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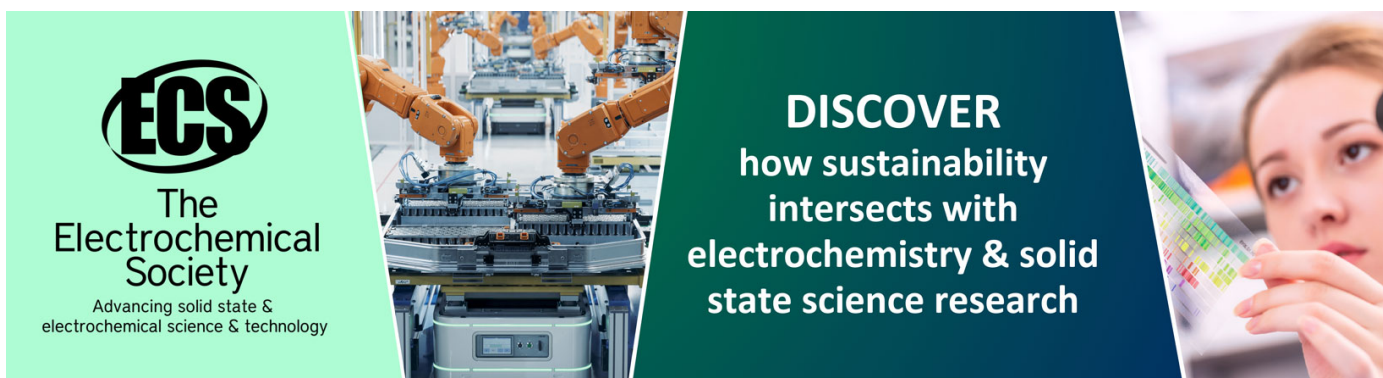
A proposed method for mathematical quantitative description of fractures from borehole wall images

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A proposed method for mathematical quantitative description of fractures from borehole wall images

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Abstract. Fractures from borehole wall images are different from those from surface outcrop in trace pattern, distribution pattern, parameters description and so on. Focus on the geometric parameters of fractures from borehole images, a method for mathematical quantitative description of these fractures is proposed. The fracture from borehole images is described into a point in spatial coordinate system, and this point is defined Fracture Feature Point. The key parameters including dip direction, dip angle and position depth of fractures from borehole wall images are converted into coordinates of fractures feature points. The related calculation equations are deduced in local coordinate system and spatial coordinate system. Furtherly, the space distribution regularities of the fractures based on the distribution characteristics of fracture feature points are summarized. Typical application in multiple boreholes of this description method is studied and an analysis method for any two fractures' connectivity is established based on Fracture Feature Point. The results of an engineering example research show that the method is effective in fractures connectivity analysis. The mathematical quantitative description method of fractures from borehole wall images will provide new ideas of the research on deep rock mass structure.

1. Introduction

The fractures of different sizes, complex shapes and intricate distributions exist in the rock mass which intersect each other to form the rock mass structure. The rock mass structure controlled by fractures determines the mechanical properties of the rock mass such as deformation, strength, seepage and so on. It is indispensable to first study the geometric and spatial distribution characteristics of fractures when dealing with rock mass engineering problems. However, only a small part of the fractures expose in the rock mass surface or artificial excavation surface, a large number of fractures are distributed inside the rock mass and are difficult to observe directly ^[1-5].

Drilling is a necessary means to explore the internal structure of rock mass. Based on the drilled borehole, borehole camera technology provides a new and effective method to obtain the information of the internal fractures in rock mass. By this technology, we can observe rock mass in borehole wall and obtain 360° borehole wall panoramic image (borehole wall image) ^[6-7]. The fracture geometric trace can be acquired on the borehole image based on the recognition technology. The geometric parameters such as orientation, aperture and depth can be calculated from borehole image which provide the basic data for study of the internal fractures in rock mass. However, the fractures in borehole image are different from fractures in outcrop in the form of expression: (1) the fracture trace obtained from the borehole image is a sinusoid, and the fracture trace obtained from the outcrop is



