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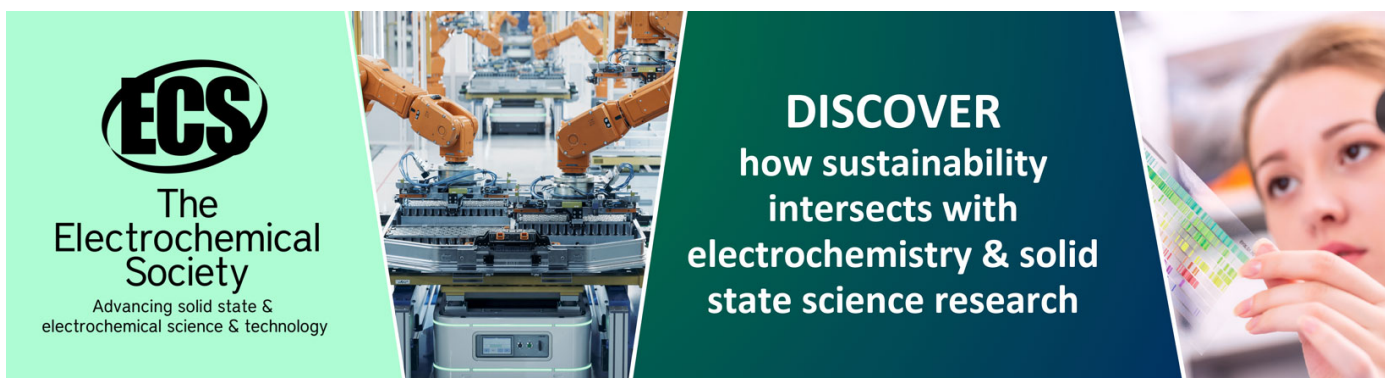
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Study of the frequency response of the block–rock mass with bimodulus characteristics

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Abstract. The pendulum-type wave phenomenon is a unique mechanical phenomenon in deep rock mass. It propagates at low frequencies and low speeds in the rock mass, and can cause deep rock engineering disasters such as rockbursts. For determining the low-frequency characteristics of a pendulum-type wave, the frequency response of the pendulum-type wave in block-rock mass with bimodulus characteristics is studied under a half-sine impact load. The dynamic pendulum-type wave model in a block–rock mass is used to determine the low frequency characteristics of the wave. The improved central difference method is applied, and the influence of the modulus ratio on frequency response is analyzed. Furthermore, the natural frequency of the system is inverted, through the frequency response of the block, and its correctness is verified. The results show that a decrease in the modulus ratio causes an obvious increase in the number of frequency response peaks of the block. The amplitude of the frequency response peaks decreases and the position of peaks moves to a low frequency range. The low-frequency characteristics of the pendulum-type wave in the block–rock mass with bimodulus characteristics are more obvious. The natural frequency of the system can be inverted by the fact that it corresponds to the peak frequencies of the frequency response. This approach provides a new method for the determination and measurement of the natural frequency of nonlinear systems.

1. Introduction

The recent rapid development in the world economy has caused an increase in the depth of underground mining because of increasing demand for energy and natural resources. Consequently, the use of traditional continuum mechanics is becoming increasingly unsuitable for deep mining applications. Deep rock engineering disasters such as rockbursts occur frequently, seriously threatening the safety of human life and mining operations. The pendulum-type wave phenomenon reflects the incompatible and discontinuous dynamic characteristics of rock masses. It is one of the critical mechanical phenomena reported in deep rock mass engineering and is of great significance to the prevention and control of rockbursts and other engineering disasters.

Kurlenya [1-3] observed the phenomenon of the sign-alternating reaction in rock mass for the first time and named it a “pendulum-type” wave. Aleksandrova [4-6] proposed a propagation model of the pendulum-type wave in rock masses with a viscoelastic interlayer. They performed theoretical and experimental research on the propagation law of pendulum-type waves. Saraikin [7-9] extended the



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model to two dimensions. In China, Qian et al. [10-12] introduced and investigated deep rock mechanics phenomena. They conducted a series of theoretical and experimental studies on the pendulum-type wave phenomenon. Pan and Wang [13] and Wang et al. [14,15] theoretically studied the propagation and frequency response of the pendulum-type wave in the block-rock mass.

