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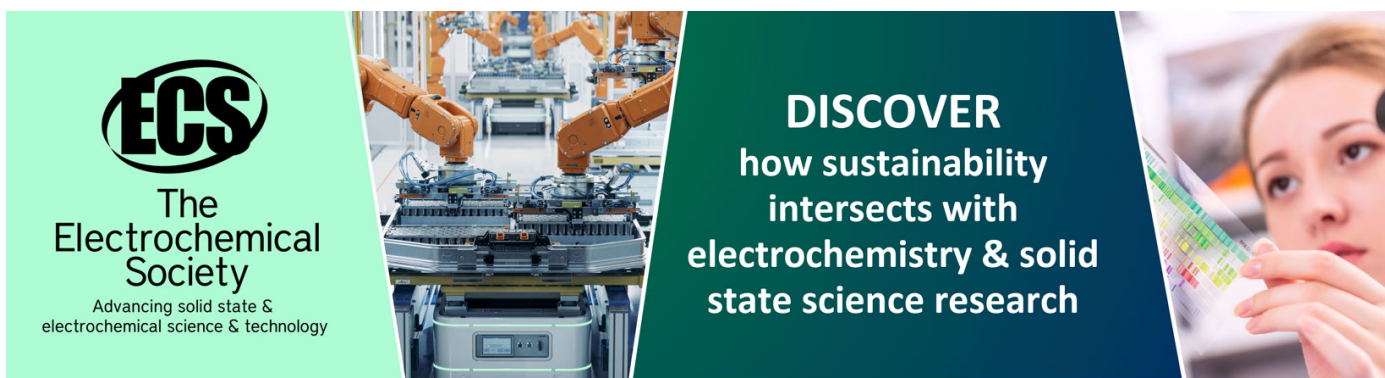
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Static and Dynamic Analysis of Viscoelastic Structure

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Abstract. Generally, the incremental superposition method is used to analyze the stress and deformation of viscoelastic structures. In the calculation, the total time is divided into several intervals. It is assumed that the internal stress remains unchanged in each time interval. The creep deformation generated in this period is taken as the initial strain at the next moment, and the displacement increment is solved according to the elastic problem together with the load, and the total displacement is obtained gradually by superposition. In the study of this method, the integral constitutive relation is used in most cases, and the relaxation modulus is expressed by Prony series. Therefore, the convolution can be expressed by recursion formula, and the memory can be greatly reduced.

1. Introduction

Because of the difficulty of qualitative analysis, a lot of research work is focused on the numerical calculation of viscoelastic problems. In recent years, at home and abroad Many scholars have done a lot of work in solving viscoelastic problems. At present, finite element method, boundary element method and Galerkin method are commonly used to analyze the mechanical properties of viscoelastic structures. The finite element method can be divided into two categories: one is to directly solve the viscoelastic problem in the time domain. Generally, the incremental superposition method is used to analyze the stress and deformation of viscoelastic structure. The displacement increment is solved according to the elastic problem, and the total displacement is obtained by superposition step by step. In the study of this method, most of them adopt integral constitutive relation, and the relaxation modulus is expressed by Prony series, so convolution can be expressed by recurrence formula, and the amount of memory can be greatly reduced[1].

The other is to use the elastic viscoelastic correspondence principle. After the Laplace transformation, problems in time domain can be discussed in phase space [2]. However, there are many difficulties in inverse transformation, so Laplace inverse transformation has become an important research topic. In fact, it is difficult to obtain the inverse transformation of analytical solutions for most problems, so only numerical methods can be used [3]. More researches combine the finite element method with Laplace transformation [4, 5].

The finite element method is also commonly used in viscoelastic dynamic problems. Duan and Hansen [6] proposed a substructure synthesis method in time domain.

When studying the stability, buckling and long-term mechanical behavior of viscoelastic structures, such as chaotic phenomena, if the linear or nonlinear integro partial differential equations are discretized directly by using the finite element method and the boundary element method, a set of



integral ordinary differential equations with very high dimensions will be obtained, which is time-consuming and laborious. Galerkin truncation is often used to study the dynamic behavior of structures, especially the nonlinear dynamic behaviors. Galerkin truncation transforms the integral partial differential equation into integral ordinary differential equation.

