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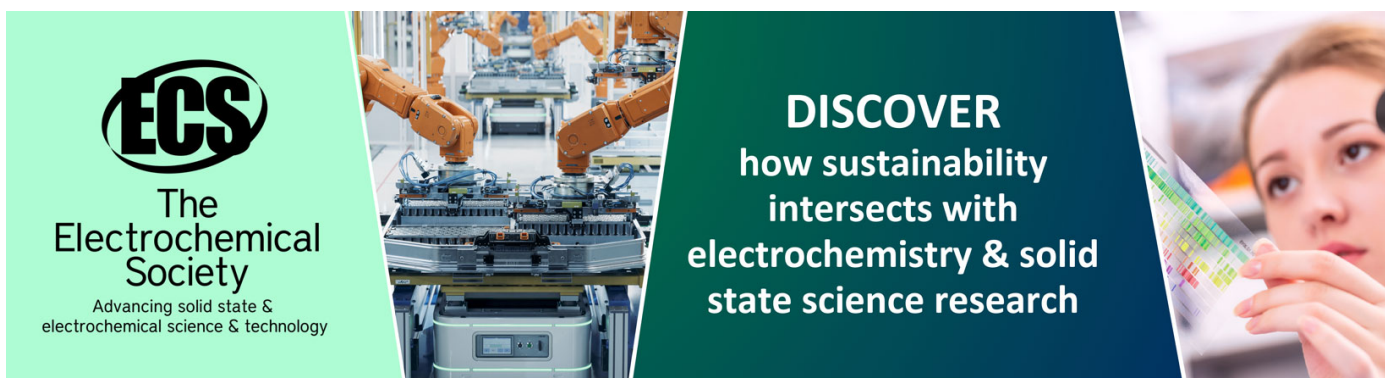
## Students' characteristics of students' obstacles in understanding the definition of a function

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# Students' characteristics of students' obstacles in understanding the definition of a function

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**Abstract.** Prior knowledge and experience on the concept of definition's function gave an important rule on the students' understanding. Their prior knowledge about function relate to their understanding of sets, ordered pairs, Cartesian product, and relations. Since a concept of function is the concept related to another topics, it will develop new knowledge about the formal definition of a function. Students' obstacle in understanding those concepts rose because they found some complicity in connecting their prior knowledge to the recent one. This research is a case study to characterize the student's obstacles in understanding those concepts to form a formal definition of a function. The subjects of this research were four out of nineteen students based on the criteria specified. The result showed that there were two characteristics, namely *refung present* as the first characteristic and *hirefung present* as the second characteristic. This study makes easier to create teaching learning process based on characteristics of students' obstacles. It can be a references for lecturers to arrange a learning trajectory based on the features of students' obstacles.

## 1. Introduction

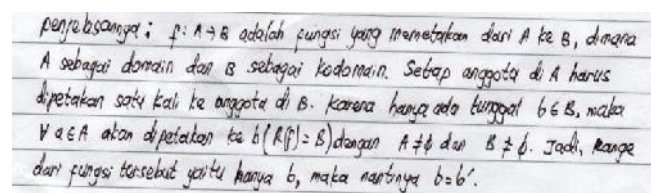
At the present time, students commonly have some difficulties in understanding formal definition of a function. These happen to students at senior high school and students at University. This gives negative affects on their learning formal definition of a function. Bishop et.al [1] discussion about obstacles and affordances for integer reasoning assert that the obtacles forces students to modify and adjust some aspects of their thinking to resolve their contradiction. It makes students difficult to understand formal definion of a function. Kumsa et.al [2] studying of students' understanding in concept states that the students obstacles caused by internal and external factors. The obstacles happen to students when they have difficulties in learning process. In addition, they happen when the students have misunderstanding in formal definition of function or they do not write a formal definition of function in their mathematical language. Their obstacles are also come from the method of teacher in teaching and learning process of formal definition of a function.

The application function's concept is mostly found in many curriculums, especially in Math Curriculum. These concepts are often linked not only in Math but also in another lesson. Because of this reason, this concept is considered as a base in Math, physics, engineering, astronomy, etc. [3]. In accordance to the importance of the concept of function in many lessons, some scientists agree that the concept of function is very crucial in the curriculum especially in learning of math so it becomes the main focus for the mathematics education research community [4, 5, 6, 7].



Definition of a function is given from high school to college level; the definition that has been understood by students is still a common concept. They think that a function  $f$  from a set  $A$  into a set  $B$  is a rule of correspondence that assigns to each element  $x$  in  $A$  a uniquely determined element  $f(x)$  in  $B$  [8].