Researchers have, over the years, conducted a series of studies on the pendulum-type wave, but their characteristics are not fully understood. Previous studies have focused on the dynamic response of the block-rock mass in the time domain. For the pendulum-type wave, the most significant characteristics that distinguish it from the classical elastic wave are its low-frequency and low-speed properties. Therefore, this study focuses on the study of low-frequency characteristics of the pendulum-type wave. Moreover, many engineering materials, such as graphite, reinforced composite materials, metal alloys, ceramics, glass, and cast iron, have bimodulus characteristics (i.e., different tensile and compression elastic moduli). For structures such as beam structures and elastic planes, their bimodulus characteristics have been studied [16]. The block-rock mass model is a simplified representation of rock blocks and the interlayer between them with viscoelastic properties. In previous studies, the interlayer is replaced by a damper and a linear spring. This simplified mechanical model does not account for the heterogeneity and bimodulus characteristics of the rock mass. Therefore, in this study, the bimodulus characteristics of the rock mass are considered. Specifically, the interlayer has a different stiffness under tension than under compression. The modulus ratio a is defined as the ratio of the tension to the compression stiffness. Based on the time-domain dynamic response behavior of the block-rock mass, the influence of the modulus ratio on the frequency response of the pendulum-type wave in the block system under impact loading is studied. Moreover, the natural frequency of the system is inverted from the frequency response of the block-rock mass.

2. Propagation model and calculation method of the pendulum-type wave

2.1. Mechanical model of the pendulum-type wave

Figure 1 shows the dynamic model [4] of the pendulum-type wave in the block-rock mass model used in this study. The model comprises rock blocks and interlayers that have viscoelastic properties. The interlayer is replaced by a damper and a nonlinear spring. The right end of the system is fixed. In the figure, the mass of the i -th rock block is m_i , and its displacement is x_i . The spring stiffness of the i -th interlayer is k_i , while its damping coefficient is c_i . The applied external load is $F(t)$.

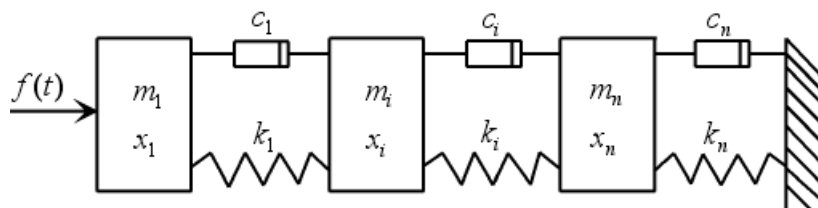


Figure 1. Dynamic model of the pendulum-type wave in the block-rock mass

In matrix form, the motion equation of the block-rock mass has the following form:

$$M\ddot{X}(t) + C\dot{X}(t) + KX(t) = F(t) \quad (1)$$

where $M = \begin{bmatrix} m_1 & & \\ & m_2 & \\ & & \ddots \\ & & & m_n \end{bmatrix}$,

$$C = \begin{bmatrix} c_1 & -c_1 & & & & \\ -c_1 & (c_1 + c_2) & -c_2 & & & \\ & \ddots & \ddots & \ddots & & \\ & & -c_{i-1} & (c_{i-1} + c_i) & -c_i & \\ & & & \ddots & \ddots & \ddots \\ & & & & -c_{n-1} & (c_{n-1} + c_n) \\ & & & & -c_{n-1} & (c_{n-1} + c_n) \end{bmatrix},$$

$$K = \begin{bmatrix} k_1 & -k_1 & & & & \\ -k_1 & (k_1 + k_2) & -k_2 & & & \\ & \ddots & \ddots & \ddots & & \\ & & -k_{i-1} & (k_{i-1} + k_i) & -k_i & \\ & & & \ddots & \ddots & \ddots \\ & & & & -k_{n-1} & (k_{n-1} + k_n) \\ & & & & -k_{n-1} & (k_{n-1} + k_n) \end{bmatrix}.$$

The matrix M is the mass matrix, C is the damping matrix, K is the stiffness matrix, calculated according to the tension and compression state of each spring; $X(t)=[x_1, x_2, \dots, x_n]^T$ is the displacement vector of each block in the block-rock mass; and $F(t)=[F_1(t), F_2(t), \dots, F_n(t)]^T$ is the vector of the external load.

