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The Impact of Sampling Period Change on Dynamic Phasor Traceability Algorithm Using Least Squares

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Abstract. In recent years, the electrical signal in Power System becomes more and more complex because of large-scale applications of power electronic equipment. To accurately trace the dynamic phasor, the algorithm based on least squares is used. It is important to develop the dynamic phasor traceability algorithm with high accuracy, good real-time performance and satisfied dynamic property. In this study, the applicability of least squares in dynamic phasor traceability is studied. The relationship between the sampling period and the fundamental period is deduced based on the formula. The accuracy is verified when the sampling period is three quarters of fundamental period. The simulation results show that when the sampling period is three quarters of fundamental period, the frequency response amplitude of this algorithm is flat near the fundamental frequency. Thus, the dynamic phasor can be accurately traced.

1. Introduction

The voltage and current need to be traced to a sine wave in the AC Power System[1]. However, the voltage and current of the Power System are dynamic in actual operation. Meanwhile, electrical signal in Power System becomes more and more complex because of large-scale applications of power electronic equipment[2]. The spectrum of electrical signal also becomes complex and the dynamic phenomena become complex[3]. So, it is very important to trace the dynamic phasor accurately and quickly. The value of the dynamic phasor can be traced from the sampling result through the least squares method[4-6]. But various sampling periods have effects on the tracing results. As far as we know, the relationship between the sampling period and frequency response has not been studied. The relationship between the sampling period and dynamic phasor traceability algorithm using least squares has not been studied in the published literatures.

In this work, the requirements of sampling period when the least squares is applied to trace the dynamic phasor in the frequency domain is studied. Theoretical analysis shows that when the sampling time are an integer multiple of the fundamental period, the least squares is equivalent to the DFT. When the sampling time is three quarters of the fundamental period, the least squares phasor traceability algorithm can still trace the fundamental phasor accurately. In this case, the amplitude of frequency response near the fundamental frequency is flat. The least squares algorithm is insensitive to the frequency change near the fundamental frequency. It has good frequency response and wonderful dynamic performance. So, it is suitable for the traceability of dynamic phasor. The simulation results show that the three quarters fundamental period least squares is the best in frequency domain.

2. Analysis of phasor traceability algorithm using least squares

The dynamic phenomena generally include amplitude phase modulation and frequency ramp change in Power System[7]. Amplitude phase modulation is equivalent to other frequency components of the



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observed signal near the fundamental frequency. The frequency ramp change is also the frequency of the observed signal changes near the fundamental frequency. Therefore, the ideal dynamic phasor traceability algorithm should be accurate near the fundamental frequency. That is to say, the frequency response amplitude is flat near the fundamental frequency.

2.1. The principle of dynamic phasor traceability algorithm using least squares

The voltage and current signals in the power system can be expressed as:

$$s(t) = a \cos(2\pi ft + \varphi) \quad (1)$$

In Equation (1): $s(t)$ is the voltage or current signal. a is the signal amplitude. φ is the signal phase. f is the fundamental frequency of the signal. Equation (1) can be expressed by phasor:

$$s(t) = \frac{1}{2} (pe^{j2\pi ft} + \bar{p}e^{-j2\pi ft}) \quad (2)$$

In the formula: $p = ae^{j\varphi}$ is the phasor form of the input signal. $\bar{p}$ is the conjugate phasor of p .

It is assumed that the signal sampling time window is a fundamental wave period. If N is the number of sampling points per fundamental wave period, we can know from equation (2) that the relationship between the discrete form of $s(t)$ and the phasor can be expressed as

$$\begin{pmatrix} s(0) \\ s(1) \\ \vdots \\ s(N-1) \end{pmatrix} = \frac{\mathbf{B}}{2} \begin{pmatrix} p \\ \bar{p} \end{pmatrix} \quad (3)$$

$$\mathbf{B} = \begin{pmatrix} e^{j\omega} & e^{-j\omega} \\ e^{j2\omega} & e^{-j2\omega} \\ \vdots & \vdots \\ e^{j(N-1)\omega} & e^{-j(N-1)\omega} \end{pmatrix} \quad (4)$$

ω is the fundamental frequency.

When the sampling period is not equal to the fundamental period, the phasor can also be traced to approximate the real phasor at the fundamental frequency through using least squares. The phasor can be traced by equation (5):

$$\begin{pmatrix} \hat{p} \\ \hat{\bar{p}} \end{pmatrix} = 2(\mathbf{B}_{sq}^H \mathbf{B}_{sq})^{-1} \mathbf{B}_{sq}^H \mathbf{s} \quad (5)$$

2.2. Applicability of sampling time to phasor traceability algorithm using least squares

In conventional studies, the solution of equation (3) by least squares is equivalent to the DFT. However, we can know from the formula that the least squares is equivalent to the DFT only when the sampling period is twice of the fundamental period.