Students still find some difficulties in understanding the definition. The definition has not yet been associated with formal mathematics; it still represents simple word. The disadvantage of this definition is to interpret the phrase "correspondence rule" [8]. To clear up this disadvantage, then the definition of function was related to sets, ordered pairs, Cartesian product, and relations. They were called formal definition of function. The formal definition of the function is that let  $A$  and  $B$  be sets. Then a function from  $A$  to  $B$  is a set  $f$  of ordered pairs in  $A \times B$  such that for each  $a \in A$  there exists a unique  $b \in B$  with  $(a, b) \in f$ . (In other words, if  $(a, b) \in f$  and  $(a, b') \in f$  then  $b = b'$ ) [6]. This definition would be the central reference in this study.



penyelesaian:  $f: A \rightarrow B$  adalah fungsi yang memetakan dari  $A$  ke  $B$ , dimana  $A$  sebagai domain dan  $B$  sebagai kodomain. Setiap anggota di  $A$  harus dipetakan satu kali ke anggota di  $B$ . Karena hanya ada tunggal  $b \in B$ , maka  $\forall a \in A$  akan dipetakan ke  $b$  ( $R(f) = B$ ) dengan  $A \neq \emptyset$  dan  $B \neq \emptyset$ . Jadi, Range dari fungsi tersebut yaitu hanya  $b$ , maka hasilnya  $b = b'$ .

**Figure 1.** Student's work explained the formal definition of function using their own language and their own understanding.

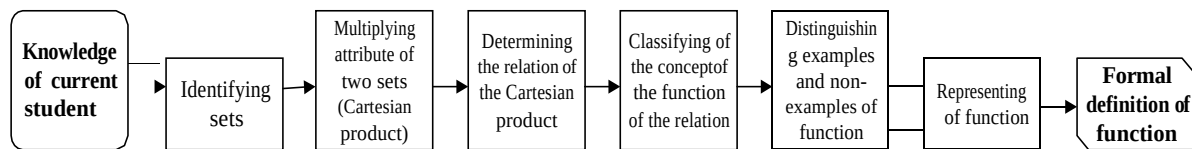
### In English Version:

The explanation:  $f: A \rightarrow B$  is a function mapping from  $A$  to  $B$  with  $A$  as domain and  $B$  as codomain. Each member in  $A$  must be mapped once to a member in  $B$ . Since there is only single  $b \in B$  then  $\forall a \in A$  will be mapped to  $b$  ( $R(f) = B$ ) with  $A \neq \emptyset$ ,  $B \neq \emptyset$ . So the range of the function is just  $b$ , meaning  $b = b'$ .

Figure 1 shows students' difficulties in understanding the formal definition of function. Their understanding used in high schools interferes their mind although they have already possessed the knowledge of formal form-functional definition. Set, ordered pair, Cartesian product, and relation are students' prior knowledge to form formal definitions of function. Students can apply those knowledges to learn new topics and to solve new problems [9].

Understanding a certain concept is the main goal in learning a new lesson. NCTM asserts the definition of function plays an important rule to build a concept of function itself [10]. Students should be able to understand that definition is to solve not only a certain problem about function but also another topics that still use its application. This reason becomes a main goal in studying function. To reach the learning goal, the students' understanding about the formal definition of function could be formed by their prior knowledge. In the process of forming up their new concept, it was started by correlating between their prior knowledge and experience with the new one [11]. Students' understanding the formal of a function can be checked through their learning flow by providing serious task. It makes students to define functions in their own language, make examples and non-examples and represent functions in various ways (tables, graphics, verbal, and algebra) to achieve an understanding of the concept of formal definition of a function. In completing the task, students usually found some obstacles in understanding the definition.

Obstacle is a knowledge to solve a certain problem, but if they are applied to a new problem or context, the knowledge is insufficient or creates a contradiction that prevents student to learn more [12, 13, 14, 15]. The obstacles in understanding the formal definition of function prevent the students to learn more about it. This kind of obstacles may occur because students have some trouble in connecting their prior knowledge with the new knowledge [16].



**Figure 2.** The learning flow of students in general in understanding the formal definition of a function.

Where:

-  : Knowledge of current student
-  : Development of student understanding
-  : Student learning flow
-  : Learning objectives

Students who have some obstacles and get stuck in completing the task can be supported by scaffolding. It could be in form of questions or instruction to put them at ease in understanding the formal definition of function. Scaffolding is a logical and structured arrangement of ideas to be understood in a sequence that makes students think farther and faster than they do themselves [17]. In this case, appropriate mathematical guidances are needed by providing scaffolding in the form of questions and instructing so the formal definition of a function could be well organized to achieve learning objectives.

Those obstacles also happen to students at Mathematics department of Madura University. They do not understand the concepts of ordered pairs, Cartesian product, and relation. Students of Mathematics Department at Madura University are still bringing their understanding definition of a function when they are in senior high school, without having to do their prior knowledge (set, ordered pairs, Cartesian product, and relation) to formal definition of a function. They fail to understand formal definition of a function completely. They have difficulties to relate their prior knowledge to their new knowledge since the students think that the concepts of formal definition of a function taught by the teachers at University are the same as the concepts in senior high schools. Therefore, it is interesting to discuss more about students' obstacles in understanding formal definition of a function. This study aims to characterize the students' obstacles in understanding formal definition of a function at Mathematics Department of Madura University.

