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Investigation on vibration characteristics of ship shafting under uncertainty

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Investigation on vibration characteristics of ship shafting under uncertainty

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Abstract. In order to investigate the influence of random uncertainty on the vibration characteristics of the ship shafting system, a nonparametric model that adds the data uncertainty and model uncertainty was constructed, and the system's frequency response function and vibration response were calculated. The results show that the random uncertainty will make the frequency response function and vibration characteristics of the ship shafting system fluctuate. With the increase of random uncertainty, the degree of fluctuation of the system gradually strengthens. The study demonstrates that it is essential to consider the influence of random uncertainty during ship shafting designing and vibration calculation.

1. Introduction

The propulsion system is the core component of the ship's power plant. During the ship's navigation, the propulsion system will be affected by many dynamic nonlinear factors, such as hull deformation, hydrodynamics, dynamic oil film force [1~4]. The influence of these dynamic factors is mostly ignored in the calculation of the former ship propulsion shafting. Nevertheless, recent studies have shown that the impact of these dynamic factors often results in the substantial uncertainty of actual operation state in the propulsion shafting, and it is difficult to establish accurate vibration calculation models [5~8]. Besides, it cannot obtain a uniform description of the model based on a single method. These lead to significant errors between the existing calculation results and the actual ship test results and cannot accurately guide the design and installation of the propulsion shafting.

Uncertainty refers to the fact that the occurrence of an event or result cannot be accurately determined. There is variability and inconsistency inside and outside the system, and ambiguity in the decision-making and designing process. In the study of shafting vibration and alignment calculation considering uncertainty, Zhou [5] studied the influence of hull deformation, stern bearing fulcrum position and oil film stiffness on reasonable alignment of shafting, and developed integrated calculation and analysis software for shafting vibration and alignment; He et al. [9] established a coupling alignment model of the vibration damping system and the shaft system. Tian [10] and Ma et al. [11] established the dynamic model of large ship propulsion shaft system considering the hull deformation excitation and stern bearing elevation from the overall dynamics of the ship shaft system, and studied the influence of dynamic factors on the shaft system vibration.

The above studies indicate that the inaccuracies can be alleviated by considering the influence of dynamic factors in the calculation of shafting vibration. However, the current research only considers

the uncertainty in the process of data input or model establishment, and cannot yet incorporate data uncertainty and model uncertainty into the dynamic model. In this way, the unified description of the model cannot be obtained based on a single method. Therefore, it is also necessary to find new methods to solve the uncertainties in the research of ship shafting vibration modeling.

The nonparametric modeling proposed by Soize [12] is an emerging method for studying the effects of uncertainty in dynamic systems and can carry out the calculations and simulations without identifying the type and number of uncertainties. Therefore, it is appropriate to introduce the nonparametric modeling into the study of ship shafting uncertainty.

Based on this, in this paper, the non-parametric modeling method was introduced and the self-built ship shaft testbed was taken as the research object to establish a non-parametric dynamic model of the shaft system that consider both data uncertainty and model uncertainty. By analyzing the nonlinear characteristic of shafting output vibration response, the influence of uncertainty on shafting operation and vibration is revealed.

2. Nonparametric dynamics model of ship shafting

2.1. Customary dynamics model

In this paper, the shafting of the test bench is composed of the shaft, front bearing, intermediate bearing, stern bearing, and counterweight disk. All three bearings are oil-lubricated sliding bearings. The counterweight disc is used to simulate the actual propeller. The horizontal direction is the x-direction, the vertical direction is the y-direction, and the axis is the z-direction. According to the structural characteristics of the system, considering the unbalanced excitation force and nonlinear oil film force, the dynamic model of the shafting system was established, as shown in Figure 1. In the model, only the transverse vibration of the system is considered, and the influence of torsional vibration and the gyroscopic effect is ignored.

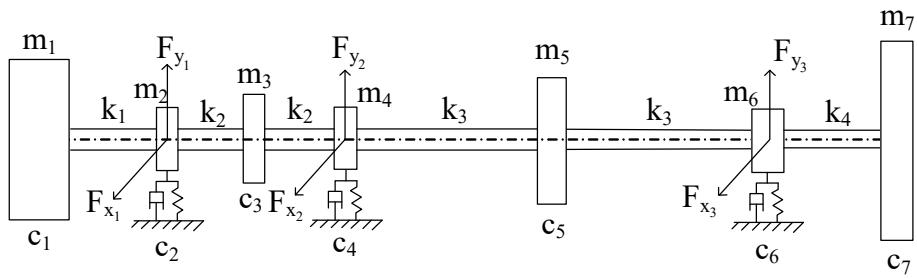

Figure 1. The ship shafting dynamic model

In Figure 1, the m_i（i=1~7）are seven equivalent mass disks, m1 is the equivalent lumped mass of the drive end of the shaft, m_2, m_4, and m_6 are the equivalent lumped mass at the front bearing, intermediate bearing, and stern bearing, m_3 and m_5 are the equivalent lumped mass of the shaft between the three bearings, and m_7 is the equivalent lumped mass at the load end of the shafting. The disks are connected by massless elastic shafts.

