PREFACE

Symmetries and integrability of difference equations

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Symmetries and integrability of difference equations

The concept of integrability was introduced in classical mechanics in the 19th century for finite dimensional continuous Hamiltonian systems. It was extended to certain classes of nonlinear differential equations in the second half of the 20th century with the discovery of the inverse scattering transform and the birth of soliton theory. Also at the end of the 19th century Lie group theory was invented as a powerful tool for obtaining exact analytical solutions of large classes of differential equations. Together, Lie group theory and integrability theory in its most general sense provide the main tools for solving nonlinear differential equations.

Like differential equations, difference equations play an important role in physics and other sciences. They occur very naturally in the description of phenomena that are genuinely discrete. Indeed, they may actually be more fundamental than differential equations if space-time is actually discrete at very short distances. On the other hand, even when treating continuous phenomena described by differential equations it is very often necessary to resort to numerical methods. This involves a discretization of the differential equation, i.e. a replacement of the differential equation by a difference one.

Given the well developed and understood techniques of symmetry and integrability for differential equations a natural question to ask is whether it is possible to develop similar techniques for difference equations. The aim is, on one hand, to obtain powerful methods for solving 'integrable' difference equations and to establish practical integrability criteria, telling us when the methods are applicable. On the other hand, Lie group methods can be adapted to solve difference equations analytically. Finally, integrability and symmetry methods can be combined with numerical methods to obtain improved numerical solutions of differential equations.

The origin of the SIDE meetings goes back to the early 1990s and the first meeting with the name 'Symmetries and Integrability of Discrete Equations (SIDE)' was held in Estérel, Québec, Canada. This was organized by D Levi, P Winternitz and L Vinet. After the success of the first meeting the scientific community decided to hold bi-annual SIDE meetings. They were held in 1996 at the University of Kent (UK), 1998 in Sabaudia (Italy), 2000 at the University of Tokyo (Japan), 2002 in Giens (France), 2004 in Helsinki (Finland) and in 2006 at the University of Melbourne (Australia). In 2008 the SIDE 8 meeting was again organized near Montreal, in Ste-Adèle, Québec, Canada. The SIDE 8 International Advisory Committee (also the SIDE steering committee) consisted of Frank Nijhoff, Alexander Bobenko, Basil Grammaticos, Jarmo Hietarinta, Nalini Joshi, Decio Levi, Vassilis Papageorgiou, Junkichi Satsuma, Yuri Suris, Claude Vialet and Pavel Winternitz. The local organizing committee consisted of Pavel Winternitz, John Harnad, Véronique Hussin, Decio Levi, Peter Olver and Luc Vinet. Financial support came from the Centre de Recherches Mathématiques in Montreal and the National Science Foundation (through the University of Minnesota).

Proceedings of the first three SIDE meetings were published in the *LMS Lecture Note* series. Since 2000 the emphasis has been on publishing selected refereed articles in response to a general call for papers issued after the conference. This allows for a wider author base, since the call for papers is not restricted to conference participants. The SIDE topics thus are represented in special issues of *Journal of Physics A: Mathematical and General* **34**

(48) and *Journal of Physics A: Mathematical and Theoretical*, **40** (42) (SIDE 4 and SIDE 7, respectively), *Journal of Nonlinear Mathematical Physics* **10** (Suppl. 2) and **12** (Suppl. 2) (SIDE 5 and SIDE 6 respectively).

The SIDE 8 meeting was organized around several topics and the contributions to this special issue reflect the diversity presented during the meeting. The papers presented at the SIDE 8 meeting were organized into the following special sessions: geometry of discrete and continuous Painlevé equations; continuous symmetries of discrete equations — theory and applications; algebraic aspects of discrete equations; singularity confinement, algebraic entropy and Nevanlinna theory; discrete differential geometry; discrete integrable systems and isomonodromy transformations; special functions as solutions of difference and q-difference equations.

This special issue of the journal is organized along similar lines. The first three articles are topical review articles appearing in alphabetical order (by first author). The article by Doliwa and Nieszporski describes the Darboux transformations in a discrete setting, namely for the discrete second order linear problem. The article by Grammaticos, Halburd, Ramani and Viallet concentrates on the integrability of the discrete systems, in particular they describe integrability tests for difference equations such as singularity confinement, algebraic entropy (growth and complexity), and analytic and arithmetic approaches. The topical review by Konopelchenko explores the relationship between the discrete integrable systems and deformations of associative algebras. All other articles are presented in alphabetical order (by first author). The contributions were solicited from all participants as well as from the general scientific community.

The contributions published in this special issue can be loosely grouped into several overlapping topics, namely:

- Geometry of discrete and continuous Painlevé equations (articles by Spicer and Nijhoff and by Lobb and Nijhoff).
- Continuous symmetries of discrete equations — theory and applications (articles by Dorodnitsyn and Kozlov; Levi, Petrera and Scimiterna; Scimiterna; Ste-Marie and Tremblay; Levi and Yamilov; Rebelo and Winternitz).
- Yang–Baxter maps (article by Xenitidis and Papageorgiou).
- Algebraic aspects of discrete equations (articles by Doliwa and Nieszporski; Konopelchenko; Tsarev and Wolf).
- Singularity confinement, algebraic entropy and Nevanlinna theory (articles by Grammaticos, Halburd, Ramani and Viallet; Grammaticos, Ramani and Tamizhmani).
- Discrete integrable systems and isomonodromy transformations (article by Dzhamay).
- Special functions as solutions of difference and q-difference equations (articles by Atakishiyeva, Atakishiyev and Koornwinder; Bertola, Gekhtman and Szmigielski; Vinet and Zhedanov).
- Other topics (articles by Atkinson; Grünbaum; Nagai, Kametaka and Watanabe; Nagiyev, Guliyeva and Jafarov; Sahadevan and Uma Maheswari; Svinin; Tian and Hu; Yao, Liu and Zeng).

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Decio Levi, *Università degli Studi di Roma Tre, Rome*
Peter Olver, *University of Minnesota, Minneapolis*
Zora Thomova, *SUNY Institute of Technology, Utica*
Pavel Winternitz, *Université de Montréal, Montreal*
Guest Editors