2.2. Calculation method

The improved central difference method [17] is used to solve the matrix differential equation of the block-rock mass with bimodulus characteristics (i.e., equation (1)). To obtain the dynamic response of the block-rock mass in the time-domain, the frequency response of the block-rock mass is obtained using Fourier transforms. The formula of the Fourier transform is given by:

$$F_i(\omega) = \int_{-\infty}^{+\infty} \ddot{x}_i(t) \cdot e^{-j\omega t} dt \quad (2)$$

The classical central difference method can only be used to solve the matrix differential equation with fixed parameters. However, the improved central difference method can be used to solve the matrix differential equation with parameters changing with time. It is applicable to identifying the solution of complex dynamic equations of nonlinear systems. In this study, the recurrence formula of the improved central difference method is as follows:

$$\left(\frac{M}{\Delta t^2} + \frac{C}{2\Delta t}\right)X_{i+1} = F(t_i) - \left(K - \frac{2M}{\Delta t^2}\right)X_i - \left(\frac{M}{\Delta t^2} - \frac{C}{2\Delta t}\right)X_{i-1} \quad (3)$$

$$\dot{X}(t_i) = \frac{1}{2\Delta t}(X_{i+1} - X_{i-1}) \quad (4)$$

$$\ddot{X}(t_i) = \frac{1}{\Delta t^2}(X_{i+1} - 2X_i + X_{i-1}) \quad (5)$$

The stiffness matrix K is continuously changing with time when solving the dynamic equation. The stiffness matrix K in each time step corresponds to current stress state at that moment. For example, when solving for the displacement of the block at time t_{i+1} , it is necessary to determine the tension and compression state of all the springs at time t_i so as to select the corresponding spring stiffness to form the stiffness matrix K_i in every time step.

The central difference method is conditionally stable. Hence, it is necessary to check the stability of the improved central difference method. The stability condition is as follows:

$$\frac{\Delta t}{T} \leq \frac{1}{\pi} \approx 0.318$$

where Δt is the time step selected during the calculation, and T is the natural vibration period of the system. To achieve sufficient accuracy, the condition $\Delta t / T \leq 0.1$ is often taken as a stable condition for the central difference method in general calculations.

3. The frequency response of the block–rock mass

3.1. Calculation parameters

In this study, k_t is the spring stiffness coefficient under tension and k_c is the spring stiffness coefficient under compression. The modulus ratio a is defined as the ratio of tension stiffness to compression stiffness, i.e., $a = k_t / k_c$. According to the test parameters of Aleksandrova [5], the original calculation parameters are: $n = 10$, $m_i = 1\text{kg}$, $k_c = 2 \times 10^5 \text{ kg/s}^2$, $k_t = a \times k_c$, $c_i = 20\text{kg/s}$, and $i = 1, 2, 3, \dots, 10$. The initial velocity and the initial displacement are both zero. The time step for the calculation is $\Delta t = 0.0005$.

3.2. Selection of impact load

A half-sine impact load is applied to the block–rock mass with bimodulus characteristics. Figure 2 shows the force–time curve recorded by the sensor when the vertical impact load is applied to the model by a vertical vibrator [18]. The force–time curve is very close to the half-sine impact load; therefore, it is reasonable to use the half-sine impact load.

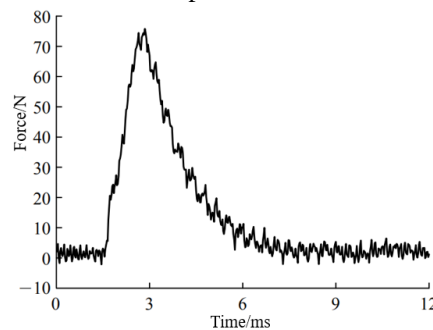


Figure 2. Force–time curve recorded by the sensor

Similar to engineering structures, dynamic resonance occurs in a block rock mass. To eliminate the influence of resonance on the frequency response of the block–rock mass, its natural frequencies should first be obtained, after which the appropriate frequency of the half-sine impact load can be determined. The complex modal analysis method was used to solve the matrix differential equation, and ten complex frequencies of the block–rock mass were obtained as follows: $0.22 \pm 66.84i$, $1.98 \pm 199.02i$, $5.34 \pm 326.73i$, $10.00 \pm 447.10i$, $15.55 \pm 557.45i$, $21.49 \pm 655.31i$, $27.31 \pm 738.51i$, $32.47 \pm 805.20i$, $36.52 \pm 853.91i$, and $39.11 \pm 883.57i$. To prevent the occurrence of low-frequency resonance, a half-sine wave $F(t)$ with frequency $\omega = 100$ is selected to simulate the external impact load:

$$F(t) = \begin{cases} 100 \cdot \sin(100t) & 0 < t \leq \pi/100 \\ 0 & \pi/100 < t \end{cases}$$

3.3. Checking the stability

In this section, the stability of the improved central difference method is verified. With a reduction in spring stiffness, the natural frequencies of the system decrease. Hence, the natural vibration period of the system increases and the ratio $\Delta t / T$ decreases. Therefore, it can be verified if the stability meets the requirements when the system stiffness is at its maximum. When the tension and compression stiffness are equal, the stiffness coefficient of the system is at a maximum, and the calculated ratios $\Delta t / T$ this time are 0.070, 0.068, 0.064, 0.059, 0.052, 0.044, 0.036, 0.026, 0.016, and 0.005. Thus, the ratio satisfies the stability condition of Section 2.2.

3.4. The frequency response

Because of damping, the amplitude of dynamic response of the block–rock mass is relatively large in a short time period in the beginning, and then gradually attenuates. Therefore, the most critical period of the system in the short time period after the application of the impact load. In this section, the calculation time of frequency response is selected as 1 s, which reflects the frequency response of the block–rock mass in a short time period after the application of impact load:

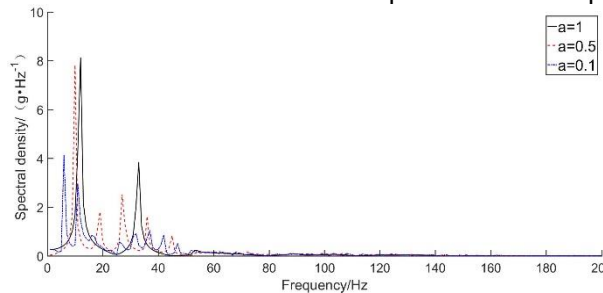


Figure 3. Frequency response of block 2

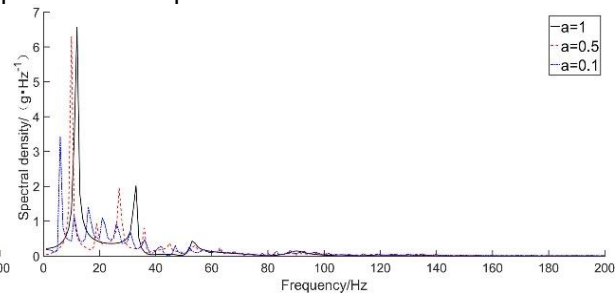


Figure 4. Frequency response of block 5

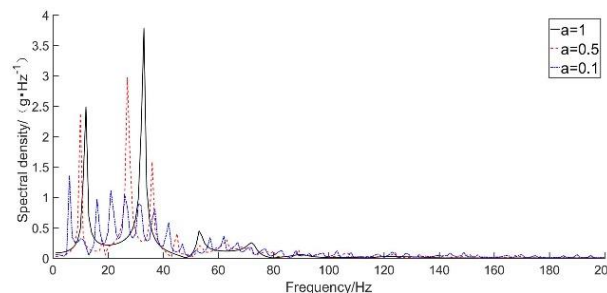


Figure 5. Frequency response of block 9

The frequency response curves of blocks 2, 5, and 9 are presented in Figure 3, 4, and 5. These figures show that impact loading of the block–rock mass causes the modulus ratio to decrease as the number of frequency response peaks in the block–rock mass increase. Moreover, the amplitudes of the peaks of the frequency response decrease and the position of the peaks moves to a low frequency range. The low-frequency characteristics of the pendulum-type wave in the block–rock mass with bimodulus characteristics are obvious.