usually a line; (2) The fracture in borehole image contains the depth of the drilling position, and fracture in outcrop often ignores the position information. Due to the above differences, the conventional analytical method cannot be directly applied to the fractures from borehole images. Therefore, it is necessary to carry out special research on fractures in borehole images and many scholars have carried out related research. Waleed ^[8] and Wang ^[9] studied automatic recognition and parameters extraction of fractures in borehole image based on image processing. Huang ^[10] and Han ^[11] studied calculation method of geometric parameters of fractures in borehole images and realized mathematical description. Wang ^[12] studied on fractures connectivity analysis method based on adjacent boreholes images. Mauldon ^[13], Martel ^[14] and Wang ^[15] used statistical and stereological methods to study the spatial distribution characteristics of fractures in borehole image. In 1978, the International Society of Rock Mechanics (ISRM) proposed a suggested method for fracture description ^[16], which used 11 parameters including the occurrence, aperture and spacing etc. to describe the fracture. Considering the three-dimensional spatial distribution characteristics of fractures, an analytical approach of fractures spatial distribution from borehole wall images is proposed in this paper. A new representation method of fractures from borehole wall images is put forward which can comprehensively analyze the parameters such as dip direction, dip and depth. The study in this paper will provide new ideas and methods to the analysis of fractures, especially fractures from borehole wall image.

2. Fractures in borehole wall images

According to Suggested Methods for the Quantitative Description of Discontinuities in Rock Masses by ISRM, fractures can be described qualitatively. Orientation of the fracture refers to the state of the fracture in three-dimensional space, usually expressed by the strike, dip and dip direction as shown in Fig. 1. The strike refers to the direction indicated by the two ends of the intersection of fracture and the horizontal plane. The dip direction refers to the direction indicated by the projection line of the straight line drawn downward along the plane along the strike line on the horizontal plane. The dip refers to the angle between the inclined line and its projection line on the horizontal plane. The difference between the strike and the dip direction is 90° . The parameters of dip direction α and the dip β are usually used to describe the fracture orientation.

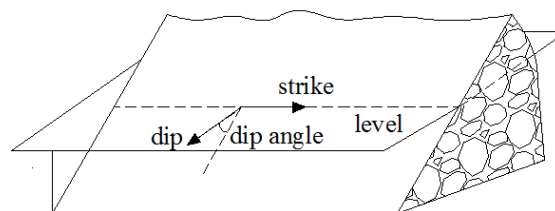


Figure 1. Diagram indicating the strike, dip and dip direction of fracture

Borehole image usually refers to the panoramic optical image of borehole wall obtained by borehole camera technology. It is a 360° borehole wall panoramic image as shown in Fig.2. The geographic orientation information is described directly in the top of the borehole image, and the depth information is described on the left side of the borehole image. According to the definition of orientation and virtual core image, the line connecting the highest point and the lowest point of the fracture trace is the inclined line, and the direction indicated by the projection AB of inclined line on the horizontal plane is the dip direction α . In borehole wall expanded image, the trace of fracture is a sine curve and the direction indicated by AB is the orientation Q corresponding to the lowest point of the sine curve, as follows:

$$\alpha = Q \quad (1)$$

The dip β is expressed by the vertical distance h between the highest point and the lowest point in the sine curve and the diameter d of the borehole, as follows:

$$\beta = \arctan \frac{h}{d} \quad (2)$$

The position depth is defined as the depth of middle position of sine curve, and can be expressed by the intermediate value between the highest point depth and the lowest point depth of the fracture curve in the expanded image.

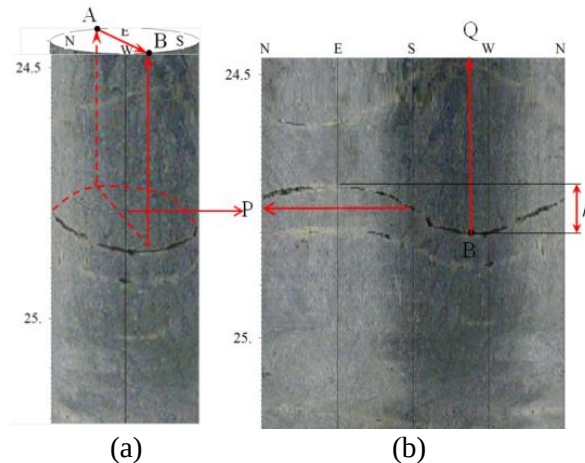


Figure 2. Parameters of fracture can be extracted in borehole image which included two forms: (a) virtual core image; (b) expanded image