Considering the effect of gravity G_g, unbalanced excitation force F_e, and nonlinear oil film force F at the shafting, the description of each force is as follows:

(1) Gravity G_g acts on the entire shaft and acts equivalently on each lumped mass disk. The direction of it is downward in the y-direction.

(2) The unbalanced excitation force $F_e = m_i e w^2$ acts on each lumped mass disk, and the direction changes periodically. The e is the mass eccentricity and ω is the rotational angular velocity.

(3) The calculation model used for the nonlinear oil film force F on the shaft at the front bearing, intermediate bearing, and stern bearing is the unsteady nonlinear oil film force model proposed by Capone in 1991. The details can refer to the literature [13].

Taking the position state x, y of the center point of each disk as the degree of the system's freedom, according to the centroid theorem in vibration mechanics, the differential equation of the shaft system is obtained:

$$\begin{cases}[\underline{M}]\ddot{x}+[\underline{C}]\dot{x}+[\underline{K}]x=G_x\\ [\underline{M}]\ddot{y}+[\underline{C}]\dot{y}+[\underline{K}]y=G_y\end{cases} \tag{1}$$

Where, x and y are generalized coordinate vectors, $[\underline{M}]$ is the mass matrix, $[\underline{K}]$ is the stiffness matrix, $[\underline{C}]$ is damping matrix, G_x and G_y are the excitation force matrices in the x and y directions. At this time, the dynamic model is deterministic, which can be regarded as modeling based on mean, and the result obtained is also the mean model.

2.2. Nonparametric modeling

In the actual navigation of the ship, due to the influence of many dynamic factors, it will bring substantial uncertainty to the shafting. Besides, there are always errors between the design parameters and the actual operating conditions. All these indicate that the mean model does not accurately reflect the actual situation. In order to represent the actual operating state, the uncertainties must be considered. Therefore, the random matrix model has more considerable practical significance.

The nonparametric approach was proposed by Professor Christian Soize in 2000. By introducing the stochastic matrix theory in statistical physics, it provides a new method for the dynamic analysis of uncertain systems. The following key equations are introduced to explain the theory and implementation process of nonparametric modeling.

Assuming that the matrix [A] is a $n\times n$ random matrix of the mass, damping, or stiffness, according to the statistical characteristics of the random matrix, it can be easily deduced that the probability density function of [A] must fulfill the following three basic constraints:

$$\begin{cases}\int\limits_{M_n^+(R)} p_{[A]}([A])\tilde{d}A=1\\ \int\limits_{M_n^+(R)} [A]p_{[A]}([A])\tilde{d}A=[\underline{A}]\in M_n^+(R)\\ \int\limits_{M_n^+(R)} \ln(\det[A])p_{[A]}([A])\tilde{d}A=v,|v|<+\infty\end{cases} \tag{2}$$

Where, $M_n^+(R)$ is constituted by positive definite symmetric matrices.

In order to ensure the physical significance in the probability density function of the above random matrix, the entropy is introduced to represent the degree of uncertainty of the system, and the maximum entropy principle is used to construct the Lagrange function. Then the probability density function can be expressed as

$$p_{[A]}([A])=c_A\times(\det[A])^{\lambda-1}\times\exp(-\frac{(n-1+2\lambda)}{2}tr\{[\underline{A}]^{-1}[A]^T\}) \tag{3}$$

Where, $[\underline{A}]$ is the mathematical expectation of the random matrix $[A]$, c_A is a positive constant calculated by

$$c_A=\frac{(2\pi)^{-n(n-1)/4}(\frac{n-1+2\lambda}{2})^{n(n-1+2\lambda)/2}}{\left\{\prod_{l=1}^{n}\Gamma(\frac{n-l+2\lambda}{2})\right\}(\det[\underline{A}])^{(n-1+2\lambda)/2}} \tag{4}$$

Where $\Gamma(x)$ is the Gamma function defined by $\Gamma(x)=\int_0^{+\infty} t^{x-1}e^{-x}dt.(x>0)$. The variance of the random matrix $[A]$ can be expressed as

$$\sigma_{jk}=\frac{1}{n-1+2\lambda}\left\{\underline{A}_{jk}^2+\underline{A}_{jj}A_{kk}\right\} \tag{5}$$

