^2}p^2 q_2 = 2J_1(A_{(q_2)})p^2 q_2, q_{11}1(A_{(q_2)}, \Omega) = 2J_1(A_{(q_2)})\Omega^2. \tag{23}$$

Thus, the first equation of the system can be written as follows:

$$Q_1(p)q_1 = s_1\frac{q_{11}(A_{(q_2)}, \Omega)}{\Omega_2}p^2 q_2 - s_1 q_{12}(A_{(q_2)}, \Omega)q_2. \tag{24}$$

Consider the nonlinear function $F_{22}\left(q_2, p^2 q_1\right)$ in the second equation of the system (13). Symbolize the equalities (19):

$$q_1 = A_{q_1}e^{-j(\Omega t - \Theta)},\ q_2 = A_{q_2}e^{-j\Omega t},\ \text{j- imaginary unit.} \tag{25}$$

Substitute (25) in the harmonically linearized equation (24) at $p = j\Omega$
to find that

$$A_{q_2} = A_{q_1}\left|-\frac{s_1}{Q_1(j\Omega)}\right|(q_{11}(A_{q_2}, \Omega) + q_{12}(A_{q_2}, \Omega)) \tag{26}$$

$$\Theta = -\arg\left\{-\frac{s_1}{Q_1(j\Omega)}\right\}. \tag{27}$$

Thus,

$$q_1 = A_{q_2}\left(u_1(\Omega)\sin\Omega t + v_1(\Omega)\cos\Omega t\right)\left(q_{11}\left(A_{q_2}, \Omega\right) + q_{12}\left(A_{q_2}, \Omega\right)\right), \tag{28}$$

$$u_1(\Omega) = Re\left(-\frac{s_1}{Q_1(j\Omega)}\right) = \frac{-s_1\left(k_1^1 - P_1\Omega^2\right)}{\left(k_1^1 - P_1\Omega^2\right)^2 + \left(k_2^1 + k_c^1\right)^2\Omega^2}, \tag{29}$$

$$v_1(\Omega) = Im\left(-\frac{s_1}{Q_1(j\Omega)}\right) = \frac{s_1\left(k_2^1 + k_c^1\right)\Omega}{\left(k_1^1 - P_1\Omega^2\right)^2 + \left(k_2^1 + k_c^1\right)^2\Omega^2}. \tag{30}$$

$$k_c^i = \frac{k_{\text{V. TR}}^i}{n_i},\ P_i = \frac{I_{\rho i} n_i^2 + a_i}{n_i},\ i = 1, 2, 3. \tag{31}$$

Thus, the function $F_{22}(q_2, p^2 q_1)$ can be written as:

$$F_{22}\left(q_2, p^2 q_1\right) = -A_{q_2}\Omega^2\left(u_1(\Omega)\sin\Omega t + v_1(\Omega)\cos\Omega t\right)$$
$$\left(q_{11}\left(A_{q_2}, \Omega\right) + q_{12}\left(A_{q_2}, \Omega\right)\right)\cos\left(A_{q_2}\sin\Omega t\right). \tag{32}$$

Expand this function into a Fourier series and discard the higher harmonics of such expansion to obtain the linearized expression:

$$F_{22}\left(q_2, p^2 q_1\right) = -\left(u_1(\Omega)\ q_{11}\left(A_{q_2}, \Omega\right)q_2 + \frac{v_1(\Omega)}{\Omega}\ q_{22}\left(A_{q_2}, \Omega\right)pq_2\right)$$
$$\left(q_{11}\left(A_{q_2}, \Omega\right) + q_{12}\left(A_{q_2}, \Omega\right)\right) \tag{33}$$

$$q_{22}\left(A_{q_2}, \Omega\right) = (J_1\left(A_{q_2}\right) + J_1\left(A_{q_2}\right))\Omega^2 \tag{34}$$

These transforms produce a linearized equation system:

$$Q_1(p)q_1 = s_1\frac{q_{11}\left(A_{q_2}, \Omega\right)}{\Omega^2}p^2 q_2 - s_1 q_{12}\left(A_{q_2}, \Omega\right)q_2, \tag{35}$$

$$Q_2(p)q_2 = -s_2\left(u_1(\Omega)\ q_{11}\left(A_{q_2}, \Omega\right)q_2 + \frac{v_1(\Omega)}{\Omega}\ q_{22}\left(A_{q_2}, \Omega\right)pq_2\right)$$
$$\left(q_{11}\left(A_{q_2}, \Omega\right) + q_{12}\left(A_{q_2}, \Omega\right)\right) \tag{36}$$

$$Q_3(p)q_3 = 0. \tag{37}$$

To find the amplitude and frequency of the periodic solution $q_2(t)$, write down the characteristic equation of the system:

$$\begin{vmatrix} Q_1(\lambda) & -s_1\left(\frac{q_{11}}{\Omega^2}\lambda^2 + q_{12}\right) & 0 \\ 0 & Q_2(\lambda) + s_2\left(u_1(\Omega)\ q_{11}q_2 + \frac{v_1(\Omega)}{\Omega}\ q_{22}\lambda\right)\left(q_{11} + q_{12}\right) & 0 \\ 0 & 0 & Q_3(\lambda) \end{vmatrix} = 0 \tag{38}$$