According to the definition of $\mathbf{B}$

$$(\mathbf{B}^H \mathbf{B})^{-1} = \frac{1}{|\mathbf{B}^H \mathbf{B}|} \begin{bmatrix} N & -\sum_{k=1}^N e^{-j2k\omega} \\ -\sum_{k=1}^N e^{j2k\omega} & N \end{bmatrix} \quad (6)$$

$$|\mathbf{B}^H \mathbf{B}| = N^2 - \sum_{k=1}^N e^{j2k\omega} \sum_{k=1}^N e^{-j2k\omega}$$

$\omega = 2\pi / N$. N is the number of sampling points in a fundamental period. The sum of all sampling points in the whole period of the fundamental wave is zero, so

$$\begin{cases} \sum_{k=1}^N e^{j2k\omega} = \sum_{k=1}^N e^{j2k\frac{2\pi}{N}} = 0 \\ \sum_{k=1}^N e^{-j2k\omega} = \sum_{k=1}^N e^{-j2k\frac{2\pi}{N}} = 0 \end{cases} \quad (7)$$

Equation (6) can be simplified as:

$$(\mathbf{B}^H \mathbf{B})^{-1} = \frac{1}{N^2} \begin{bmatrix} N & 0 \\ 0 & N \end{bmatrix} = \frac{1}{N}$$

Substituting the above formula into the formula for estimating the phasor by the LS method:

$$\begin{pmatrix} \hat{p} \\ \hat{\bar{p}} \end{pmatrix} = 2(\mathbf{B}^H \mathbf{B})^{-1} \mathbf{B}^H \mathbf{s} = \frac{2}{N} \mathbf{B}^H \begin{bmatrix} a \cos(\omega + \varphi) \\ a \cos(2\omega + \varphi) \\ \vdots \\ a \cos(N\omega + \varphi) \end{bmatrix} \quad (8)$$

The right side of the second equal sign of equation (8) is the formula for tracing the phasor of the DFT. From the above analysis, it can be seen that the equivalent of the least squares and the DFT depends on equation (7). The sum of all sampling points in the sampling time should be zero. Therefore, the least squares and the DFT are equivalent only when two times of the sampling time are an integer multiple of fundamental period. Otherwise, the phasor traceability results of the least squares and the DFT are different.

2.3. Analysis of three-quarter fundamental period least squares

When the sampling time is three quarters of fundamental period, the twice of it is not equal to an integer multiple of fundamental period. Therefore, the traceability result of the least squares is different from the DFT when the sampling time is three-quarter fundamental period. The formula is used to derive the accuracy of three-quarter fundamental period least squares.

$\omega = 2\pi / N$ is adopted in equation (5):

$$\begin{aligned} (\mathbf{B}_{sq}^H \mathbf{B}_{sq})^{-1} &= \frac{1}{|\mathbf{B}_{sq}^H \mathbf{B}_{sq}|} \begin{bmatrix} \frac{3}{4}N & -\sum_{k=1}^{\frac{3}{4}N} e^{-j2k\frac{2\pi}{N}} \\ -\sum_{k=1}^{\frac{3}{4}N} e^{j2k\frac{2\pi}{N}} & \frac{3}{4}N \end{bmatrix} \\ |\mathbf{B}_{sq}^H \mathbf{B}_{sq}| &= (\frac{3}{4}N)^2 - \sum_{k=1}^{\frac{3}{4}N} e^{-j2k\frac{2\pi}{N}} \sum_{k=1}^{\frac{3}{4}N} e^{j2k\frac{2\pi}{N}} \end{aligned} \quad (9)$$

The sum of all sampling points of $e^{j\omega t}$ in a fundamental period is zero in equation (9):

$$\begin{cases} \sum_{k=1}^{\frac{3}{4}N} e^{-j2k\frac{2\pi}{N}} = \sum_{k=1}^{\frac{1}{4}N} e^{-j2k\frac{2\pi}{N}} = -j \sum_{k=1}^{\frac{1}{4}N} \sin(2k\frac{2\pi}{N}) \\ \sum_{k=1}^{\frac{3}{4}N} e^{j2k\frac{2\pi}{N}} = \sum_{k=1}^{\frac{1}{4}N} e^{j2k\frac{2\pi}{N}} = j \sum_{k=1}^{\frac{1}{4}N} \sin(2k\frac{2\pi}{N}) \end{cases}$$