Since $E\left\{\left\|[A]-[A]\right\|_F^2\right\}=\Sigma_j\Sigma_k\sigma_{jk}^2$, if defining a dispersion control parameter δ_A as

$$\delta_A=\left\{\frac{E\left\{\left\|[A]-[\underline{A}]\right\|_F^2\right\}}{\left\|\underline{A}\right\|_F^2}\right\}^{1/2}=\left\{\frac{1}{n-1+2\lambda}(1+\frac{(tr[\underline{A}])^2}{tr([\underline{A}])^2})\right\}^{1/2} \tag{6}$$

Then, the parameter λ in Eqs. (3~6) can be deduced as

$$\lambda=\frac{1}{2\delta^2}(1-\delta^2(N-1)+\frac{(tr[\underline{A}]^2)}{tr([\underline{A}])^2}) \tag{7}$$

It should be noted that for a stochastic system determined by the dimension n, the λ increases as δ_A decrease. If $\lambda\to+\infty$, then $\sigma_{jk}\to 0$, $\delta_A\to 0$ and the random matrix $[A]$ will approach in the mean matrix $[\underline{A}]$.

Therefore, the construction of the random matrix $[A]$ is mainly related to the dispersion control parameter δ_A. The random matrix can be obtained by controlling the dispersion control parameters and performing Monte Carlo simulation on the mean matrix.

For any positive symmetric matrix $[\underline{A}]$, it can be decomposed by the Cholesky factorization into a product form of a lower triangular matrix and an upper triangular matrix:

$$[\underline{A}]=[L_A]^T[L_A] \tag{8}$$

Where, L_A is the upper triangular real matrix. Assuming λ is a positive integer, let $m_A=n-1+2\lambda$, and the random matrix $[A]$ can be simulated by

$$[A]=\frac{1}{m_A}\sum_{j=1}^{m_A}\left([L_A]^TU_j\right)\left([L_A]^TU_j\right)^T \tag{9}$$

Where, U_j is the vector composed of independent Gaussian random variables with zero mean and unit variance. Then, the random samples that fluctuate in a certain range can be obtained through the mean matrix $[\underline{A}]$, and the random matrix model of the system can be established.

For the ship shafting system in section 2.1, the Monte Carlo method is used to simulate the random matrix of the mass, stiffness, and damping, and the system's Random matrix model can be obtained as

$$\begin{cases}[M]\ddot{x}+[C]\dot{x}+[K]x=G_x\\ [M]\ddot{y}+[C]\dot{y}+[K]y=G_y\end{cases} \tag{10}$$

Where, $[M],[C]$ and $[K]$ are the random matrices corresponding to the mean mass, damping, and stiffness matrices, respectively. The dispersion control parameters δ_M, δ_C and δ_K determines the sample space of the random matrix. At this time, the dynamic model can represent the vibration state of the ship shafting system that includes both the data uncertainty and the model uncertainty so that the dynamic response characteristics of this system can be studied.

3. Vibration characteristics analysis

According to the dynamic model in Figure 1 and the structural characteristics of the ship shafting testbed, the main parameters of the calculation are set as follows: the masses and damping coefficients of the equivalent mass discs are m_1=5 kg, m_2=2.53 kg, m_3=2.78 kg, m_4=7.05 kg, m_5=9.05 kg, m_6=7.94 kg, m_7=35 kg, c_1=1050 N·s/m, c_2=2100 N·s/m, c_3=1050 N·s/m, c_4=2100 N·s/m, c_5=1050 N·s/m, c_6=2100 N·s/m, c_7=1050 N·s/m, the stiffness coefficients of the shaft segments are k_1=3.13.1×10^9 N/m, k_2=7.8×10^8 N/m, k_3=3.42×10^7 N/m, k_4=3.9×10^8 N/m，the radius and width of the front bearing are R_1=21.5×10^{-3} m, L_1=100×10^{-3} m, the radius and width of the intermediate bearing are R_2=21.5×10^{-3} m, L_2=100×10^{-3} m, the radius and width of the stern bearing are R_3=21.5×10^{-3} m, L_3=300×10^{-3} m, the average oil film thickness c=0.1×10^{-3} m, the dynamic viscosity of lubricating oil μ_n=0.12 N·s/m^2, the radius clearance of bearing δ=0.1×10^{-3} m, the unbalanced eccentricity e=0.2×10^{-3} m, and the acceleration of gravity g=9.8 m/s^2.

