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Optimization of Control Points Number at Coordinate Measurements based on the Monte-Carlo Method

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Abstract. Improving the quality of products causes an increase in the requirements for the accuracy of the dimensions and shape of the surfaces of the workpieces. This, in turn, raises the requirements for accuracy and productivity of measuring of the workpieces. The use of coordinate measuring machines is currently the most effective measuring tool for solving similar problems. The article proposes a method for optimizing the number of control points using Monte Carlo simulation. Based on the measurement of a small sample from batches of workpieces, statistical modeling is performed, which allows one to obtain interval estimates of the measurement error. This approach is demonstrated by examples of applications for flatness, cylindricity and sphericity. Four options of uniform and uneven arrangement of control points are considered and their comparison is given. It is revealed that when the number of control points decreases, the arithmetic mean decreases, the standard deviation of the measurement error increases and the probability of the measurement α-error increases. In general, it has been established that it is possible to repeatedly reduce the number of control points while maintaining the required measurement accuracy.

1. Introduction

Coordinate measurement traditionally involves measuring individual points on the workpiece surfaces, replacing the cloud of control points with the nominal geometry of the workpiece, and comparing with the tolerance for the corresponding dimensions. As a result, a decision is made to accept or reject the defective workpiece. Evaluation of the error in the form of the surface consists of several components: the form error after machining, the systematic and random measurement errors. The systematic measurement error is related to the error of basing and the sampling error (the number of control points). The random measurement error primarily depends on the errors of the touch sensor of the coordinate measuring machine.

Obviously, increasing the accuracy requires the use of a large number of control points on the surfaces. However, the measurement productivity is significantly reduced. Therefore, the question arises of the optimal choice of the number of control points. The idea of the need to reduce sample sizes in the measurement and the possibility of a priori estimating the possible error or uncertainty of measurement was repeatedly mentioned in a number of papers [1–12]. The information given in the studies is of unquestionable scientific and practical value, but only concern certain surfaces or aspects of the problem. Therefore, in practice this issue is solved directly by the CMM operator and the result largely depends on its qualification. In this paper, we propose a new optimization method for determining the number and location of control points on surfaces, which is based on statistical simulations Monte Carlo.

2. The formulation of the problem

The initial problem is formulated as follows – to measure the given surface with a certain error and maximum productivity. The maximum productivity is achieved, firstly, by minimizing the number of control points, and secondly, the location of control points, which ensures their best sequencing. As an estimate of measurement error can be used absolute or relative error values, the rejection value for a given tolerance, the probabilities of measuring I and II errors.

The new strategy is based on computer modeling of measurement errors for batches of workpieces using the Monte Carlo method. The fundamentals of the application of the Monte Carlo method for various applied problems are presented in [13]. The advantage of using the Monte Carlo method is to obtain non-point, but interval error estimates for certain batches of workpieces.

Simulation is carried out in the following sequence. At the first stage, sequences of pseudo-random numbers with given correlation and probability distribution laws modeling the random values of error components at each test are generated. With a stable technological process, the error distribution law does not change, and its parameters change insignificantly. Therefore, it becomes possible to determine the law and the error distribution parameters of various surfaces on the basis of a small sample from a batch of workpieces. At the second stage, multiple calculations are performed for surfaces with simulated deviations of the form, determining the values of the size, form or location errors. The number of iterations in modeling can be related to the number of workpieces in a batch or taken large for statistical stability. At the third stage, statistical processing of the simulation results is performed - establish the law and determine the distribution parameters or calculate the statistical moments. Thus, an interval estimation of the measurement error is obtained to compare the variants with different numbers and locations of control points on the surfaces.

The statistical modeling technique was previously described in detail in our paper [14].

3. Mathematical Modeling

As objects of research three types of surfaces are chosen: flatness, cylinder, sphere. It is these surfaces that make up the overwhelming majority on various workpieces of machines and mechanisms. The surfaces are described in a parametric form, where the simulated errors δ_1, δ_2, δ_3 are additionally introduced into the equations.

The equation of the flatness with errors is:

$$\left.\begin{aligned} X_1 &= x_i; \\ Y_1 &= y_i; \\ Z_1 &= c + \delta_1; \end{aligned}\right\} \tag{1}$$

where $x_i = [x_1; x_2]$, $y_i = [y_1; y_2]$ is the linear parameters that determine the dimensions of the workpiece; c is the position of the plane.

The equation of the cylinder with errors is:

$$\left.\begin{aligned} X_2 &= (r_2 + \delta_2)\cos\nu; \\ Y_2 &= (r_2 + \delta_2)\sin\nu; \\ Z_2 &= z_i; \end{aligned}\right\} \tag{2}$$

where $z_i = [z_1; z_2]$ is length of the cylinder; $\nu = [0; 2\pi]$ is angular parameter; r_2 is radius of the cylinder.