## 2. Method

This research a case study which explains or describes a uniqueness of certain phenomenon happen to research subjects. This study describe in details of students' obstacles in understanding formal definition of a function at Mathematics Department of Madura University. They have no necessary parts, elements, and steps in understanding formal definition of a function. They do not follow the learning flow in understanding the formal definition of a function. This research was conducted to 19 students of Mathematics Education Study Program of Madura University who had taken calculus, introduction to mathematics and real analysis courses. Therefore, they had the basic concepts forming the formal definition of a function namely; set, ordered pairs, Cartesian product, relation and function. A task was given to those students to re-explain the formal definition of function using their own language and understanding. Tasks with enough information and well arrangement make students easier to understand the relations or patterns, so they can come out their problems [18]. From the population, 15 students defined the function based on the prior knowledge they had acquired in senior high school, while the rest defined it partially. The subjects of this research were four students who defined the definition of function in uncompleted way because of some obstacles they got.

This study applied observation, documentation, and interview to collect the data. The researcher looked in details their understanding formal definition of function and manuscript it. The answers of the research subject were analyzed based on the errors they made in understanding and representing the formal definition of function. They usually do not match with the learning flow of general definition of a function. These learning flow include (a) Identifying sets, (b) Multiplying attribute of two sets (Cartesian product) (c) Determining the relation of the Cartesian product, (d) Classifying of the concept of the function of the relation, (e) Distinguishing examples and non-examples of functions, and (f) Representing the function. Based on the learning flow, students found some difficulties in defining that concepts because they have no enough knowledge and unable to link between the concepts to the real example. The subjects of this research were analyzed based on the scaffolding in form of questions and instructing so the learning goal would be reached. The objective of this research was to know the characteristic of students' obstacle in understanding the concepts of function to make the formal definition of function.

### 3. Result and Discussion

Based on tests, observation, and interview, students of mathematics departments of Madura University experience some obstacles in understanding formal definition of a function. Table 1 describes the students' obstacles and characteristics in understanding formal definition of a function.

**Table 1.** The students' obstacles and characteristics in understanding formal definition of a function.

| Students | Obstacles                                                                                                                    | Characteristics         |
|----------|------------------------------------------------------------------------------------------------------------------------------|-------------------------|
| S1       | The first subject is not able to link the concept of a relation with a Cartesian product to defining a function (Concept 3)  | <i>refung present</i>   |
|          | S1 has not classified member functions based on relation to form a formal definition yet (Concept 4)                         |                         |
|          | S1 also represents a wrong example of a function in algebraic form (Concept 6)                                               |                         |
| S2       | The second subject is not able to link the concept of a relation with a Cartesian product to defining a function (Concept 3) | <i>refung present</i>   |
|          | S2 has not classify function's members based on relation to form a formal definition yet (Concept 4)                         |                         |
|          | S2 also represents a wrong example of a function in algebraic form (Concept 6)                                               |                         |
| S3       | There is no between the definitions symbolically and verbally                                                                | <i>hirefung present</i> |
|          | The third subject multiplied two sets (Cartesian product) and relation (concept 2 and concept 3)                             |                         |
|          | S3 classify members from function $A$ to $B$ based on relation (concept 4)                                                   |                         |
| S4       | S3 also represents a wrong example of a function in algebraic form (Concept 6)                                               | <i>hirefung present</i> |
|          | The fourth subject do not understand the definition of Cartesian product (concept 1 and concept 2)                           |                         |
|          | S4 is not able to link the relation to the function (Concept 3)                                                              |                         |
|          | S4 has not classified the examples based on the relation's concept on each function (Concept 4)                              |                         |
|          |                                                                                                                              |                         |

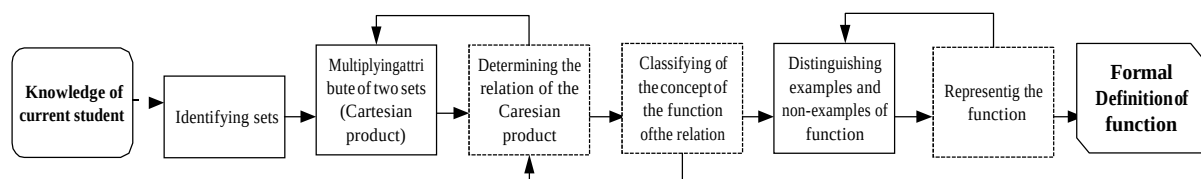
| Students | Obstacles                                                                      | Characteristics |
|----------|--------------------------------------------------------------------------------|-----------------|
|          | S4 also represents a wrong example of a function in algebraic form (Concept 6) |                 |

Table 1 shows that students of Mathematics Department of Madura University have obstacles in understanding formal definition of a function. They are multiplying attributes of two sets (Cartesian product), determining the relation of the Cartesian product, classifying of the concept of the function of the relation, representing the function. S1 and S2 experiencing obstacles in determining the relation of the Cartesian product, classifying the concept of the function of the relation, and representing the function. S3 dan S4 have obstacles in multiplying attributes of two sets (Cartesian product), determining the relation of the Cartesian product, classifying the concept of the function of the relation, and Representing the function.