Since the shafting is coupled by the gravity, unbalanced excitation force, and nonlinear oil film force, in this paper, the Runge-Kutta method is used to solve the Eq. (10), and the dynamic response of the system is computed. The time step is set as 1/50 of the period, and 1100 periods were simulated. The data of the first 1000 periods were discarded, and the results of the last 100 periods were taken for analysis.

3.1. Frequency response function

The frequency response function is an important indicator to measure the performance of the system. It represents the relationship between the output and input of frequency and can describe a nonlinear system's essential characteristics. The frequency response function of the system can be expressed as:

$$h(w) = (-w^2[M] + iw[C] + [K])^{-1} \tag{11}$$

And it is generally converted into decibels:

$$dB(w) = 10\log_{10}(w^4 \left\|[h(w)]\right\|_F^2) \tag{12}$$

Where, $\| \ \|_F$ represents the Frobenius norm.

According to the shafting system dynamic model, the system's frequency response function with several random uncertainties is calculated, as shown in Figure 2. Figure 2(a) shows the frequency response function without considering the uncertainty. It can be seen that the frequency response function shows a gradual upward trend with increasing rotating speed. Figures 2 (b ~ d) show the frequency response functions for different dispersion control parameters, i.e. $\delta_M = \delta_C = \delta_K = 0.2$, $\delta_M = \delta_C = \delta_K = 0.5$, and $\delta_M = \delta_C = \delta_K = 0.8$, and the number of samples is 32. By comparing it with Figure 2(a), the frequency response function presents certain volatility. It is because after considering the uncertainty, the mass, stiffness, and damping matrix of the system are transformed from the deterministic matrix to random samples, which is distributed within a specific range. Moreover, from the variations in Figure 2(b) to Figure 2(d), it can be seen that as the uncertainty increases, the degree of fluctuation of the system's frequency response function also strengthens.

The above analysis can be concluded that uncertainty will change the essential characteristics of the ship shafting. When the uncertainty is considerable, it will have a certain fluctuation effect on the system performance.

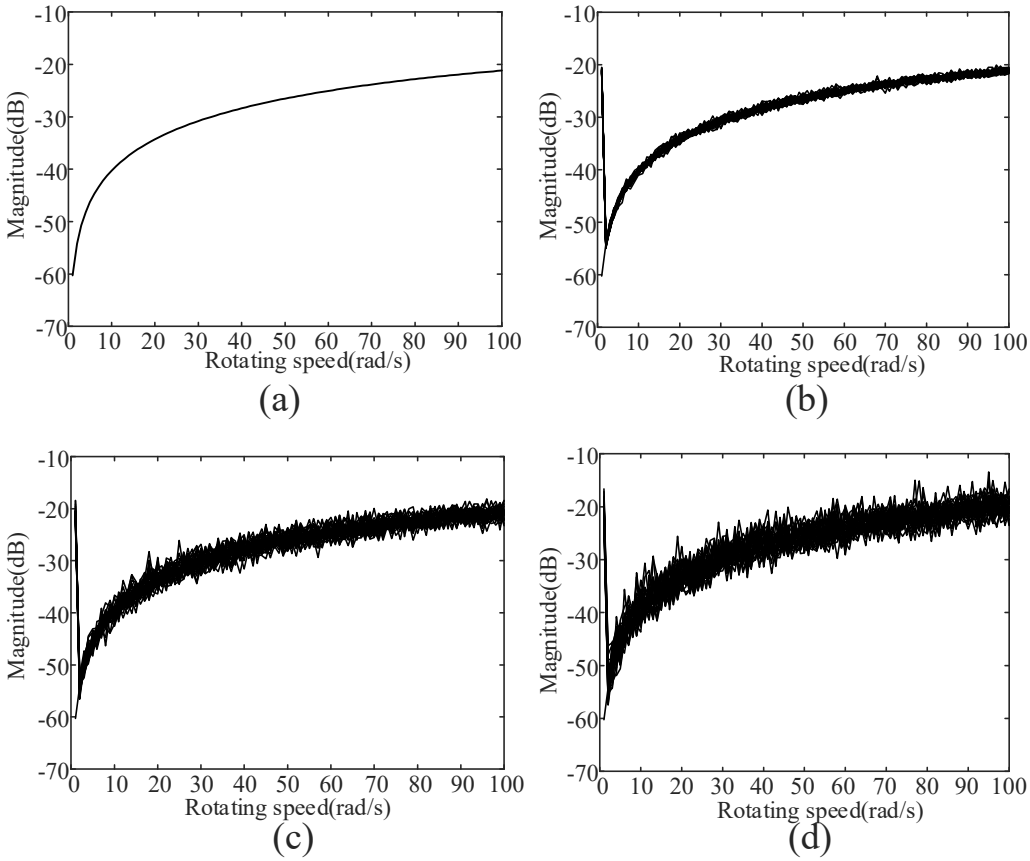

Figure 2. The frequency response function of the system with several random uncertainties
(a) $\delta_M = \delta_C = \delta_K = 0$ (b) $\delta_M = \delta_C = \delta_K = 0.2$
(c) $\delta_M = \delta_C = \delta_K = 0.5$ (d) $\delta_M = \delta_C = \delta_K = 0.8$