The equation of the sphere with errors is:

$$\left.\begin{aligned} X_3 &= (r_3 + \delta_3)\cos\vartheta\cos\varphi; \\ Y_3 &= (r_3 + \delta_3)\cos\vartheta\sin\varphi; \\ Z_3 &= (r_3 + \delta_3)\sin\vartheta; \end{aligned}\right\} \tag{3}$$

where $\vartheta = [0; 2\pi]$, $\varphi = [0; \pi]$ are angular parameters; r_3 is radius of the sphere.

The initial data for the simulation was the results of measuring the workpieces on the mobile CMM Faro Arm Edge (FARO Swiss Holding GmbH, Switzerland). The measurements were carried out with the following parameters: flatness of size 100×100 mm, a cylinder 50 mm in diameter and 100 mm in length, a hemisphere of radius 60 mm. Was used uniform grid for linear and angular coordinates for control points: 121 for the plane, 110 for the cylinder, and 101 for the sphere. For this, three-dimensional models of workpieces with measurable surfaces with marking of control points were created in the program Power Inspect.

The measurement was made at given points 30 times. The re-basing procedure was carried out 3 times. For the flatness, the array of measured points of all the experiments is shown in Fig. 1. The deviation of the shape at the point of the flatness is the distance between the nominal coordinates of the points and the measured coordinates taken with the sign taken into account. The deviation of the shape for the flatness as a whole is the difference between the maximum and minimum deviation of the shape at the points of the flatness. In the case of ideal basing, the deviations of the shape are the coordinates along the *Z*-axis (this axis is the normal to the flatness). In Fig. 2 shows the calculated deviations in 110 points of 30 experiments after elimination of the error of basing.

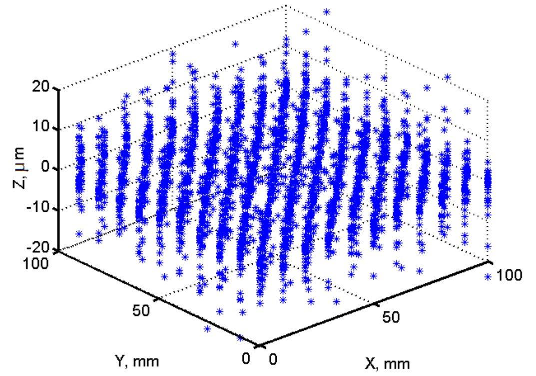

Figure 1. An array of measured points for the flatness.

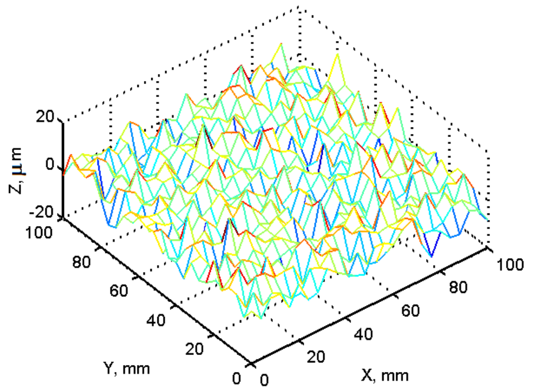

Figure 2. The errors of the flatness after elimination of the error of basing.

On the basis of the data obtained, Monte Carlo simulations were carried out for batches of 100 workpieces with uniform and normal distribution of error components. Then, according to the standard method, the error values are calculated: flatness, cylindricity, sphericity. Modeling and processing of results was carried out according to the developed program in the Matlab environment.

In the simulation, four options of the location of the control points are considered:

- option 1 – uniform grid of linear and angular parameters of the surface, the number of control points 121 for the plane, 110 for the cylinder and 101 for the sphere;
- option 2 – uneven mesh, sparse for the plane along the *X* axis, for the cylinder along the length, for the sphere along the zenith angle, the number of control points 66 for the plane, 60 for the cylinder, 61 for the sphere;

- option 3 – non-uniform grid, sparse for the plane along the *Y* axis, for the cylinder in the angle of the circle, for the sphere in the azimuth angle, the number of control points 66 for the plane, 55 for the cylinder, 51 for the sphere;
- option 4 – uniform grid, sparse in angular and linear coordinates, the number of control points 36 for the plane, 30 for the cylinder, 31 for the sphere.

Option 1 was considered to be a basic one for modeling. All other variants were obtained from the basic by eliminating a number of control points according to the above algorithm. The results of one of the simulation implementations are shown in Fig. 3–5. Here, for a cylinder and a hemisphere, an error scale of 500 times is used.