As the propagation distance of the pendulum-type wave increases, the spectral density gradually decreases, and the low-frequency components are more significant. However, close to the fixed end, the high-frequency content increases because of the reflection of the wave at the fixed end.

3.5. Test verification

The numerical solution demonstrates that the number of frequency response peaks in the block–rock mass with bimodulus characteristics increases. Figure 6 shows the results of experiments performed by Jie et al. [18] for the frequency response in a 1D block–rock mass. The number of blocks in the test was six. According to the traditional model and calculation method, the number of peaks of the frequency response should be six; however, the number of peaks obtained by the experiment is much more than six. The experimentally observed phenomena are consistent with the solution of the numerical results, which further explain the rationality and necessity of considering bimodulus characteristics of the block–rock mass.

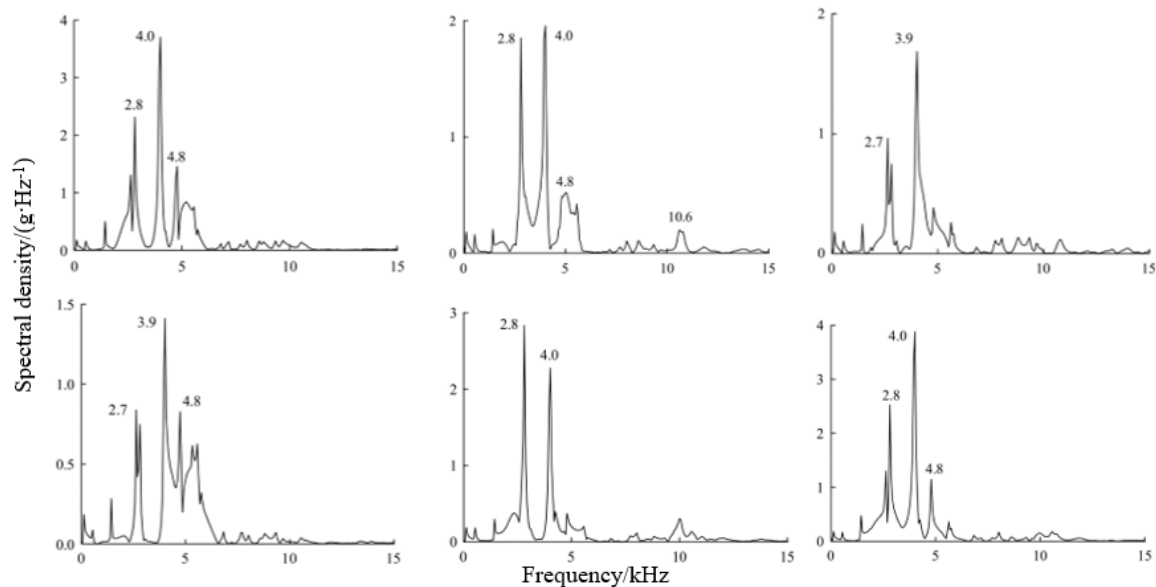


Figure 6. Experimental results of frequency response of one-dimensional block-rock mass [18]

4. Inversion of natural frequency of system

In this section, we study the relationship between the peak frequencies of the frequency response and the natural frequency of the system. The natural frequency of the system is further inverted by the frequency response. We note that this method has been verified by the calculation results.

4.1. Definitions

Here, we will distinguish several commonly used concepts.

The natural frequencies λ of the system are the natural circular frequencies of vibration with the unit in rad/s . The natural frequencies are only related to the properties of the system itself. Resonance occurs when the external load frequencies are close to the natural frequencies. Corresponding to the natural circular frequencies λ of vibration are the natural cyclic frequencies f_n of vibration with units in Hz . Here, $f_n = 1/T = \lambda / (2\pi)$. The frequencies corresponding to the peaks of the frequency response of the block are called the peak frequencies F_H .

4.2. Study on the inversion of natural frequencies

The calculation time of the block's frequency response selected in this section is 20 s. The frequency response curves for block 2, block 5, and block 9 are shown in Figure 7, 8, and 9, respectively. Figure 10 is a supplementary curve of the frequency response of block 2 in the range 50—100 Hz.

