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Vibration optimization of laminated composite plates using genetic algorithm with various discrete fiber angles

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Abstract. The mechanical characteristics of a laminated plate can be tailored by adjusting the fiber orientation angle of each orthotropic lamina. To find the fiber orientation angle of each lamina that maximizes the fundamental frequency, previous works analyses using various optimization algorithms have been performed so far. However, the solution space becomes significantly larger as the number of combinations in orientation angles increases. Thus the optimization process takes much more time. This paper sets up the orientation angles to discrete design variables, such as 15° , 30° , etc. Genetic algorithm is employed to find the optimal solutions. The CFRP rectangular composite plates with 8, 12 and 16 layers are considered as an example using the Ritz method for finding the eigenvalues. Two sets of classical boundary conditions are used as edge conditions. It was revealed that the fundamental frequency makes about 5% difference from the optimal value if it is in 30° steps and about 24% in 90° steps.

1. Introduction

The need for high stiffness and weight reduction is increasing for transportation equipment like automobiles, aircraft and rockets. Fiber-Reinforced Plastic (FRP) has been widely used in such fields as a composite material because it is lighter in weight, higher in specific strength and higher in specific stiffness than conventional metals. Several studies has been conducted to investigate the behavior of RC beam strengthened with FRP [1-2]. The mechanical characteristics of a laminated plate can be tailored by adjusting the fiber orientation angle of each orthotropic lamina.

To find the fiber orientation angle of each lamina that maximizes the fundamental frequency, various analyses using optimization algorithms have been performed so far. Narita [3] determined the orientation angles starting with the outermost layers by the Layerwise optimization method. Apalak et al. [4] analyzed using genetic algorithm, artificial neural networks and finite element method. Sadr et al. [5] studied using the combination of Elitist-genetic algorithm and finite strip method. Jafari et al. [6] attempted to optimise by ANN-GA method combining artificial neural networks and genetic algorithm. They treated not only square plate models but also skew ones, of which the latter is beyond the scope of the present paper. Furthermore, they compared the obtained results with [3-5].

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI. Published under licence by IOP Publishing Ltd 1 For problems like this, the solution space rapidly becomes more significant as the number of combinations in orientation angles increases, and optimisation analysis takes much more time. Besides, if laminas with limited types of orientation angles are prefabricated, that reduces manufacturing costs of laminated plates. This paper sets the orientation angles to discrete values as design variables, such as 15°, 30°, 45°, etc. This makes the solution space smaller, hence it reduces the calculation cost in comparison with continuous angle variables. Genetic algorithm is employed to find the optimal solutions. The fundamental frequencies of the optimal solutions obtained with discrete angle variables are compared to those with continuous ones. The differences of the values between the two approaches are investigated. The CFRP rectangular composite plates are analyzed using the Ritz method for finding the eigenvalues. This study is expected to clarify the setting of the proper step of angle variables to acquire a practical solution in practical time.

2. Vibration analysis of laminated plates by the Ritz method

Consider a symmetric laminated rectangular plate shown in figure 1. When the fiber direction L of the k-th layer forms an angle \prod_{k} with the x-axis, the stress-strain relationship of the layer is shown as

$$\begin{cases} \uparrow_{x} \\ \uparrow_{y} \\ \uparrow_{xy} \end{cases}^{(k)} = \begin{bmatrix} \overline{\underline{Q}}_{11} & \overline{\underline{Q}}_{12} & \overline{\underline{Q}}_{16} \\ \overline{\underline{Q}}_{12} & \overline{\underline{Q}}_{22} & \overline{\underline{Q}}_{26} \\ \overline{\underline{Q}}_{16} & \overline{\underline{Q}}_{26} & \overline{\underline{Q}}_{66} \end{bmatrix}^{(k)} \begin{cases} \mathsf{v}_{x} \\ \mathsf{v}_{y} \\ \mathsf{x}_{xy} \end{cases}$$
(1)

where $\overline{Q}_{ij}^{(k)}$ (*i*, *j* = 1,2,6) is the transformed reduced stiffness coefficient of the *k*-th layer. The strain energy and the kinetic energy are written as

$$U = \frac{1}{2} \iint_{A} \{|\}^{T} [D] \{|\} dA$$
(2)

$$T = \frac{1}{2} \dots h \iint_{A} \left(\frac{\partial W}{\partial t} \right)^{2} \mathrm{d}A$$
(3)

where $\{|\}$ is the curvature vector defined by

$$\left\{ \left| \right. \right\} = \left\{ -\frac{\partial^2 W}{\partial x^2} - \frac{\partial^2 W}{\partial y^2} - 2\frac{\partial^2 W}{\partial x \partial y} \right\}^T$$
(4)

[D] is the bending stiffness matrix, of which the components are described as

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \overline{Q}_{ij}^{(k)} \left(z_k^3 - z_{k-1}^3 \right) \quad (i, j = 1, 2, 6)$$
(5)

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Figure 1. Laminated plate and its *k*-th layer.