The characteristics of students' obstacles in understanding formal definition of a function classify into two categories those are *refung* (relation, function, and representation of function) and *hirefung* present (multiplication of two sets, relations, functions and representation of function). The characteristics of S1 and S2 include to *refung* present, but S3 and S4 are *hirefung present*. *Refung* presents have obstacles in determining the relation of the Cartesian product, classifying the concept of the function of the relation, and representing the function. However, *hirefung present* have obstacles in multiplying attributes of two sets (Cartesian product), determining the relation the Cartesian product, Classifying of the concept of the function of the relation, and Representing the function. To help students overcoming from their obstacles in understanding formal definition of a function, it is important for the teachers to support students by scaffolding. Scaffolding can be used by the teachers to benefit students achieve their learning objectives, defining function formally.

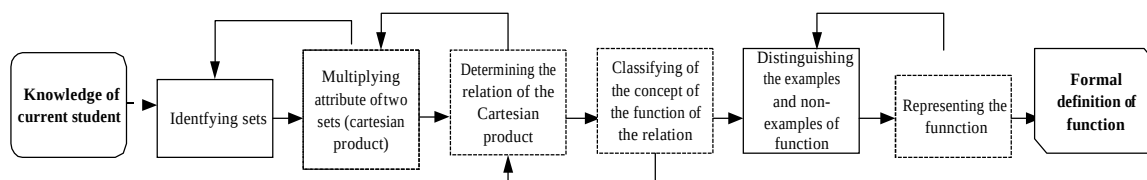
Teachers of mathematics department at Madura University give different ways of scaffolding depend on their obstacles. This way given to help them understand easily about the formal definition of function. There were six kind of strategies in scaffolding, (a) Modelling; (b) Contingency management; (c) Feedback; (d) Instructing; (e) Questioning; and (f) Cognitive structure [19]. Instructing and questioning were two kinds of scaffolding used in this research. Each question was accompanied by clear illustrations to help the students understand the questions, so they could respond with their right answer, mindset, and understanding [20].

The first and second subject got some obstacles in understanding concept 3, 4 and 6. The scaffolding type that was given to first and second subject was in form of question-related to concept 2 (Multiple attribute of two set (Cartesius product). Questions related to concept 3, determining the relation of Cartesius product, were also given to the subject when they got the obstacles in concept 4. Furthermore, when the subject got some difficulties in understanding concept 6, the scaffolding needed was in concept 5. The subject was asked to differentiate between example and non-example by drawing a diagram so it would help them represent the concept easily. Students should have a whole comprehension on the prior concept so it could guide them to comprehend the next concept. So the characteristic of learning flow of S1 and S2 degree (the first characteristic) was called *refung present* (relation, function, and function representation). Figure 3 explains the learning flow of first character.



**Figure 3.**Students' learning flow on the first character.

The third and fourth subject got some obstacles to comprehend concept 2, 3, 4 and 6. This would interfere their learning flow in understanding the formal definition of function. The third and fourth subject faced the first stuck on concept 2, that is about multiple attribute of two sets (Cartesian product). To solve this problem, scaffolding was needed. It was in form of questions about set (concept 1). Then, after the subjects had their whole-comprehension on the second concept, they proceeded to the next. A same treatment was given when the subject got some obstacles in this step. It was a scaffolding of the two-sets Cartesian product to determine the relation of two sets. While in concept 4, it took scaffolding about concept 2 because the function is part of the Cartesian product. The students' learning flow in understanding the definition of formal function should be linked to each other. Meanwhile, when the subject found some difficulties in comprehending concept 6, the scaffolding required by third and fourth subject was about concept 5 because it distinguished between examples and non-examples of functions using diagrams. They can represent it easily. The learning flow characteristics of the third and fourth subject (second characteristic) are *hirefung present* (multiplication of two sets, relations, functions and representations). Figure 4 describes the learning flow of the second characteristic.



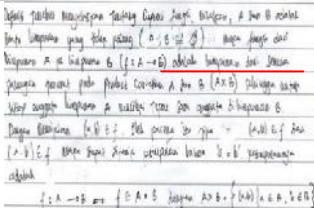
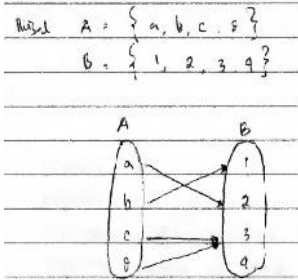
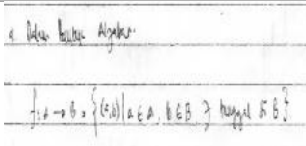
**Figure 4.** Students' learning flow on the second character.

The research indicators that would be used in measuring the students' understanding the formal definition of function were: (1) Student ideas relate to the definition of function using their own language and previous understanding; (2) His ability to make example and non-examples; and (3) The ability is to present (represent) functions in different forms, for example: verbal, numerical, visual, algebraic, and in ordered pairs. These are in line with Bishop statement that the learners' ideas about the underlying phenomenon, their ability to present the functions in different forms, and their ability to solve the problem of function from one representation to another become the indicators of someone's understanding on defining function [1]. The students were able to comprehend the whole- formal definition of function if they had reached those indicators. The formal definition of function was linked to set in order not to raise ambiguity [11].