Substituting the above formula into equation (9) gives:

$$(\mathbf{B}_{sq}^H \mathbf{B}_{sq})^{-1} = \frac{1}{|\mathbf{B}_{sq}^H \mathbf{B}_{sq}|} \begin{bmatrix} \frac{3}{4}N & j \sum_{k=1}^{\frac{1}{4}N} \sin(2k\frac{2\pi}{N}) \\ -j \sum_{k=1}^{\frac{1}{4}N} \sin(2k\frac{2\pi}{N}) & \frac{3}{4}N \end{bmatrix} \quad (10)$$

$$|\mathbf{B}_{\text{sq}}^H \mathbf{B}_{\text{sq}}| = \left(\frac{3}{4}N\right)^2 - \sum_{k=1}^{\frac{1}{4}N} \sin(2k\frac{2\pi}{N}) \sum_{k=1}^{\frac{1}{4}N} \sin(2k\frac{2\pi}{N})$$

$\sum_{k=1}^{\frac{1}{4}N} \sin(2k\frac{2\pi}{N})$ is the sum of the sampled values of the sin function at even points in the range from 0 to π . When N approaches infinity:

$$\lim_{N \rightarrow +\infty} \sum_{k=1}^{\frac{1}{4}N} \sin(2k\frac{2\pi}{N}) = \frac{1}{2} \int_0^{\pi} \sin \omega t dt = \frac{1}{\omega} = \frac{n}{2\pi} \quad (11)$$

When N approaches infinity,

$$(\mathbf{B}_{\text{sq}}^H \mathbf{B}_{\text{sq}})^{-1} = \frac{1}{\left(\frac{3}{4}N\right)^2 - \left(\frac{N}{2\pi}\right)^2} \begin{bmatrix} \frac{3}{4}N & j\frac{N}{2\pi} \\ -j\frac{N}{2\pi} & \frac{3}{4}N \end{bmatrix} \quad (12)$$

For the other part of equation (5),

$$\mathbf{B}_{\text{sq}}^H \mathbf{s} = \begin{bmatrix} e^{-j\omega} & e^{-j2\omega} & \dots & e^{-j\frac{3}{4}N\omega} \\ e^{j\omega} & e^{j2\omega} & \dots & e^{j\frac{3}{4}N\omega} \end{bmatrix} \begin{bmatrix} a \cos(\omega + \varphi) \\ a \cos(2\omega + \varphi) \\ \vdots \\ a \cos(\frac{3N}{4}\omega + \varphi) \end{bmatrix} = a \begin{bmatrix} \sum_{k=1}^{\frac{3}{4}N} [e^{-jk\omega} \cos(k\omega + \varphi)] \\ \sum_{k=1}^{\frac{3}{4}N} [e^{jk\omega} \cos(k\omega + \varphi)] \end{bmatrix} \quad (13)$$

In equation (13),

$$\sum_{k=1}^{\frac{3}{4}N} [e^{jk\omega} \cos(k\omega + \varphi)] = \frac{1}{2} \sum_{k=1}^{\frac{3}{4}N} \cos(2k\omega + \varphi) + \frac{3}{8}N \cos \varphi + j \frac{1}{2} \sum_{k=1}^{\frac{3}{4}N} \sin(2k\omega + \varphi) - j \frac{3}{8}N \sin \varphi \quad (14)$$

When N approaches infinity,

$$\lim_{N \rightarrow +\infty} \sum_{k=1}^{\frac{3}{4}N} \cos(2k\omega + \varphi) = \frac{1}{2} \int_0^{\pi/\omega} \cos(\omega t + \varphi) dt = -\frac{N}{2\pi} \sin \varphi \quad (15)$$

$$\lim_{N \rightarrow +\infty} \sum_{k=1}^{\frac{3}{4}N} \sin(2k\omega + \varphi) = \frac{1}{2} \int_0^{\pi/\omega} \sin(\omega t + \varphi) dt = \frac{N}{2\pi} \cos \varphi \quad (16)$$

From equations (15) and (16), we can see that when N approaches infinity, equation (14) can be transformed into

$$\sum_{k=1}^{\frac{3}{4}N} [e^{jk\omega} \cos(k\omega + \varphi)] = \frac{1}{2} \left[\frac{3}{4}N \cos \varphi - \frac{N}{2\pi} \sin \varphi \right] + j \frac{1}{2} \left[-\frac{3}{4}N \sin \varphi + \frac{N}{2\pi} \cos \varphi \right] \quad (17)$$

Similarly,

$$\sum_{k=1}^{\frac{3}{4}N} [e^{-jk\omega} \cos(k\omega + \varphi)] = \frac{1}{2} \left[\frac{3}{4}N \cos \varphi - \frac{N}{2\pi} \sin \varphi \right] - j \frac{1}{2} \left[-\frac{3}{4}N \sin \varphi + \frac{N}{2\pi} \cos \varphi \right] \quad (18)$$