 W, A, \dots and *h* are the bending deflection function, area of the plate, density and thickness, respectively. z_{k-1} and z_k are the z-coordinates of both sides of layer k. To simplify the analysis, dimensionless coordinates $\langle , y \rangle$ and dimensionless frequency parameter Ω are introduced as shown below.

$$<=\frac{2x}{a}, y=\frac{2y}{b}, \Omega=\check{S}a^{2}\left(\frac{...h}{D_{0}}\right)^{1/2}$$
(6)

where S is angular frequency, D_0 is reference plate stiffness expressed as

$$D_0 = \frac{E_T h^3}{12(1 - \epsilon_{LT} \epsilon_{TL})} \tag{7}$$

Deflection is defined as

$$W(\langle , \mathbf{y}, t) = \sum_{m=0}^{M-1N-1} A_{mn} X_m(\langle) Y_n(\mathbf{y}) \sin \check{S}t$$
(8)

where $X_m(<), Y_n(y)$ are described as

$$X_{m}(\varsigma) = \varsigma^{m}(\varsigma + 1)^{bc1}(\varsigma - 1)^{bc3}$$

$$Y_{n}(y) = y^{n}(y + 1)^{bc2}(y - 1)^{bc4}$$
(9)

 A_{mn} is unknown coefficient and $X_m(<), Y_n(y)$ are the displacement functions that satisfy the boundary conditions geometrically. *bc*1, *bc*2, *bc*3 and *bc*4 are boundary condition indices defined as

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$$bc1, bc2, bc3, bc4 = \begin{cases} 0 & (free) \\ 1 & (simple support) \\ 2 & (clamp) \end{cases}$$
(10)

The stationary value is obtained by

$$\frac{\partial (U_{\max} - T_{\max})}{\partial A_{mn}} = 0$$

$$m = 0, 1, \cdots, M - 1, n = 0, 1, \cdots, N - 1$$
(11)

Thus the frequency equation is formed in matrix as

$$\left[K\right] - \Omega^2 \left[M\right] = 0 \tag{12}$$

where [K] and [M] are the total stiffness matrix and the total mass matrix, respectively. The optimized frequency is obtained by solving equation (12).

3. Problem setting

Symmetrically laminated rectangular plates with 8, 12 and 16 layers are to be analyzed. Material constants are set as follows:

$$E_L = 150.0 \text{ GPa}, E_T = 10.0 \text{ GPa}, G_{LT} = 5.0 \text{ GPa}, \notin_{LT} = 0.3$$

which represent the average characteristics of CFRP. The aspect ratio (b/a) is set to 1 in figure 1. The orientation angle of each lamina is a design variable, which is set to discrete values of 15°, 30°, 45°, 60° or 90°. Genetic algorithm is used to explore the optimum solutions, with the following parameters:

 Table 1. Optimal stacking sequences and fundamental frequencies.

(a) 8 layers

B.C.	Step	Stacking sequence	opt	dif. (%)
CSFF	15°	[15/-45/30/30]s	15.87	-0.56
	30°	[30/-30/-60/30]s	15.62	-2.17
	45°	[-45/45/45/45]s	14.67	-8.09
	60°	[0/0/0]s	14.49	-9.25
	90°	[0/0/0/0]s	14.49	-9.25
	cont.	[24.6/-49.2/25.9/29.2]s	15.96	0.00
SSSS	15°	[45/-45/-45] _s	55.62	0.00
	30°	[-60/60/60/30]s	52.65	-5.34
	45°	[45/-45/-45]s	55.62	0.00
	60°	[-60/60/60] _S	52.65	-5.34