3. Results and Discussion

As known, lack of periodic external disturbance or action means that solution like (19) are only possible if the characteristic equation of the closed system has a pair of purely imaginary roots. It is thereby assumed that other roots of the equation have negative real parts [5]. Thus, assuming that in the characteristic equation, $\lambda = j\Omega$, obtain a system to find the amplitude and frequency of the desired periodic solution:

$$-K_1\Omega^6 + K_3\Omega^4 - K_5\Omega^3 + K_7 - \left(N_1\Omega^4 - N_3\Omega^2 + N_5\right) q_{11}\left(q_{11}+q_{12}\right)\frac{s_1 s_2\left(k_1^1 - P_1\Omega^2\right)}{\left(k_1^1 - P_1\Omega^2\right)^2 + \left(k_2^1 + k_c^1\right)^2\Omega^2}$$
$$+\left(N_2\Omega^4 - N_4\Omega^2\right)\frac{s_1 s_2\left(k_2^1 + k_c^1\right)}{\left(k_1^1 - P_1\Omega^2\right)^2 + \left(k_2^1 + k_c^1\right)^2\Omega^2} q_{22}\left(q_{11}+q_{12}\right) = 0, \quad (39)$$

$$K_3\Omega^5 - K_4\Omega^3 - K_5\Omega^3 + K_6\Omega - \left(-N_2\Omega^3 + N_4\Omega\right) q_{11}\left(q_{11}+q_{12}\right)\frac{s_1 s_2\left(k_1^1 - P_1\Omega^2\right)}{\left(k_1^1 - P_1\Omega^2\right)^2 + \left(k_2^1 + k_c^1\right)^2\Omega^2}.$$
$$+\left(N_1\Omega^4 - N_3\Omega^3 + N_5\right)\frac{s_1 s_2\left(k_2^1 + k_c^1\right)\Omega}{\left(k_1^1 - P_1\Omega^2\right)^2 + \left(k_2^1 + k_c^1\right)^2\Omega^2} q_{22}\left(q_{11}+q_{12}\right) = 0, \quad (40)$$

The coefficient of the systems are as follows:

$$K_1 = P_1 P_2 P_3, \quad (41)$$

$$K_2 = P_1 P_2\left(k_2^3 + k_c^3\right) + P_3 P_2\left(k_2^1 + k_c^1\right) + P_3 P_1\left(k_2^2 + k_c^2\right), \quad (42)$$

$$K_3 = P_2\left(k_2^1 + k_c^1\right)\left(k_2^3 + k_c^3\right) + P_1\left(k_2^2 + k_c^2\right)\left(k_2^3 + k_c^3\right) + P_3\left(k_2^1 + k_c^1\right)\left(k_2^2 + k_c^2\right) + P_3 P_1 k_2^1 + P_1 P_2 k_1^1, \quad (43)$$

$$K_4 = \left(k_2^1 + k_c^1\right)\left(k_2^2 + k_c^2\right)\left(k_2^3 + k_c^3\right) + P_1\left(k_1^3\left(k_2^2 + k_c^2\right) + k_1^2\left(k_2^3 + k_c^3\right)\right) + P_2\left(k_1^3\left(k_2^1 + k_c^1\right) + k_1^1\left(k_2^3 + k_c^3\right)\right)$$
$$+P_3\left(k_1^2\left(k_2^1 + k_c^1\right) + k_1^1\left(k_2^2 + k_c^2\right)\right), \quad (44)$$

$$K_5 = k_1^3\left(k_2^1 + k_c^1\right)\left(k_2^2 + k_c^2\right) + k_1^2\left(k_2^1 + k_c^1\right)\left(k_2^3 + k_c^3\right) + k_1^1\left(k_2^2 + k_c^2\right)\left(k_2^3 + k_c^3\right)$$
$$+P_1 k_1^2 k_1^3 + P_2 k_1^1 k_1^3 + P_3 k_2^1 k_1^2, \quad (45)$$

$$K_6 = k_1^1 k_1^3\left(k_2^2 + k_c^2\right) + k_1^2 k_1^3\left(k_2^1 + k_c^1\right) + k_1^1 k_1^2\left(k_2^3 + k_c^3\right), \quad (46)$$

$$K_7 = k_1^1 k_1^3 k_1^2, \quad (47)$$

$$N_1 = P_1 P_3, \quad (48)$$

$$N_2 = P_1\left(k_2^3 + k_c^3\right) + P_3\left(k_2^1 + k_c^1\right), \quad (49)$$

$$N_3 = \left(k_2^1 + k_c^1\right)\left(k_2^3 + k_c^3\right) + P_1 k_1^3 + P_3 k_1^1, \quad (50)$$

$$N_4 = k_1^3\left(k_2^1 + k_c^1\right) + k_1^1\left(k_2^3 + k_c^3\right), \quad (51)$$

$$N_5 = k_1^1 k_1^3 \quad (52)$$

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4. Conclusions

Thus, the calculations have produced analytical expressions for finding the amplitude and frequency of the desired periodic solution.

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