3. Fracture Feature Point

The fracture is a space concept. When studying and dealing with rock engineering problems, the spatial distribution characteristics of the rock mass fracture should be studied in depth. The spatial features of the fracture are usually described by geometric parameters such as dip, dip angle, spacing, and gap width. Considering that the fracture is a comprehensive reflection of various geometric parameters, a fusion analysis method for multi-parameters of fractures is proposed, and the following basic assumptions are made:

- The fracture is an infinitely extended spatial plane.
- The coordinate system adopts the three-dimensional Cartesian coordinate system of the right-hand rule. The X-axis points in the true east direction, the Y-axis points in the true north direction, and the Z-axis points vertically upward.

Any fracture in space is shown in Fig.3. The spatial coordinate system is established according to the basic assumptions. A straight line is drawn perpendicular to the plane of the fracture by the origin of the coordinate system, and the intersection point of the perpendicular line with the fracture (ie, the vertical foot) is called Fracture Feature Point. It can be seen that in the same coordinate system, the feature point have a one-to-one correspondence with the fracture, and the feature point coordinates reflect the spatial position and form of the fracture.

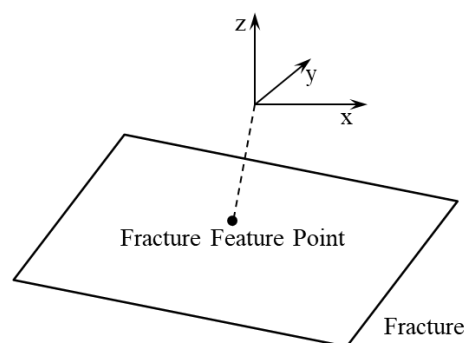


Figure 3. Schematic diagram of Fracture Feature Point

Taking the fracture in a single borehole as the object of analysis, the coordinate system is established by choosing the center point of the borehole as the origin point. The x-axis points in the east direction, the y-axis points in the true north direction, and the z-axis direction is the vertical direction. Each borehole has a corresponding coordinate system called a local coordinate system, as shown in Fig. 4. A vertical line is drawn from the origin of the coordinate system to the fracture, and the intersection point on the fracture is the characteristic point in the local coordinate system.

Assuming that the dip of the fracture is α , the dip angle is β , the position depth in the borehole is H , and the coordinates of the characteristic points of the fracture in the local coordinate system is $P_{LC}(X_{LC}, Y_{LC}, Z_{LC})$. The Equation of the plane of the fracture can be expressed by the feature point coordinates as:

$$X_{LC} \bullet (x - X_{LC}) + Y_{LC} \bullet (y - Y_{LC}) + Z_{LC} \bullet (z - Z_{LC}) = 0 \quad (3)$$

Can also be expressed as:

$$X_{LC} \bullet x + Y_{LC} \bullet y + Z_{LC} \bullet z - (X_{LC}^2 + Y_{LC}^2 + Z_{LC}^2) = 0 \quad (4)$$

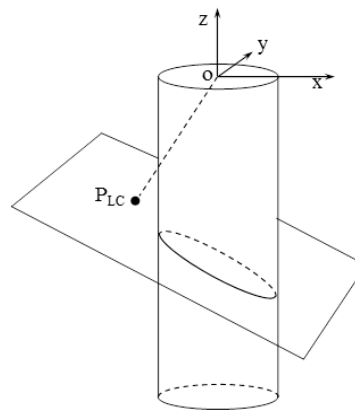


Figure 4. Schematic diagram of Fracture Feature Point in local coordinate system

According to the definition of fracture occurrence and feature points, the following transformation relationship exists between feature point coordinates and fracture dip, dip angle and position depth:

$$\begin{cases} \alpha = \arctan \frac{Y_{LC}}{X_{LC}} \\ \beta = \arctan \left(\frac{\sqrt{X_{LC}^2 + Y_{LC}^2}}{Z_{LC}} \right) \\ H = \frac{X_{LC}^2 + Y_{LC}^2 + Z_{LC}^2}{Z_{LC}} \end{cases} \quad (5)$$