The students still use the definition in senior high school. It causes the ways in which definitions appear in school mathematics vary significantly with the type of mathematics involved and with the age of the intended student [22], starting from informal situations to more formal. However, in higher education, students often are asked to memorize the definition (even if it is not understandable for them) in the course and they are given credit in examinations for being able to repeat it [23].

Based on the analysis on the students' answer, it was found that the students got some obstacles in defining the formal function. Those obstacles prevented the students to learn more about the function [21]. While the obstacles actually gave an important rule in learning process because it forced the students to modify and match their thinking aspects to solve some contradiction [13]. Obstacles can be overcome by providing assistance to students through scaffolding with the assignment to achieve the goal of learning in understand is the formal definition of a function. Adult deliberately teaches strategies which will enable the child to solve problems posed by a task [24]. Table 2 explains the students' obstacle and their scaffolding in understanding the formal definition of functions as follows.

**Tabel 2.** Students' obstacles and the scaffolding given in understanding the formal definition of function.

| Subject | Obstacles                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Scaffolding                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|---------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| S1      |  <p>In English version:<br/>It explains about definition of function. For example <math>A</math> and <math>B</math> are a non-empty set, the function of set <math>A</math> to set <math>B</math> (<math>f: A \rightarrow B</math>) is the set of all ordered pairs in Cartesian product <math>A</math> and <math>B</math> (<math>A \times B</math>) so that for each member of set <math>A</math> has exactly one member in set <math>B</math>. With such <math>(a, b) \in f</math>. Therefore if <math>(a, b) \in f</math> and <math>(a, b') \in f</math> it can be deduced that <math>b = b'</math>. The conclusion is</p> $f: A \rightarrow B \Leftrightarrow f \subseteq A \times B \text{ dengan } A \times B = \{(a, b)   a \in A, b \in B\}$ | <p>1. Determine the relation (relationship or linking) from set <math>A</math> to <math>B</math> based on the set that you've created!<br/>Answer :<br/>For example:<br/><math>A = \{a, b, c, d\}, B = \{1, 2, 3, 4\}</math><br/><i>Relation A to B</i><br/> <math display="block">\{(a, 1)\}, \{(a, 2)\}, \dots, \{(b, 2)\}, \dots \{(d, 4)\}, \{(a, 1), (b, 2)\}, \dots</math> <math display="block">= \{(a, 2), (b, 2), (c, 2), (d, 2)\}, \{(a, 1), (b, 2), (c, 3), (d, 3)\}</math> <math display="block">\{(a, 1), (b, 2), (c, 3), (d, 4)\}</math></p> <p>2. Based on the Cartesian product of the previous two sets, explain the concept of a relation!<br/>Answer:<br/><br/>Relation is a subset of Cartesian product of two sets</p> |
|         | <p>The first subject is not able to link the concept of a relation with a Cartesian product to defining a function (Concept 3)</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | <p>Classify the members of the function set based on the relation you have made!<br/>Answer:<br/><math>A \neq \emptyset, B \neq \emptyset</math><br/><math>A = \{a, b, c, d\}, B = \{1, 2, 3, 4\}</math><br/><i>function A to B</i><br/> <math display="block">\{(a, 1), (b, 1), (c, 1), (d, 1)\}, \{(a, 2), (b, 2), (c, 2), (d, 2)\}, \dots</math> <math display="block">= \{(a, 2), (b, 1), (c, 3), (d, 4)\}, \dots</math> <math display="block">\{(a, 1), (b, 2), (c, 3), (d, 4)\}</math></p>                                                                                                                                                                                                                                            |
|         |  <p>In algebraic form:</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | <p>1. If <math>f: A \rightarrow B</math>, then the algebraic form of the function is:<br/><math display="block">f(a) = b, \forall a \in A, b \in B</math></p> <p>2. Make a representation in algebraic form based on the example that you have made,</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |

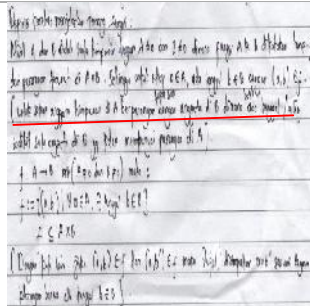


| Subject | Obstacles                                                                         | Scaffolding                                                                                                 |
|---------|-----------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------|
|         | $f: A \rightarrow B$<br>$= \{(a, b)   a \in A, b \in B \ni \text{unique in } B\}$ | by:<br>$A = \{a, b, c, d\}, B = \{1, 2, 3, 4\}$<br>Answer:<br>$f(a) = 1, f(b) = 2, f(c) = 3,$<br>$f(d) = 3$ |

S1 also represented a wrong example of a function in algebraic form (Concept 6)

The Chaacteristic of S1 are *refung present*

S2



In English version:

The definition describes the function: let  $A$  and  $B$  are non-empty set. Where the function  $A$  and  $B$  is called to function  $f$  of the ordered pair in  $A \times B$ . Thus for every  $a \in A$  there is a single  $b \in B$  with  $(a, b) \in f$ . (for each member in  $A$  pair exactly one with a member in  $B$  where there is a single / at least one member in  $B$  that has no pair in  $A$ )