3.2. Time-frequency characteristic

To further illustrate the influence of random uncertainty on the ship shafting, the vibration state of the ship shafting is explained from the perspective of time history and frequency spectrum under different degrees of uncertainty. Since it often brings more considerable uncertainty to the stern bearing than others when the ship shafting is affected by nonlinear dynamic factors, the vibration response of the shaft at the stern bearing is mainly analyzed.

Figure 3 shows the waveform, axis trajectory, and frequency spectrum of the ship shafting at the stern bearing without considering the uncertainty. In this circumstance, the speed is 60 rad/s. It can be seen that the vibration response of the system in the time domain is sinusoidal, the peak-to-peak value of the vibration is 1.6×10^{-8}m. Besides, the axis trajectory presents a standard ellipse, and the corresponding frequency spectrum is also composed of a single frequency component, which is the driving frequency of the system, indicating that the system's vibration is caused by the unbalanced excitation force.

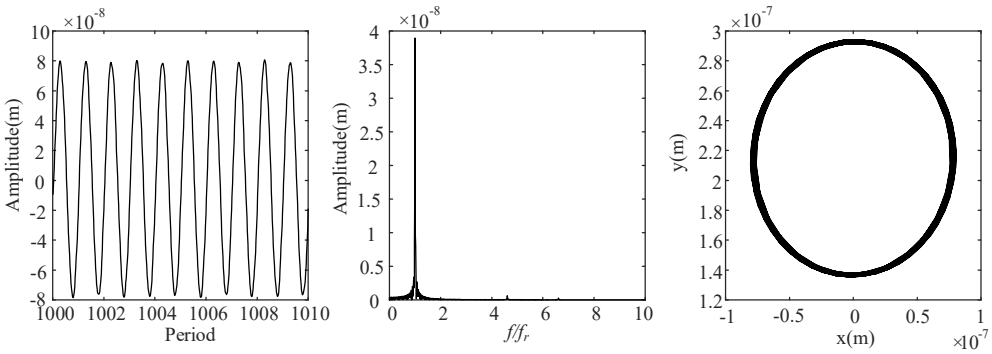

Figure 3. The time-domain waveform, axis trajectory, and frequency spectrum of the system under $\delta_M = \delta_C = \delta_K = 0$

Figures 5 to 7 show the waveform, axis trajectory, and frequency spectrum of the ship shafting at the stern bearing when the dispersion control parameters of mass, stiffness, and damping are $\delta_M = \delta_C = \delta_K = 0.2$, $\delta_M = \delta_C = \delta_K = 0.5$, and $\delta_M = \delta_C = \delta_K = 0.8$, respectively. Due to space limitations, only the calculation results of some of the 32 random samples are given.