By simplifying, the coordinates of the feature points are obtained as follows:

$$\begin{cases} X_{LC} = H \bullet \cos \beta \bullet \sin \beta \bullet \cos \alpha \\ Y_{LC} = H \bullet \cos \beta \bullet \sin \beta \bullet \sin \alpha \\ Z_{LC} = H \bullet \cos \beta \bullet \cos \beta \end{cases} \quad (6)$$

It can be seen from Equation (6) that the fracture can be represented by parameters such as dip, dip angle and position depth of the fracture in the spatial three-dimensional coordinate system. Each fracture corresponds to a unique feature point, and the geometrical parameters such as the feature point coordinates and the fracture dip, dip angle, and position depth can be mutually transformed.

In the actual drilling and surveying project, multiple boreholes are often arranged in the same field, each borehole corresponds to its own local coordinate system, and it is difficult to conduct a whole study of the fractures in the field within different coordinate systems. Therefore, in the same research area, it is necessary to establish a unified global coordinate system to normalize all fracture feature points.

Select the center of the area or the center of the representative drilling hole as the origin of the coordinate system. The X axis points in the positive direction, the Y axis points in the true north

direction, and the Z axis in the vertical direction. Let the coordinate of the local coordinate system origin O_{LC} in the global coordinate system be $O_{LC}(X_{GLO}, Y_{GLO}, Z_{GLO})$, then the coordinates of the feature point P_{LC} in the local coordinate system in the global coordinate system $(X_{GLC}, Y_{GLC}, Z_{GLC})$ have the following expressions:

$$\begin{cases} X_{GLC} = X_{GLO} + X_{LC} \\ Y_{GLC} = Y_{GLO} + Y_{LC} \\ Z_{GLC} = Z_{GLO} + Z_{LC} \end{cases} \quad (7)$$

Therefore, the Equation of the plane of the fracture in the global coordinate system can be expressed as:

$$(X_{GLO} - X_{GLC}) \cdot (X - X_{GLC}) + (Y_{GLO} - Y_{GLC}) \cdot (Y - Y_{GLC}) + (Z_{GLO} - Z_{GLC}) \cdot (Z - Z_{GLC}) = 0 \quad (8)$$

In the local coordinate system and the global coordinate system, the unit normal vector of the plane where the fracture lies is invariant. The unit normal vector of the plane can be obtained by the Equation (4) as follows:

$$\vec{n} = \left(\frac{X_{LC}}{\sqrt{X_{LC}^2 + Y_{LC}^2 + Z_{LC}^2}}, \frac{Y_{LC}}{\sqrt{X_{LC}^2 + Y_{LC}^2 + Z_{LC}^2}}, \frac{Z_{LC}}{\sqrt{X_{LC}^2 + Y_{LC}^2 + Z_{LC}^2}} \right) \quad (9)$$

A vertical line is drawn from the origin O of the global coordinate system to the plane of the fracture. According to Equation (9), the vertical equation is as follows:

$$\begin{aligned} X &= \frac{t \cdot X_{LC}}{\sqrt{X_{LC}^2 + Y_{LC}^2 + Z_{LC}^2}} \\ Y &= \frac{t \cdot Y_{LC}}{\sqrt{X_{LC}^2 + Y_{LC}^2 + Z_{LC}^2}} \\ Z &= \frac{t \cdot Z_{LC}}{\sqrt{X_{LC}^2 + Y_{LC}^2 + Z_{LC}^2}} \end{aligned} \quad (10)$$