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	90°	[0/90/90/90]s	42.55	-23.50			
	cont.	[-44.3/42.4/48.6/55.1]s	55.54	-0.15			
(b) 12 layers							
B.C.	Step	Stacking sequence	opt	dif. (%)			
CSFF	15°	[15/-45/30/30/15/30]s	15.90	-0.13			
	30°	[30/-30/30/-60/-60/30] _s	15.62	-1.90			
	45°	$[45/-45/0/-45/0/-45]_{s}$	14.85	-6.73			
	60°	[0/0/0/0/0]s	14.49	-9.02			
	90°	[0/0/0/0/0]s	14.49	-9.02			
	cont.	$[28.0/18.1/-47.9/-53.1/-40.7/6.0]_{s}$	15.92	0.00			
SSSS	15°	[45/-45/-45/-45/-45] _s	55.83	0.00			
	30°	[60/-60/-60/60/-60/-30]s	52.82	-5.38			
	45°	[-45/45/45/-45/45/45]s	55.83	0.00			
	60°	[-60/60/60/-60/60/60]s	52.82	-5.38			
	90°	[90/90/0/0/90/90]s	42.55	-23.78			
	cont.	$[47.7/-40.8/-52.4/-44.0/45.3/-58.1]_{\rm S}$	55.51	-0.57			
(c) 16 layers							
B.C.	Step	Stacking sequence	opt	dif. (%)			
CSFF	15°	[15/-45/15/30/30/30/-45/-45]s	15.90	0.00			
	30°	[30/30/-60/-30/-30/30/30/-30]s	15.62	-1.79			
	45°	[45/-45/0/-45/0/45/0/45]s	14.86	-6.54			
	60°	[0/0/0/0/0/0/0]s	14.49	-8.91			
	90°	$[0/0/0/0/0/0/0]_{s}$	14.49	-8.91			
	cont.	$[26.0/28.4/21.9/-46.9/-30.9/-62.7/-34.4/50.2]_{\rm S}$	15.78	-0.79			
SSSS	15°	[-45/45/45/-45/45/-45/-45/45] _S	55.83	0.00			
	30°	[60/-60/30/-30/-30/-60/-60]s	52.82	-5.39			
	45°	$[45/-45/-45/45/-45/45/-45]_{\rm S}$	55.83	0.00			
	60°	$[-60/60/60/-60/60/-60/-60/60]_{\rm S}$	52.83	-5.38			
	90°	[0/90/90/0/0/0/0]s	42.55	-23.78			

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cont. $[45.4/-38.8/47.9/-48.5/62.0/-58.1/-51.4/49.4]_{s}$ 55.06 -1.38

Population size 40, Crossover rate 0.07, Mutation rate 0.02 Maximum number of generations 200, Tournament size 4

Two-point crossover and tournament selection are adopted. For comparison, analyses are also performed using real value genetic algorithm with continuous angle variables. BLX- α is employed for a crossover with $\Gamma = 0.5$. CSFF and SSSS (C: clamped, S: simply supported, F: free) is set as edge boundary conditions. The number of approximate terms in the Ritz method is 8×8 .

4. Results

Table 1 presents the optimal fundamental frequencies and the stacking sequences obtained by the numerical calculation. The calculation was performed three times for each case of layers and boundary condition. The result that maximize the fundamental frequency is adopted. The difference is defined by comparing each fundamental frequency with the largest value as a reference value among cases with the same number of layers and the same boundary condition. Reference [4] gives the optimal solution for the case of 8-layer SSSS. It also shows the results of [1]-[3]. The stacking sequence of the optimal solution is indicated as $[45/-45/-45]_s$ or $[-45/45/45/48]_s$. It almost agrees with the results set at 15° and 45° steps in Table 1 (a). Furthermore, the fundamental frequency determined by the real value genetic algorithm falls within 0.15% of the optimal value, which can be regarded as reliable. From the above results, it can be seen that the difference is approximately 5% with 30° steps and about 10% with 60° steps. Whereas with 90° steps, the difference yields 24% in SSSS. It is considered this is because the angle values are selected that differ the most from those at which the optimal solution is obtained. In other words, it can be said that the difference is at most about 24% with 90° steps.

5. Conclusion

In this study, the influence was considered by restricting possible values of fiber orientation angle as discrete design variables, when the fiber orientation angles are optimized for the maximum fundamental frequency of rectangular symmetric laminated plates with 8, 12 and 16 layers. It was revealed that the fundamental frequency makes about 5% difference from the optimal value for fiber angle step of 30° and about 24% in 90°. Since the present study showed the results with only two sets of boundary conditions and the angle step of discrete value was 15°, it is expected in the future work that effects of more boundary conditions and smaller angle increment are considered.

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