1. Write the definition of Cartesian product from set  $A$  and  $B$ !

Answer:

$A \times B$ :

$= \{(a, b) | a \in A \text{ and } b \in B\}$

2. Find the relation member from set  $A$  to  $B$  based on the definition of Cartesian product of  $A$  and  $B$ !

If

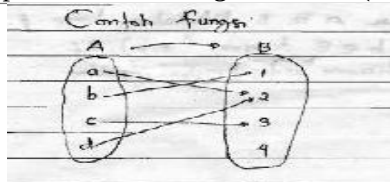
$A = \{a, b, c, d\}, B = \{1, 2, 3, 4\}$

Answer:

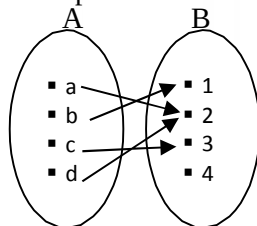
*Relation A to B*

$\{(a, 1)\}, \{(a, 2)\}, \{(a, 3)\}$   
 $\{(b, 2)\}, \dots \{(d, 4)\}, \{(a, 1), (b,$   
 $= \{(a, 2), (b, 2), (c, 2), (d, 2$   
 $\{(a, 1), (b, 2), (c, 3), (d, 3)\}$   
 $\{ \{(a, 1), (b, 2), (c, 3), (d, 4$

The second subject is not able to link the concept of a relation with a Cartesian product to defining a function (Concept 3)



Example of function:



1. Classify the function members through each member of relationships!

Answer:

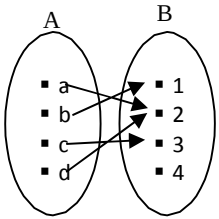
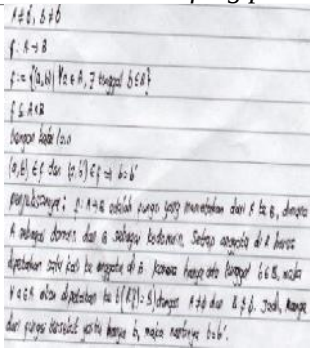
*function A to B*

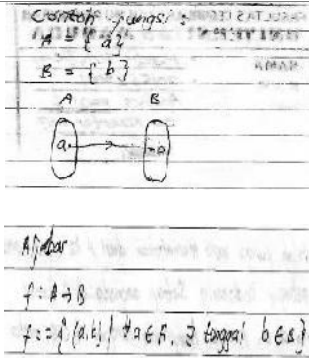
$\{(a, 1), (b, 1), (c, 1), (d, 1)\},$   
 $\{(a, 2), (b, 2), (c, 2), (d, 2)\}, \dots$   
 $= \{(a, 2), (b, 1), (c, 3), (d, 4)\}, \dots$   
 $\{ \{(a, 1), (b, 2), (c, 3), (d, 4)\}$

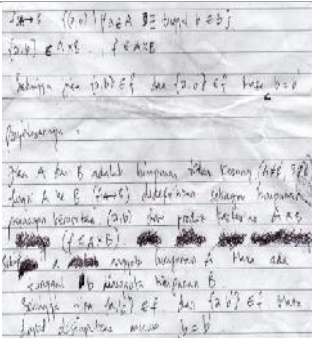
2. Define the function based on set identification, Cartesian product and relation!

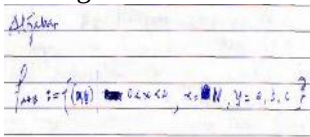
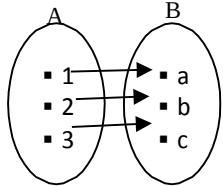
Answer:

$A$  and  $B$  set,  $A \neq \emptyset, B \neq \emptyset . A$

| Subject                                           | Obstacles                                                                                                                                                                                                                                                                                                                                                                                                                                               | Scaffolding                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |            |              |              |   |   |   |   |   |   |   |   |   |   |   |   |
|---------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|--------------|--------------|---|---|---|---|---|---|---|---|---|---|---|---|
|                                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                                         | function $A$ to $B$ is $f$ from ordered pairs in $A \times B$ such that $\forall a \in A$ , there are uniquely $b \in B$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |            |              |              |   |   |   |   |   |   |   |   |   |   |   |   |
|                                                   | S2 is not classify function's members based on relation to form a formal definition yet (Concept 4)                                                                                                                                                                                                                                                                                                                                                     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |            |              |              |   |   |   |   |   |   |   |   |   |   |   |   |
|                                                   | S1 has not classified memberfunctions based on relation to form a formal definition yet (Concept 4)                                                                                                                                                                                                                                                                                                                                                     | <p>1. draw a diagram of a function based on the set example you have made!</p> <p>Answer:</p>  <p>2. Based on the diagram above, make the table that contains the domain, the codomain and range (the result area)!</p> <p>Answer:</p> <table border="1"> <thead> <tr> <th>Domain (A)</th><th>Codomain (B)</th><th>Range <math>f(a)</math></th></tr> </thead> <tbody> <tr> <td>a</td><td>1</td><td>2</td></tr> <tr> <td>b</td><td>2</td><td>1</td></tr> <tr> <td>c</td><td>3</td><td>3</td></tr> <tr> <td>d</td><td>4</td><td>2</td></tr> </tbody> </table> | Domain (A) | Codomain (B) | Range $f(a)$ | a | 1 | 2 | b | 2 | 1 | c | 3 | 3 | d | 4 | 2 |
| Domain (A)                                        | Codomain (B)                                                                                                                                                                                                                                                                                                                                                                                                                                            | Range $f(a)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |            |              |              |   |   |   |   |   |   |   |   |   |   |   |   |
| a                                                 | 1                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |            |              |              |   |   |   |   |   |   |   |   |   |   |   |   |
| b                                                 | 2                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |            |              |              |   |   |   |   |   |   |   |   |   |   |   |   |
| c                                                 | 3                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |            |              |              |   |   |   |   |   |   |   |   |   |   |   |   |
| d                                                 | 4                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |            |              |              |   |   |   |   |   |   |   |   |   |   |   |   |
|                                                   | S2 also represents a wrong example of a function in algebraic form (Concept 6)                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |            |              |              |   |   |   |   |   |   |   |   |   |   |   |   |
| The Chaacteristic of S2 are <i>refung present</i> |                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |            |              |              |   |   |   |   |   |   |   |   |   |   |   |   |
| S3                                                |  <p>In English version:<br/>The explanation: <math>f: A \rightarrow B</math> is a function that mapped from <math>A</math> to <math>B</math> with <math>A</math> as domain and <math>B</math> as codomain. Each member in <math>A</math> must be mapped once to a member in <math>B</math>. Since there is only a single member in <math>A</math> must be mapped</p> | <p>1. If <math>A \neq \emptyset</math>, <math>B \neq \emptyset</math>, and <math>f \subseteq A \times B</math>. Write the definition of Cartesian product of the two sets!</p> <p>Answer:</p> $A \times B = \{(a, b)   \forall a \in A, b \in B\}$ <p>2. Based on the example that you have made, find the Cartesian product and relation!</p> <p>Answer:</p> $A = \{a\}, B = \{b\}$ $A \times B = \{(a, b)\}$ $\text{Relation } A \text{ to } B = \{(a, b)\}$ $\text{relation} \subseteq A \times B$                                                                                                                                         |            |              |              |   |   |   |   |   |   |   |   |   |   |   |   |

| Subject | Obstacles                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | Scaffolding                                                                                                                                                                                                                                        |
|---------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|         | <p>once to member <math>B</math>. Since there is only single <math>b \in B</math> then <math>\forall a \in A</math> will be mapped to <math>b</math> (<math>R(f)) = B</math> with <math>A \neq \emptyset</math>, <math>B \neq \emptyset</math>. So the range of the function is just <math>b</math>, meaning <math>b = b'</math></p> <p>There is no match between the definitions symbolically and verbally</p> <p>The third subject multiplied two sets (Cartesian product) and relation (concept 2 and concept 3)</p> | <p>3. Based on the example that you have made, find the Cartesian product and relation!</p> <p>Answer:</p> $A = \{a\}, B = \{b\}$ $A \times B = \{(a, b)\}$ $\text{Relation } A \text{ to } B = \{(a, b)\}$ $\text{relation} \subseteq A \times B$ |
|         | <p>S3 classify members from function <math>A</math> to <math>B</math> based on relation (concept 4)</p>                                                                                                                                                                                                                                                                                                                                                                                                                 | <p>4. Classify function members from <math>A</math> to <math>B</math></p> <p>Answer: <math>\text{function } A \text{ to } B = \{(a, b)\}</math></p>                                                                                                |
|         |                                                                                                                                                                                                                                                                                                                                                                                                                                       | <p>5. If <math>f: A \rightarrow B</math>, then the algebraic form of the function is:</p> $f(a) = b, \forall a \in A, b \in B$ <p>6. Make a representation in algebraic form from the example that you have made!</p> <p>Answer:</p> $f(a) = b$    |
|         | <p>In English version:</p> <p>Algebra</p> $f: A \rightarrow B$ $f := \{(a, b)   \forall a \in A, \exists \text{ unique } b \in B\}$ <p>S3 also represents a wrong example of a function in algebraic form (Concept 6)</p> <p>The Characteristic of S3 are <i>hirefung present</i></p>                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                    |