The feature point of the fracture in the global coordinate system is defined as the intersection of the vertical line from the origin of the coordinate system to the plane of the fracture, as shown in Fig. 5, and the coordinates are set to $P_{GC}(X_{GC}, Y_{GC}, Z_{GC})$. According to the Equation of the plane of the fracture and the perpendicular equation, the simultaneous Equation (10) and the Equation (8), the coordinates of the feature points of the fracture in the global coordinate system are:

$$\begin{aligned} X_{GC} &= X_{LC} \cdot \left(1 + \frac{X_{LC} \cdot X_{GLO} + Y_{LC} \cdot Y_{GLO} + Z_{LC} \cdot Z_{GLO}}{X_{LC}^2 + Y_{LC}^2 + Z_{LC}^2} \right) \\ Y_{GC} &= Y_{LC} \cdot \left(1 + \frac{X_{LC} \cdot X_{GLO} + Y_{LC} \cdot Y_{GLO} + Z_{LC} \cdot Z_{GLO}}{X_{LC}^2 + Y_{LC}^2 + Z_{LC}^2} \right) \\ Z_{GC} &= Z_{LC} \cdot \left(1 + \frac{X_{LC} \cdot X_{GLO} + Y_{LC} \cdot Y_{GLO} + Z_{LC} \cdot Z_{GLO}}{X_{LC}^2 + Y_{LC}^2 + Z_{LC}^2} \right) \end{aligned} \quad (11)$$

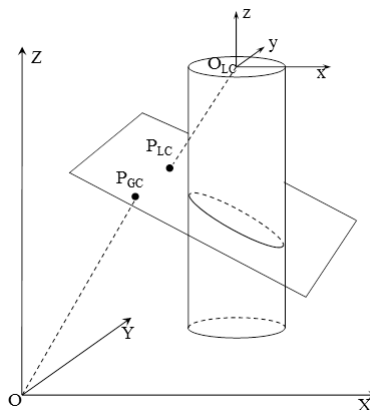


Figure 5. Schematic diagram of Fracture Feature Point in the global coordinate system (P_{LC} is Fracture Feature Point in the local coordinate system; P_{GC} is Fracture Feature Point in global coordinate system)

In the same spatial coordinate system, the fracture corresponds to the coordinates of the feature points one by one, and the distribution of the feature points in the coordinate system also reflects the spatial distribution law of the fracture. Combining with the theory of spatial geometry, the following space distribution regularities of the fractures based on fracture feature points are summarized:

(1) In the global coordinate system, the feature points of the same fracture measured in different boreholes coincide, as shown in Fig.6, indicating two fractures of the same fracture or coplanar. Therefore, the distance between the feature points of the fracture can be used to describe the connected or coplanar features of the fracture.

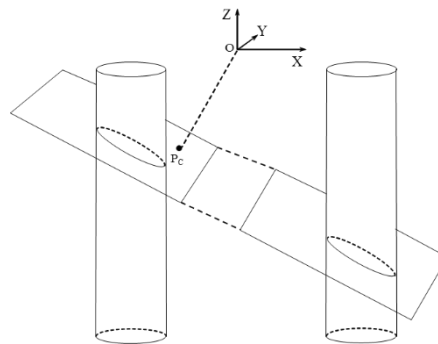


Figure 6. Schematic diagram of fracture feature point coincide

(2) The feature points of fractures P_{c1} , P_{c2} and so on with the same occurrence (i.e. the same dip and dip angle) are collinear and pass through the origin of coordinates, as shown in Fig.7. On this straight line, statistical analysis can be performed to describe the spatial distribution characteristics of the fractures, and it can also be used to divide the statistical homogeneous zones.

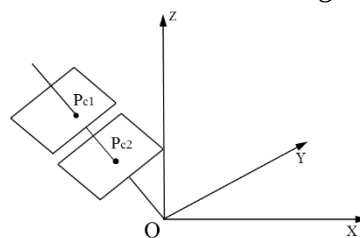


Figure 7. Schematic diagram of fracture feature point collinear

(3) A straight line on any non-XY plane passing through the origin of the coordinate, a conical surface formed by rotating around the Z-axis, and the fracture (plane) described at any fracture feature point thereon has the same dip angle as shown in Figure 8.