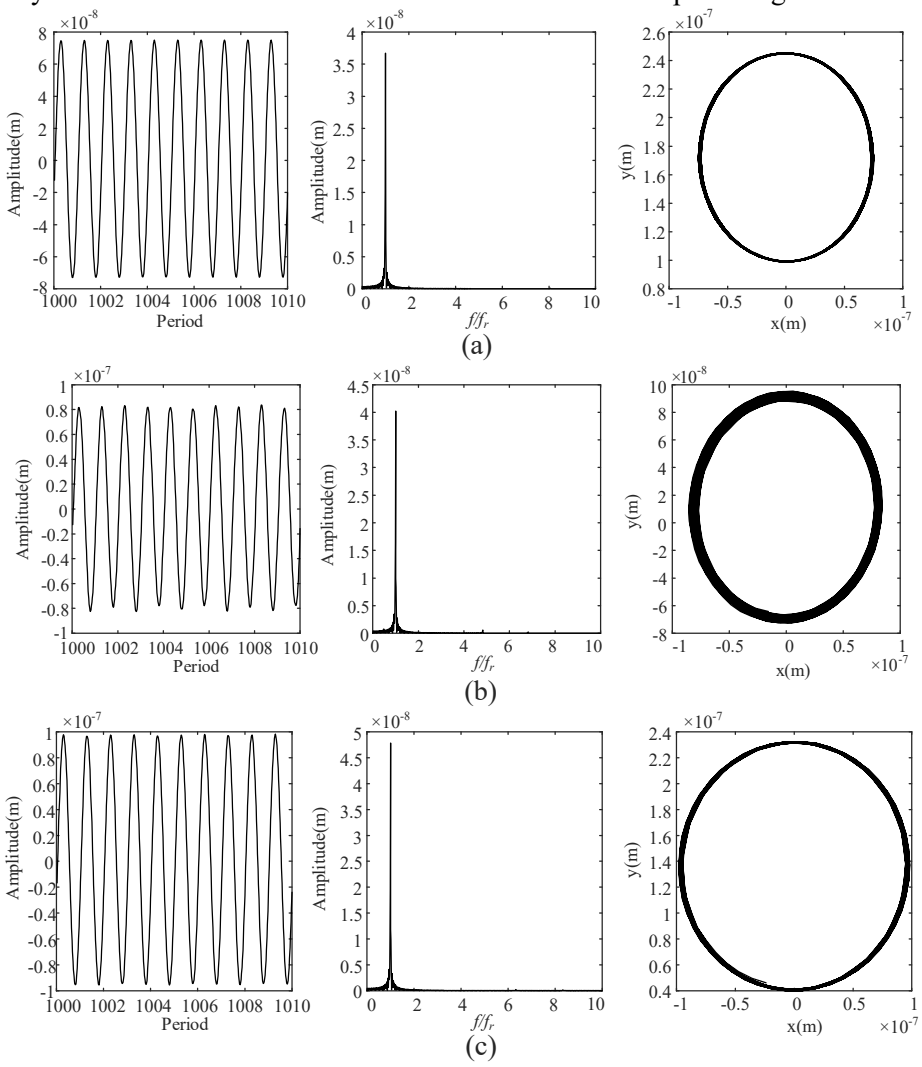

Figure 4. The time-domain waveform, axis trajectory, and frequency spectrum of the system under