| Subject | Obstacles                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Scaffolding                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| S4      |  <p>In English version:<br/>         If <math>A</math> and <math>B</math> are not empty set (<math>A \neq \emptyset</math>, <math>B \neq \emptyset</math>) from <math>A</math> to <math>B</math>. <math>f: A \rightarrow B</math> is defined as an ordered pairs of Cartesian product <math>A \times B</math> (<math>f \in A \times B</math>). Each <math>a \in A</math> then there is a single <math>b \in B</math>. so if <math>\{a, b\} \in f</math> and <math>\{a, b'\} \in f</math> then <math>b = b'</math></p> | <ol style="list-style-type: none"> <li>1. Determine the requirement of two sets so they can be multiplied!<br/>             Answer :<br/> <math>A \neq \emptyset</math>, and <math>B \neq \emptyset</math></li> <li>2. Write the definition of Cartesian product!<br/>             Answer :<br/> <math>A \times B</math><br/> <math>= \{(a, b)   \forall a \in A, b \in B\}</math></li> <li>3. Determine the members of Cartesian product based on the set that you have made!<br/>             Answer :<br/> <math>A = \{1, 2, 3\}, B = \{a, b, c\}</math><br/> <math>(1, a), (1, b), (1, c),</math><br/> <math>A \times B = \{(2, a), (2, b), (2, c), \dots,</math><br/> <math>(1, a), (2, b), (3, c)\}</math></li> </ol> |
|         | The fourth subject did not understand the definition of Cartesian product (concept 1 and concept 2)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|         | S4 was not able to link the relation to the function (Concept 3)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | <ol style="list-style-type: none"> <li>1. Which are members of a Cartesian product, in form of sets or ordered pairs?<br/>             Answer :<br/>             Ordered pairs</li> <li>2. What do you know about relation?<br/>             Answer :<br/>             Relation is a subset of Cartesian product</li> <li>3. Determine the members of the relation which is related to Cartesian product!<br/>             Answer<br/> <i>relation A to B</i><br/> <math>\{(1, a), \{(1, b), \dots,</math><br/> <math>= \{(1, a), (1, b), \{(2, a), (3, b)\}, \dots,</math><br/> <math>\{(1, a), (2, c)(3, b)\},</math></li> </ol>                                                                                          |
|         | S4 has not classified the examples based on the relation's concept on each function (Concept 4)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | Classify the function member based on the relation's members known!<br>Answer :<br><i>function A to B</i><br>$\{(1, a), (2, b), (3, c)\},$<br>$\{(1, b), (2, a), (3, c)\},$<br>$= \{(1, c), (2, b), (3, a)\}$<br>$\dots,$<br>$\{(1, b), (2, c), (3, a)\}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |

| Subject                                                                                                                                                                                                 | Obstacles | Scaffolding                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |        |          |                 |   |   |   |   |   |   |   |   |   |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------|----------|-----------------|---|---|---|---|---|---|---|---|---|
| <p>Numeric :</p>  <p>Algebraic :</p>  |           | <p>1. Draw a diagram of a function based on the set example you have made!<br/>Answer:</p>  <p>2. Based on the diagram above, make the table that contains the domain, the codomain and range (the result area)!</p> <p>Answer:</p> <table border="1"> <thead> <tr> <th>Domain</th><th>Codomain</th><th>Range<br/><math>f(a)</math></th></tr> </thead> <tbody> <tr> <td>1</td><td>a</td><td>a</td></tr> <tr> <td>2</td><td>b</td><td>b</td></tr> <tr> <td>3</td><td>c</td><td>c</td></tr> </tbody> </table> <p>3. If <math>f: A \rightarrow B</math>, then the algebraic form of the function is:<br/><math>f(a) = b, \forall a \in A, b \in B</math></p> <p>4. Make a representation in the algebraic form of the example you have made, with:<br/><math>A = \{1, 2, 3\}, B = \{a, b, c\}</math></p> <p>Answer:</p> <p>The algebraic representation is :<br/><math>f(1) = a, f(2) = b, f(3) = c</math></p> | Domain | Codomain | Range<br>$f(a)$ | 1 | a | a | 2 | b | b | 3 | c | c |
| Domain                                                                                                                                                                                                  | Codomain  | Range<br>$f(a)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |        |          |                 |   |   |   |   |   |   |   |   |   |
| 1                                                                                                                                                                                                       | a         | a                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |        |          |                 |   |   |   |   |   |   |   |   |   |
| 2                                                                                                                                                                                                       | b         | b                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |        |          |                 |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                       | c         | c                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |        |          |                 |   |   |   |   |   |   |   |   |   |
| S4 also represents a wrong example of a function in algebraic form (Concept 6)                                                                                                                          |           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |        |          |                 |   |   |   |   |   |   |   |   |   |
| The Chaacteristic of S4 are <i>hirefung present</i>                                                                                                                                                     |           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |        |          |                 |   |   |   |   |   |   |   |   |   |

#### 4. Conclusion

University experienced several obstacles in formulating formal definition of function. Research results show the characteristics of the students' obstacle in forming the formal definition of function are *refung present* (relation, function, and representation of function) and *hirefung present* (multiplication of two sets, relations, functions and representation of function). When the students are able to express

their ideas using their own words, able to relate the example and non-example around their surrounding with the formal definition of function using their own prior knowledge, able to represent functions in different forms (verbal, numerical, visual, algebraic, and ordered pairs), automatically they understood the concept well.

The results of this study can be used to make learning trajectory students in learning the formal definition of a function. It implies that the teachers are creating a learning design emphasizes on students' understanding on concepts based on the characteristics of learning obstacles.

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