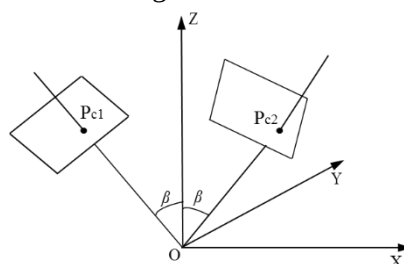


Figure 8. Schematic diagram of fracture feature points distribution with the same dip angles

4. Application in fractures from multiple boreholes

The fracture is a geological interface with a certain extension direction and length. In geological drilling, the same fracture may be penetrated by multiple boreholes. An extremely consistent fracture trace is formed in these images of the borehole wall that is penetrated, and this feature is referred to as the connectivity of the fracture. In the global coordinate system, each fracture corresponds to a unique feature point. According to the spatial distribution law of structural characteristics (1), the connectivity of fractures can be analyzed and studied.

According to the definition and basic law of the feature points of the fracture, if the two fracture feature points in the adjacent boreholes coincide, the fractures corresponding to the two feature points should belong to the same plane. It may be that the same fracture is penetrated by two boreholes at the same time, and then judged according to the structural features of the rock mass near the fracture to further determine whether it is the same fracture, that is, whether it has connectivity characteristics.

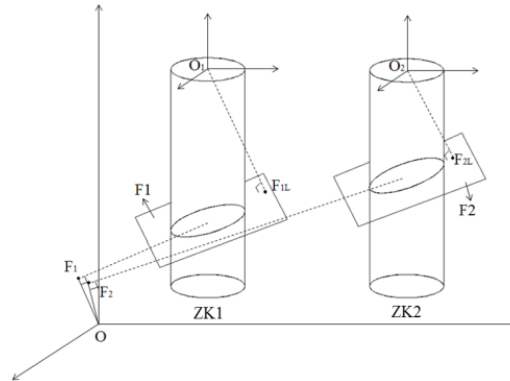


Figure 9. Schematic diagram of two feature points from two boreholes coincide

As shown in Figure 9, there is a fracture F1 in the borehole image of the borehole ZK1, and the feature point coordinates in the global coordinate system are $F_1(X_1, Y_1, Z_1)$. There is a fracture F2 in the borehole image of the borehole ZK2, and the feature point coordinates in the global coordinate system are $F_2(X_2, Y_2, Z_2)$. According to the relationship between the fracture connectivity and the feature points, the distance between the feature points of the two fractures is used to describe the connectivity of the fracture. If the distance between two feature points is zero, that is, the two feature points coincide, it indicates that the two structural faces corresponding thereto have connectivity. In practical engineering, due to measurement errors and complex shape of fracture, there is almost no complete coincidence of the two feature points. Therefore, a minimum value should be set as a criterion for judging connectivity, as shown in Equation (12).

$$D_{F_1F_2} = \sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2 + (Z_1 - Z_2)^2} < \varepsilon \quad (12)$$

$|F_1F_2|$ is the distance between the characteristic points of two fractures. If $|F_1F_2|$ is less than the set minimum ε , it indicates that the two fractures are located on the same plane, that is, they are connected. If $|F_1F_2|$ is larger than the set minimum value ε , it indicates that the two fractures are not on the same plane, that is, they are not connected. And ε can be set according to the analysis accuracy requirements.

After the Equation (12) judges the two-two fractures with connectivity, it is necessary to further judge according to the morphological features of the fractures in the borehole image. It mainly includes the structure plane trace shape, the rock features on both sides of the fracture, the opening degree and the filling condition, etc., to further judge the correctness of the connectivity analysis results.

Summarize the analysis process of fracture connectivity in adjacent boreholes as shown in Figure 10.