$$\delta_M = \delta_C = \delta_K = 0.2$$

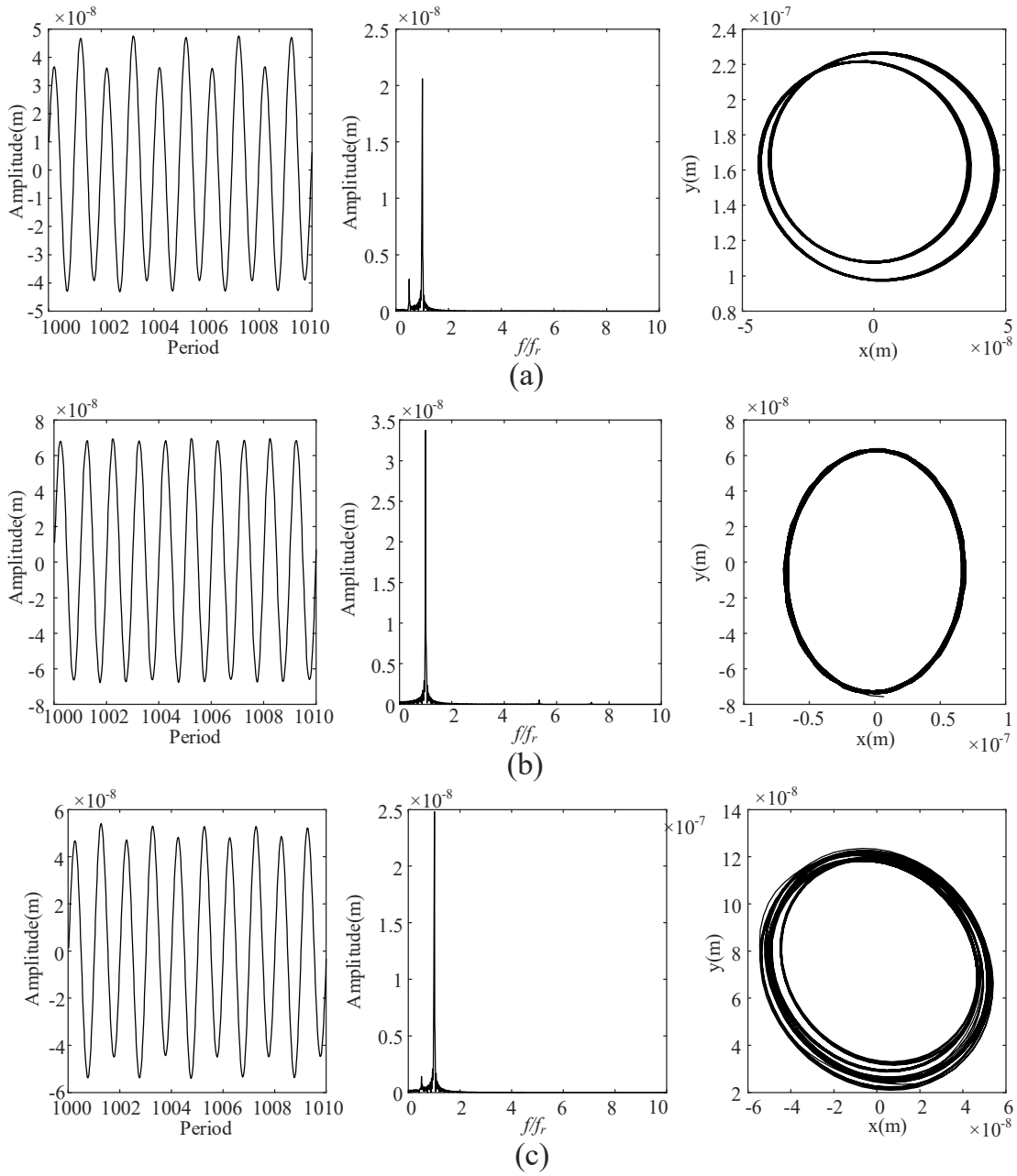

Figure 5. The time-domain waveform, axis trajectory, and frequency spectrum of the system under $\delta_M = \delta_C = \delta_K = 0.5$

