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Study on reflection of one dimensional stress wave

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Abstract. Impact load is a common form of dynamic load. Because of the time effect and the inertia and deformability of the structure, the impact load will propagate and reflect in the form of stress wave in the structure. In this paper, the form of stress wave transferred from the impact end to the structure is analyzed, and the wave equation of axial compression stress wave is derived.

1. Introduction

In a series of practical problems in the fields of production technology, military technology and scientific research, people will encounter a variety of impact load problems, and it can be observed that the mechanical response is often significantly different from that under static load. For example, when flying stone strikes the window glass, it often causes fragmentation and collapse on the back of the glass first. It is a dynamic problem because the micro element of the medium is in a dynamic process changing rapidly with time under the condition of impact dynamic load. In fact, when the external load is applied to the surface of a certain part of a deformable solid, only the medium particles of the surface part directly affected by the external load leave the initial equilibrium position [1-3]. Because of the relative motion between the media particles and the adjacent media particles, they will certainly be affected by the force given by the adjacent media particles, but at the same time, they will also react to the adjacent media particles, so that they also move away from the initial balance position. However, due to the inertia of medium particles, the motion of adjacent medium particles lags behind that of surface medium particles [4, 5].

The study of structural dynamic buckling can be divided into two kinds of problems: one is parametric buckling, which means that the structure with periodic loading function produces structural resonance in a certain buckling mode, and the loading function is the parameter of displacement in the differential equation of motion. The other main characteristic of parametric buckling is that the load amplitude required for structural collapse is lower than that of the corresponding static buckling collapse. At present, the theoretical and experimental researches on the impact buckling are mainly focused on the buckling problems caused by the ideal pulse load and step load [6, 7]. The buckling problem caused by ideal pulse load is called ideal pulse buckling, and the buckling problem caused by step load with constant amplitude and infinite duration mainly refers to the dynamic jump buckling of a kind of structure with static instability post buckling path under the action of step load.

2. Fundamental eigensolutions

Consider an elastic straight bar in a coordinate system (x, y), It's The kinetic energy density and potential energy density are

$$T = \frac{1}{2}\rho A(\frac{\partial u}{\partial t})^2 = \frac{1}{2}\rho A\dot{u}^2 \tag{1}$$

and

$$V = \frac{1}{2}EA(\frac{\partial u}{\partial x})^2 = \frac{1}{2}EA(\partial_x u)^2 \tag{2}$$

respectively. According to the Lagrange function

$$L = \int (T - V)dx = \int [\frac{1}{2}\rho\dot{u}^2 - \frac{1}{2}E(\partial_x u)^2]dx \tag{3}$$

and the variational principle

$$\delta J = \delta \int\int Ldxdt = 0 \tag{4}$$

we have

$$\dot{u}^2 - C^2\partial_x^2 u = 0, \quad C^2 = \frac{E}{\rho} \tag{5}$$

where C is velocity of stress wave. Let $q = u$, and the dual variable can be obtained as

$$p = \frac{\delta L}{\delta \dot{q}} = \rho\dot{q} \tag{6}$$

According to Hamiltonian variational principle

$$\delta\int Ldx = \delta\int [p\dot{q} - H(q,p)]dx = 0 \tag{7}$$

we get

$$\dot{q} = \frac{\delta H}{\delta p} = \frac{1}{\rho}p, \quad \dot{p} = -\frac{\delta H}{\delta q} = E\partial_x^2 u \tag{8}$$

Eq. (8) can also be expressed as

$$\begin{Bmatrix} \dot{q} \\ \dot{p} \end{Bmatrix} = \begin{bmatrix} 0 & 1/\rho \\ E\partial_x^2 & 0 \end{bmatrix} \begin{Bmatrix} q \\ p \end{Bmatrix} \tag{9}$$

Using the characteristic equation method, we get the following equations

$$\phi^{(\alpha)} = \begin{Bmatrix} q^{(\alpha)} \\ p^{(\alpha)} \end{Bmatrix} = \begin{Bmatrix} 1 \\ \rho\mu \end{Bmatrix} (c_1 e^{\frac{\mu}{c}x} + c_2 e^{-\frac{\mu}{c}x}) \tag{10}$$

$$\therefore \quad \phi^{(\alpha)} e^{\mu t} = \begin{Bmatrix} 1 \\ \rho u \end{Bmatrix} (c_1 e^{\frac{\mu}{c}(x+ct)} + c_2 e^{-\frac{\mu}{c}(x-ct)}) \tag{11}$$

$$\phi^{(\beta)} = \begin{Bmatrix} q^{(\beta)} \\ p^{(\beta)} \end{Bmatrix} = \begin{Bmatrix} 1 \\ -\rho\mu \end{Bmatrix} (c_3 e^{-\frac{\mu}{c}x} + c_4 e^{\frac{\mu}{c}x}) \tag{12}$$

and

$$\therefore \quad \phi^{(\beta)}e^{-\mu t} = \begin{Bmatrix} 1 \\ -\rho u \end{Bmatrix}(c_3 e^{-\frac{\mu}{c}(x+ct)} + c_4 e^{\frac{\mu}{c}(x-ct)}) \tag{13}$$

Therefore

$$\psi = \int \begin{Bmatrix} 1 \\ \rho\mu \end{Bmatrix}(c_1 e^{\frac{\mu}{c}(x+ct)} + c_2 e^{-\frac{\mu}{c}(x-ct)})d\mu + \int \begin{Bmatrix} 1 \\ -\rho u \end{Bmatrix}(c_3 e^{-\frac{\mu}{c}(x+ct)} + c_4 e^{\frac{\mu}{c}(x-ct)})d\mu \tag{14}$$

3. Results and discussion

Fig.1 shows the curve of the minimum critical buckling load for the first ten eigenvalues. It can be seen from the figure that at the beginning of the impact load, with the propagation of the axial stress wave, the critical buckling load of the shell decreases rapidly. When the stress wave propagates to a certain position, the critical load curve of each stage gradually tends to be gentle.

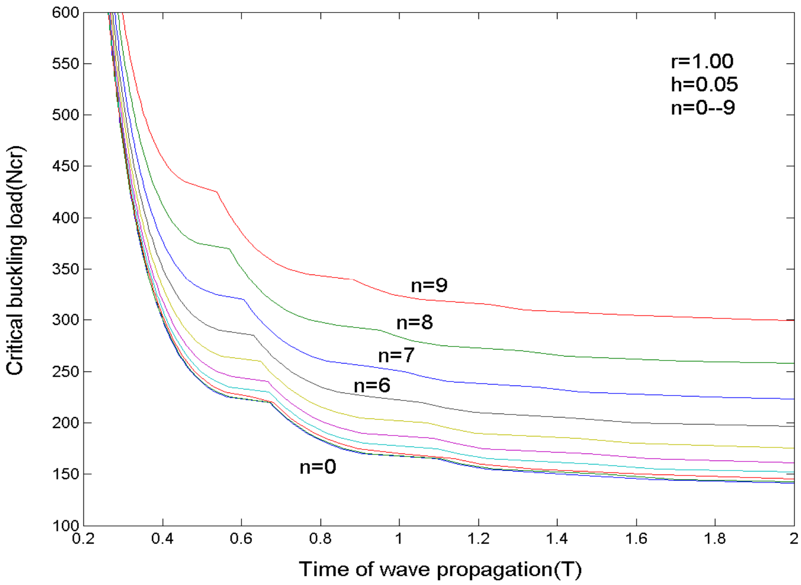

Figure 1. The first ten orders of critical buckling loads with time of wave propagation.

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