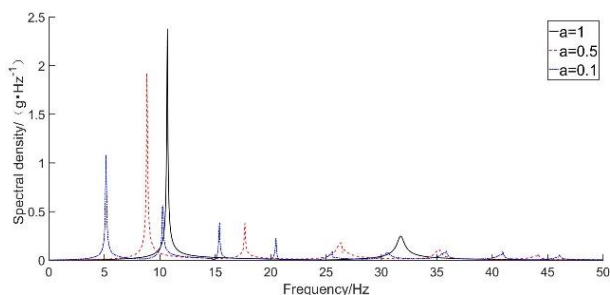


Figure 7. Frequency response of block 2

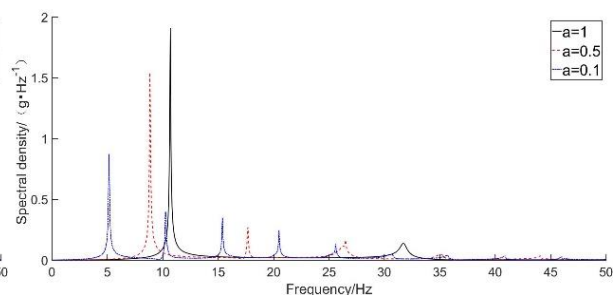
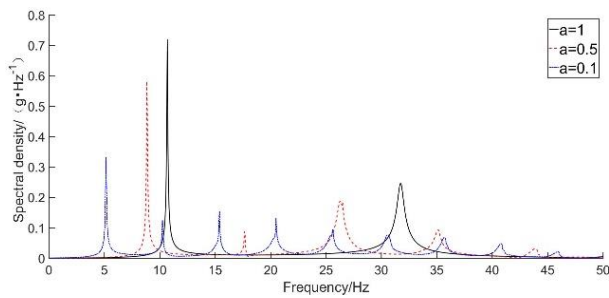
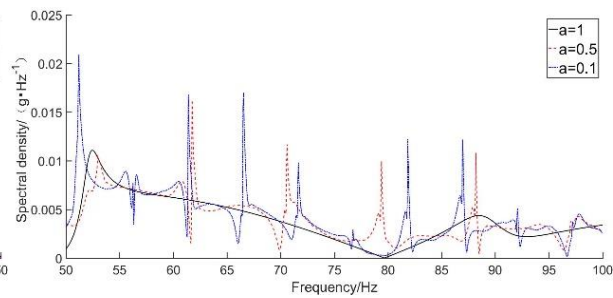


Figure 8. Frequency response of block 9

**Figure 8.** Frequency response of block 5**Figure 10.** Supplementary curve of the frequency response of block 2 in the range of 50–100 Hz

The above figures show that, the frequency responses of the different blocks are almost the same, and the peak frequency f_H of the curves are similar. From the ten natural frequencies of the block–rock mass obtained in Section 3.2, the cyclic frequencies f_n of vibration of the block–rock mass were determined as 10.64, 31.67, 52.00, 71.16, 88.72, 104.30, 117.54, 128.15, 135.90, and 140.62. By comparing the curves of the modulus ratio, $a = 1$ in Figure 7, 8, and 9, it can be seen that the calculated natural cyclic frequencies f_n of vibration are similar to the peak frequencies f_H in the frequency response curve. Moreover, the natural frequencies of the system are in correspondence with the peak frequencies of the frequency response. This is an indication that there is a correspondence between the natural frequencies λ and the peak frequencies f_H of the system peak frequency response, i.e., $\lambda = 2\pi \cdot f_H$.

For the block–rock mass with bimodulus characteristics and other nonlinear structures, the solution and measurement of the natural frequency of the system are very complex, which is difficult to generalize and apply. Drawing inspiration from the above research, the natural frequency can be inverted through the frequency response of the system as per the correspondence between the natural frequencies of the system and the peak frequencies.

4.3. Verification

To confirm the rationality and correctness of this method, the peak frequencies f_H of the frequency response of the blocks were extracted when the modulus ratio a is 0.5 and 0.1. The natural frequencies λ of the block–rock mass were then inverted. Furthermore, the sinusoidal load corresponding to the natural frequency λ is selected to load the block–rock mass with bimodulus characteristics to confirm whether the resonance phenomenon of the block–rock mass occurs.