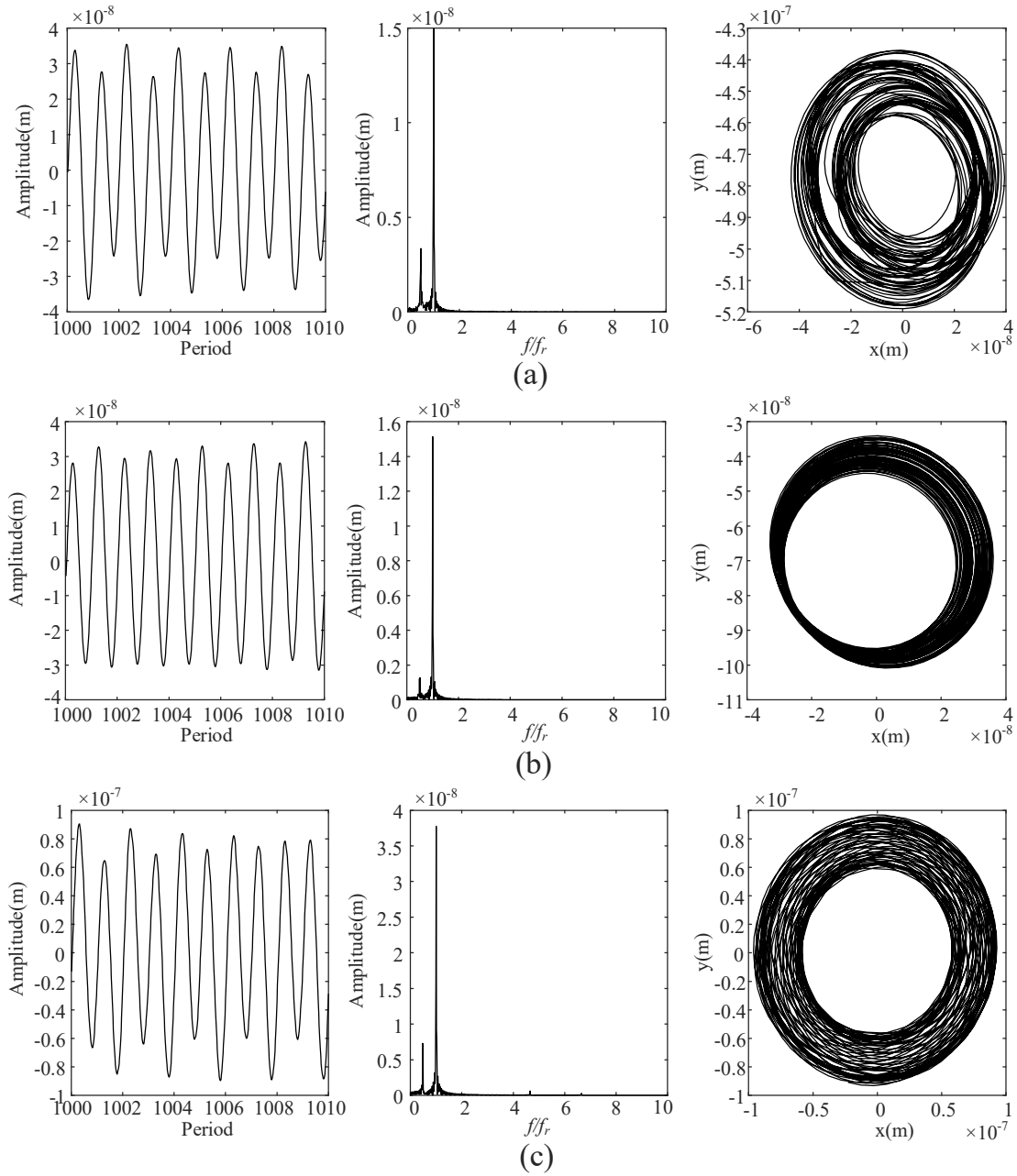

Figure 6. The time-domain waveform, axis trajectory, and frequency spectrum of the system under $\delta_M = \delta_C = \delta_K = 0.8$

Comparing Figures 4-6 with Figure 3, it can be seen that after considering the influence of uncertainty, the system's vibration characteristics have changed significantly. Although the vibration response of the shafting system at the stern bearing is still periodic signal, it is no longer a single sinusoidal curve, and other components appear. Besides, it can also be discerned from the frequency spectrum that a one-half octave component appears in the system's vibration response. Moreover, in addition to a single ellipse, the system's axis trajectory also exhibited inner loops, multiple loops, and chaotic trajectories, indicating that the uncertainty caused the system to exhibit nonlinear behavior.

It can be seen from the changes in Figures 4 to Figure 6 that as the uncertainty increases, the degree of disorder in the system's waveform and axis trajectory gradually strengthens, and the amplitude of the one-half octave on the spectrum also has raised. Figure 7 shows the amplitude of the fundamental

frequency in the vibration response of 32 random samples in the x and y directions under the three dispersion control parameters. As the uncertainty increases, the range of the system's vibration response fluctuations also gradually expands, which further proves that when the ship's shafting is in operation, its vibration should be a random process that fluctuates within a certain range.

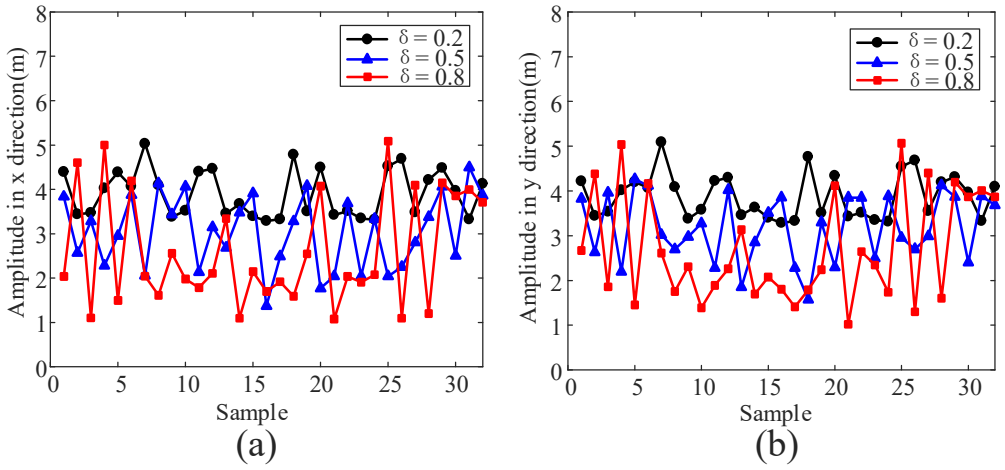

Figure 7. The amplitude of the fundamental frequency in the x and y directions

4. Conclusion

Due to the inevitable random uncertainties, there will be some problems such as the excessive amplitude of vibration and frequent failure in the shafting during operation. In order to investigate the influence of random uncertainties on the operating characteristics and vibration response of the shafting, the nonparametric dynamic model which considering data uncertainty and model uncertainty was constructed, and the calculation results were analyzed. The following conclusions can be drawn from the study:

(1) The random uncertainty will bring instability to the operating characteristics of the ship shafting, and the degree of fluctuation of the system's frequency response function strengthens with the increase of the uncertainty.

(2) The vibration response of the ship shafting is not a certain value, but it is continuously changing within a certain range, and it will also excite other frequency components besides the fundamental frequency, such as one-half octave. As the uncertainty increases, the fluctuation of the amplitude of the system's vibration response will gradually strengthen.

Acknowledgments

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