Substituting equations (17) and (18) into equation (13):

$$\mathbf{B}_{\text{sq}}^H \mathbf{s} = \frac{a}{2} \cdot \begin{bmatrix} \left(\frac{3N \cos \varphi}{4} - \frac{N \sin \varphi}{2\pi}\right) - j \left(\frac{N \cos \varphi}{2\pi} - \frac{3N \sin \varphi}{4}\right) \\ \left(\frac{3N \cos \varphi}{4} - \frac{N \sin \varphi}{2\pi}\right) + j \left(\frac{N \cos \varphi}{2\pi} - \frac{3N \sin \varphi}{4}\right) \end{bmatrix} \quad (19)$$

Substituting equations (12) and (19) into equation (5) gives:

$$\begin{bmatrix} \hat{p} \\ \hat{p}^* \end{bmatrix} = a \begin{bmatrix} \cos \varphi + j \sin \varphi \\ \cos \varphi - j \sin \varphi \end{bmatrix} = \begin{bmatrix} a e^{j\varphi} \\ a e^{-j\varphi} \end{bmatrix} \quad (20)$$

It can be seen from equation (20) that the accurate phasor estimation value can be obtained by using the three-quarter fundamental period least squares.

3. Simulation

In order to observe the characteristics of spectrum at different sampling time using least squares clearly, simulations are conducted. 0.001Hz is set as the sweep step and the spectrum of different sampling time using least squares is shown in Figure. 1. The amplitude of the traceable signal in Figure.1 is 1, the frequency is 50Hz and the phase are zero. When the amplitude, frequency and phase are other values, similar results can be obtained. It can be seen from Figure. 1 that the least squares traceability algorithm with different sampling time have a spectrum component of 1 at the fundamental frequency. The attenuation amplitude of the spectrum component of other least squares increases on both sides of the fundamental frequency with the increase of the estimation period. However, the LS algorithm has a relatively flat characteristic near the fundamental frequency when the sampling time is three-quarter fundamental period. This phenomenon is more obvious in Figure. 2, when the frequency changes between 47-53hz, the change of spectrum amplitude of three-quarter fundamental period least squares is less than 1%, which is far less than other algorithm.

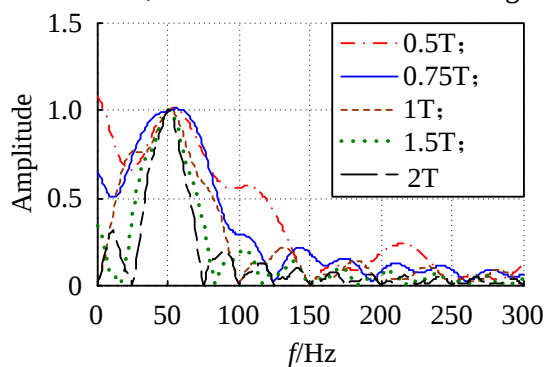


Figure.1 Spectrum analysis of LS in different sampling time

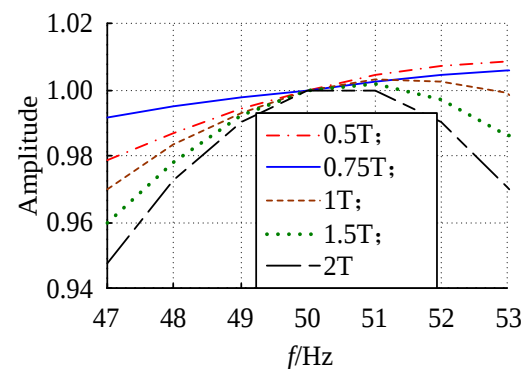


Figure.2 Frequency response near fundamental frequency

For the applications based on steady-state fundamental component, such as the relay protection, accurate extraction of the fundamental component is the purpose of the dynamic phasor traceability. Thus, the frequency spectrum of the traceability algorithm should decay rapidly around the fundamental frequency. In this case, as shown in Figure. 1, the least squares with longer sampling time is better. But for the dynamic phasor, the frequency change should be taken into consideration. So the frequency response around the fundamental frequency should be as flatter as possible. Thus, as shown in Figure. 2, the three quarters of fundamental frequency period is the best choice for the sampling time for the least squares algorithm.

4. Conclusions

In this study, the applicability of least squares in dynamic phasor traceability is studied. The fitting method is adopted in the traceability algorithm based on the least squares. This traceability algorithm can get rid of the limit that the sampling time must be an integer multiple of fundamental period. The least squares equals to the DFT only when the two times of sampling time is an integer multiple of fundamental period. When the sampling time is three-quarter the fundamental period, the frequency response of the least squares around the fundamental frequency is relative flat. The results indicate that the three-quarter fundamental period least squares is suitable for dynamic phasor estimation and traceability. The simulation results show that the least squares achieve the best traceability when the sampling time is three-quarter fundamental period.

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