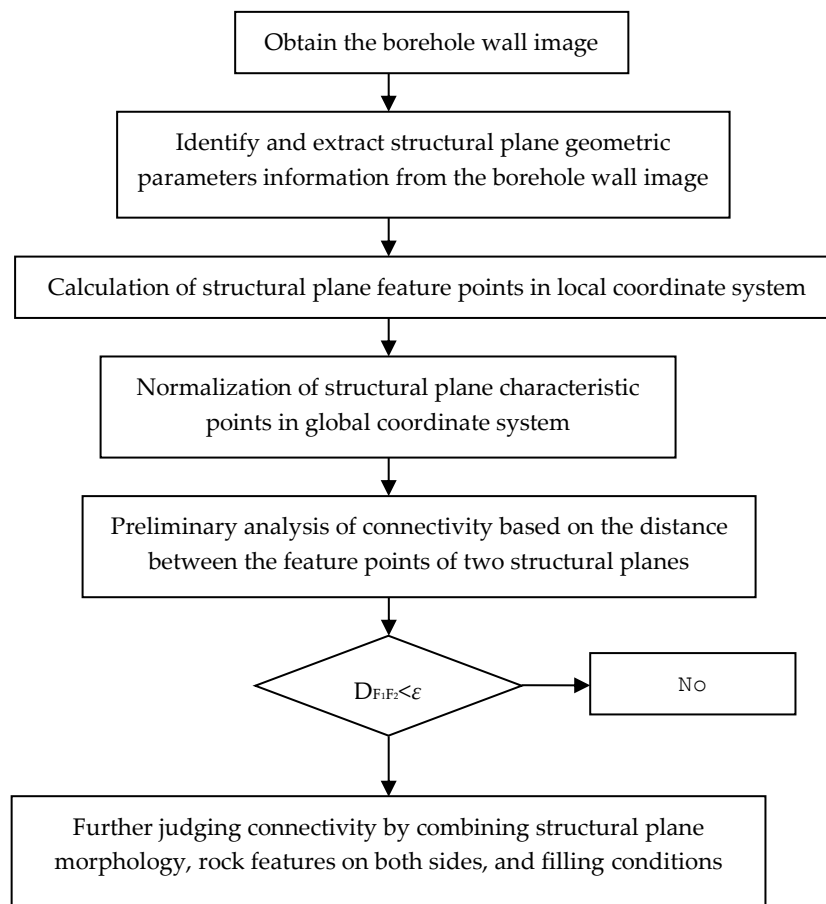


Figure 10. Flow chart of fractures connectivity analysis based on fracture feature points

There are two boreholes 5m apart in the exploration site of dam foundation rock mass of hydropower station. The borehole camera test is carried out in the two boreholes respectively, and the borehole image and the structural data of the bore wall are obtained. Among them, 22 fractures are collected in the borehole ZK1, and 25 fractures are collected in the borehole ZK2. The fractures in the two boreholes are analyzed by the fracture connectivity analysis method based on the feature points. As shown in Fig. 11, the local coordinate system and the global coordinate system are respectively established, and the coordinate points of the feature points corresponding to each fracture are calculated.

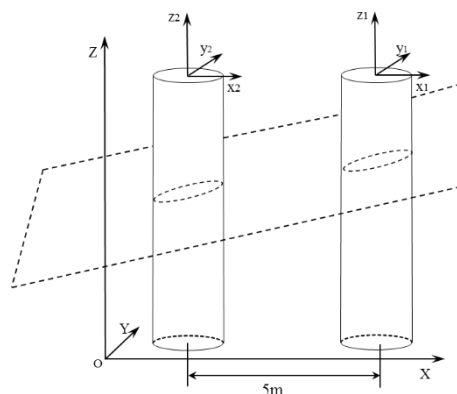


Figure 11. Schematic diagram of the calculation case

According to Equation (12), the fracture feature points collected in the two boreholes are combined in pairs, the distance between the two combined points is calculated, and the connectivity of the fracture is preliminarily judged. In order to avoid missing connective fractures, ε is analyzed according to the standard of 1m. The number of fractures whose distance between feature points is less than 1m is 6 groups. By analyzing the corresponding borehole images, it is found that there are four groups of fractures that are consistent in terms of occurrence and surrounding rock characteristics, as shown in Fig. 12.