2. Innovation points

(1) The solution of the problem is reduced to zero eigenvalue eigensolution and non-zero eigenvalue eigensolution, and the symplectic orthogonal normalization relationship between eigensolutions is extended from phase space to time domain, so that the problem can be directly discussed in the eigensolution space of time domain, and the unnecessary trouble caused by repeated use of Laplace Inverse Transformation is overcome.

(2) The symplectic eigenfunction expansion method is used to solve the nonhomogeneous equation directly, and a numerical method for solving various boundary conditions is constructed by means of variable substitution and symplectic orthogonal normalization.

(3) By discussing the classical problem in the space of zero eigenvalue eigensolution for viscoelasticity, we described the creep and relaxation characteristics of viscoelastic materials, analyzed the local effect problem in the whole eigensolution space, and gave the stress concentration phenomenon caused by the boundary constraints and temperature conditions.

3. Theoretical method

The solution satisfying the homogeneous governing equations and the homogeneous boundary conditions can be obtained as

$$\begin{Bmatrix} \bar{u} \\ \bar{v} \\ \bar{\sigma} \\ \bar{\tau} \end{Bmatrix} = \bar{C} \begin{Bmatrix} \bar{A}_{11} \cos(\mu y) + \bar{A}_{12} y \sin(\mu y) \\ \bar{A}_{21} \sin(\mu y) + \bar{A}_{22} y \cos(\mu y) \\ \bar{A}_{31} \cos(\mu y) + \bar{A}_{32} y \sin(\mu y) \\ \bar{A}_{41} \sin(\mu y) + \bar{A}_{42} y \cos(\mu y) \end{Bmatrix} e^{\mu x} \quad (1)$$

By substituting the eigenvalue solution into the homogeneous side condition, we can get the eigenvalue equation

$$\mu \pm \sin \mu \cos \mu = 0 \quad (2)$$

In this section, we consider the standard entity model shown in Fig. 1. Fig. 2 depicts the arrangement used for this study. It basically represents a Maxwell element connected in parallel with a Kelvin solid.

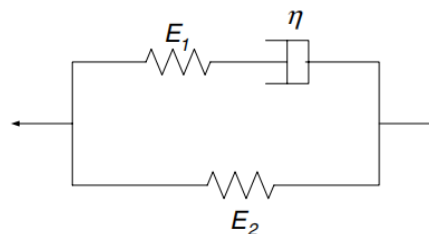


Figure 1. Three-element solid.

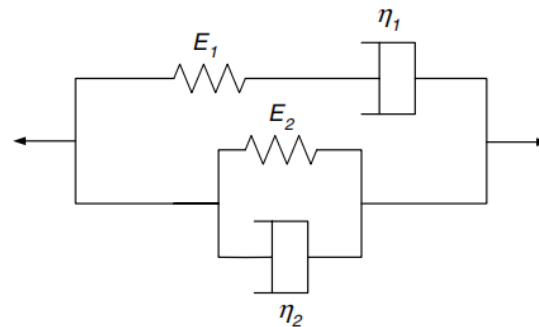


Figure 2. The modified solid model.

4. Stress relaxation

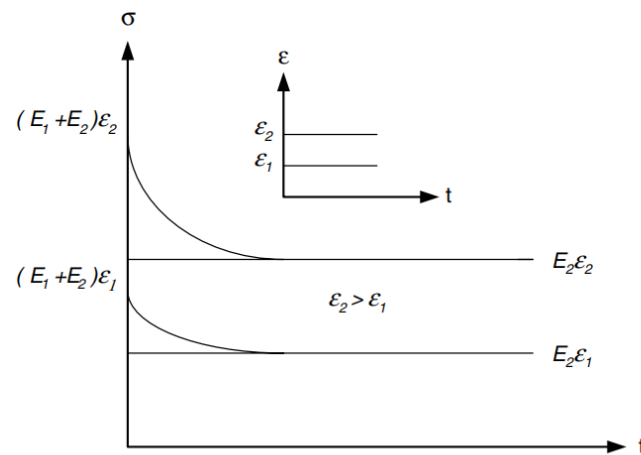


Figure 3. Relaxation response.

Fig. 3 shows the stress relaxation law of the material. However, in order to use differential forms to describe the nonlinear deformation of real polymers, it is necessary to introduce various definitions of stress, stress rate, strain and strain rate.

Acknowledgments

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