When the external load frequencies are close to the low-order natural frequencies, the resonance phenomenon of the system is more obvious and harmful. Therefore, we were more concerned about the low-order natural frequency for the practical engineering. Taking the inversion of the previous third-order natural frequency as an example, Table.1 shows the peak frequencies and the natural frequencies obtained by inversion when the modulus ratio a is 0.5 and 0.1 respectively.

Table 1. The peak frequency f_H and the natural frequency λ obtained by inversion

		Block 2		Block 5		Block 9		Average	
		f_H	λ	f_H	λ	f_H	λ	f_H	λ
$a = 0.5$	First-order	8.85	55.61	8.85	55.61	8.85	55.61	8.85	55.61
	Second-order	17.70	111.21	17.70	111.21	17.65	110.90	17.68	111.11
	Third-order	26.35	165.56	26.25	164.93	26.30	165.25	26.30	165.25
$a = 0.1$	First-order	5.15	32.36	5.15	32.36	5.15	32.36	5.15	32.36

0.1	Second-order	10.30	64.72	10.25	64.40	10.25	64.40	10.27	64.51
	Third-order	15.40	96.76	15.40	96.76	15.40	96.76	15.40	96.76

Table 1 shows that the error of the previous third-order natural frequency obtained by inversion is very small, and this method has very good stability. To make the resonance phenomenon more obvious, the frequency of the applied sinusoidal load is the corresponding first-order natural frequency. The sinusoidal load $f(t)$ has the form: $f(t) = 10 \cdot \sin(\omega t)$. Here, when $a = 0.5$, $\omega = 55.61$, whereas when $a = 0.1$, $\omega = 32.36$. The following figures show the displacement curves of block 2 when $a = 0.5$ and $a = 0.1$.

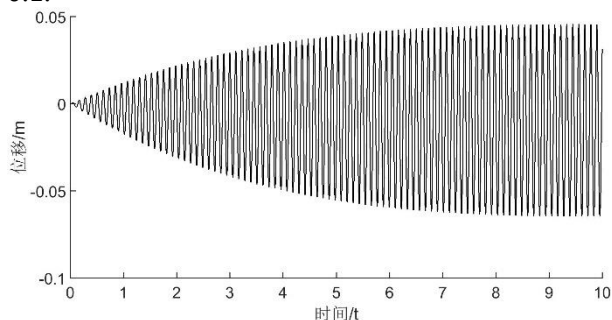


Figure 11. Displacement curve of block 2 when $a = 0.5$

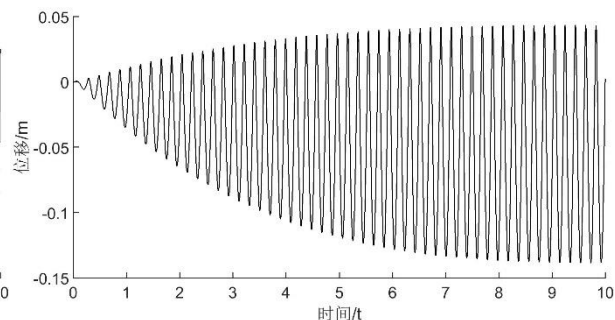


Figure 12. Displacement curve of block 2 when $a = 0.1$

It can be seen from the displacement curves in Figure 11 and 12 that resonance has occurred in the block–rock mass with different modulus ratios. The block’s displacement increases and gradually tends to a constant value. This proves the validity of the inversion. Furthermore, the natural frequency of nonlinear structures can be inverted by the frequency response. This provides a convenient method for quickly obtaining the natural frequency of nonlinear structures.

5. Conclusion

- (1) A decrease in the modulus ratio causes the number of frequency response peaks in the block–rock mass to clearly increase, which agrees with experimental observations.
- (2) Decreasing the modulus ratio causes the amplitude of the frequency response peaks to decrease and the position of the peaks moves toward a low frequency range. The low-frequency characteristics of the pendulum-type wave in the block–rock mass with bimodulus characteristics are more obvious.
- (3) There is a correspondence between the natural frequencies λ of the system and the peak frequencies f_H of the frequency response. Therefore, the natural frequencies of nonlinear systems can be inverted by the frequency response. This approach provides a new method for the determination and measurement of the natural frequency of nonlinear systems.

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