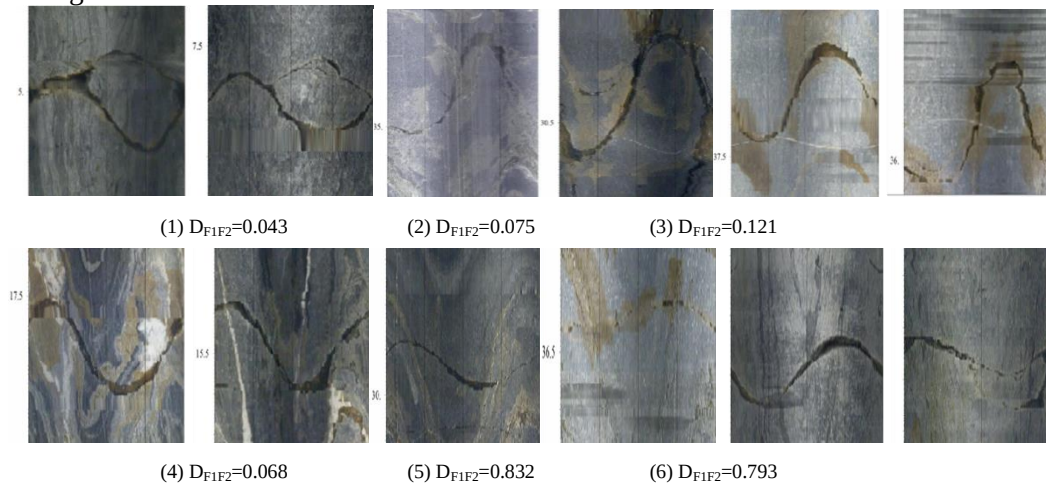


Figure 12. Results of fractures connectivity analysis

The specific calculation process takes the fracture of group (1) as an example. The occurrence of fractures in ZK1 is $NW281.5^\circ \angle 26.3^\circ$ and the coordinates of characteristic points are (-1.874, -2.518, 1.845). The occurrence of fractures in ZK2 is $NW281.5^\circ \angle 26.3^\circ$ and the coordinates of characteristic points are (-1.843, -2.536, 1.822). The distance between characteristic points of two fractures is calculated as follows:

$$D_{F1F2} = \sqrt{(-1.874 + 1.843)^2 + (-2.518 + 2.524)^2 + (1.845 - 1.832)^2} = 0.043$$

According to the connectivity criterion of D_{F1F2} less than 1m, it can be preliminarily judged that the two cracks may be on the same plane. The occurrence of the two fractures is basically the same, with an average width of 7.13mm and 7.26mm respectively. The lithology below the fracture is very similar, and they belong to dark gray rocks. Therefore, it can be judged that the two structural faces have the characteristics of connectivity, and belong to different traces formed by two drilling holes cutting the same fracture, which verifies the feasibility of the method for structural face connectivity analysis.

Different from other fracture connectivity analysis methods based on qualitative judgment, this method combines quantitative calculation with qualitative judgment. Based on the coordinates of the fracture feature points, the judgment criteria are set, and the geometric features and lithological features of the fracture are combined to make judgments, which is easy to program and improve the working efficiency of fracture connectivity analysis.

5. Conclusions

A multi-parameter fusion analysis method for fractures in borehole images is proposed, and the method is applied in fracture connectivity analysis. Based on the difference between the fractures from borehole images and the fractures from surface outcrop, this method synthetically analyses the parameters such as dip, dip angle and position depth, and puts forward the concept of Fracture Feature Point. The multiple parameters of the fracture in the borehole image are merged into the feature point coordinates in the space coordinate system, and the conversion relationship between the feature point coordinates and the geometric parameters of the fracture is established. The equations for calculating

the coordinates of the fracture feature points are carried out in the local coordinate system and the global coordinate system respectively. The coordinate of the fracture feature point is a comprehensive reflection of multiple geometric parameters of the fracture. According to the spatial geometric features of fracture feature points, the distribution law of fractures in spatial coordinate system is summarized, and the correlation between feature point distribution and fracture connectivity is studied. A method for connectivity analysis of fractures from multiple boreholes is proposed based on fracture feature points, and the method is verified by a specific example. The multi-parameters fusion analysis method proposed in this paper provides a new idea and method for analyzing fractures from borehole images. This research will promote the development of borehole camera technology, and contribute to the study of spatial distribution law of underground rock mass fractures.

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Acknowledgments

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