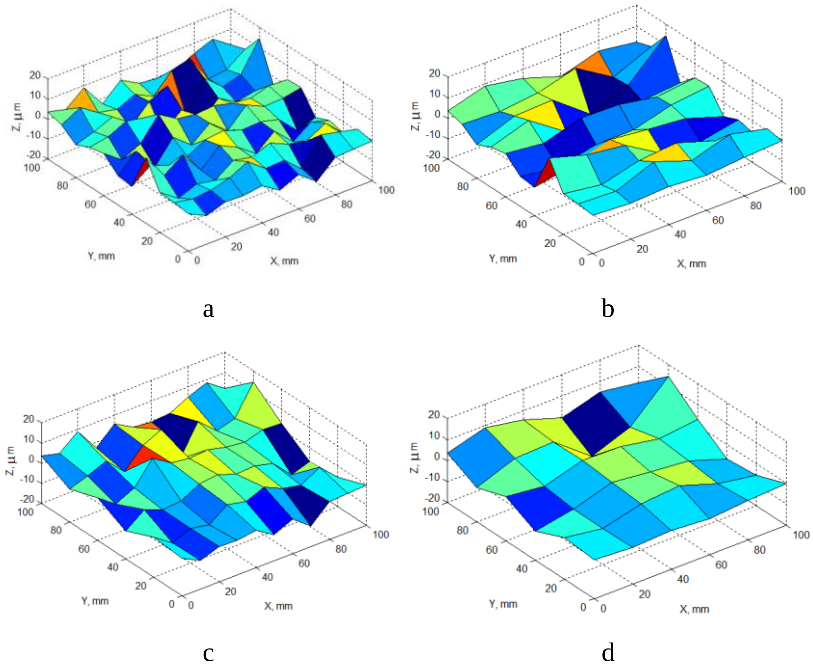

Figure 3. Simulation of the flatness error: a – option 1, b – option 2, c – option 3, d – option 4.

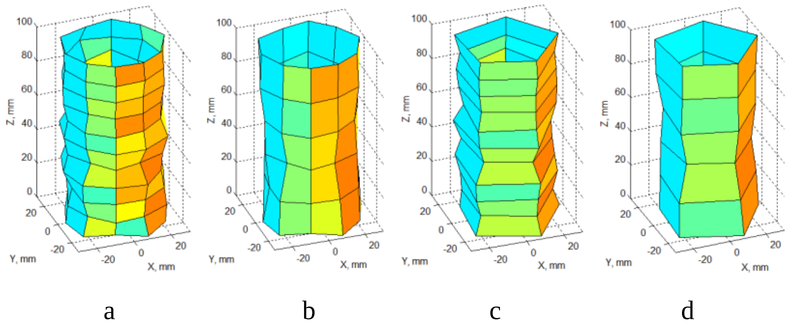

Figure 4. Simulation of the cylinder error: a – option 1, b – option 2, c – option 3, d – option 4.

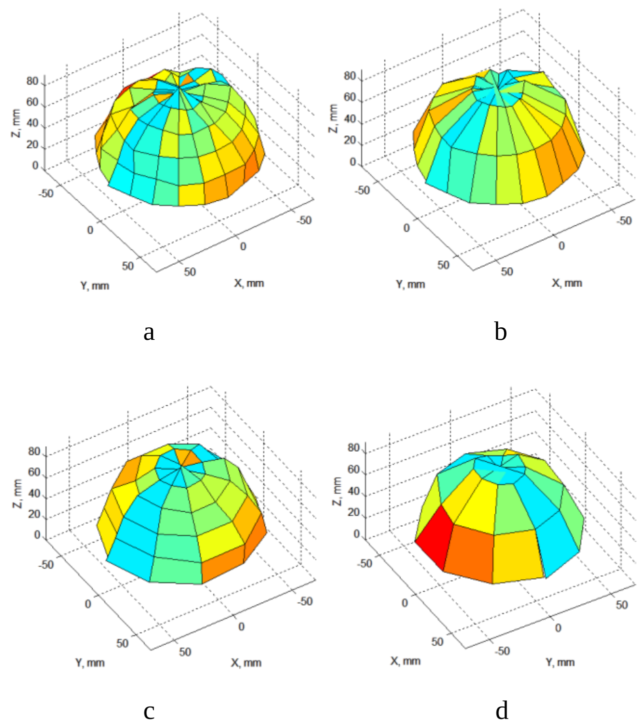

Figure 5. Simulation of the sphere error: a – option 1, b – option 2, c – option 3, d – option 4.

4. Analysis of measurement simulation results

Statistical processing of the data resulting from the simulation showed that the measurement error is well described by the normal distribution law. The evaluation was carried out by Pearson's agreement criterion with a confidence probability of 95%. As an example, Fig. 6 gives empirical distributions of the measurement error for flatness (a) and cylindricity (b).

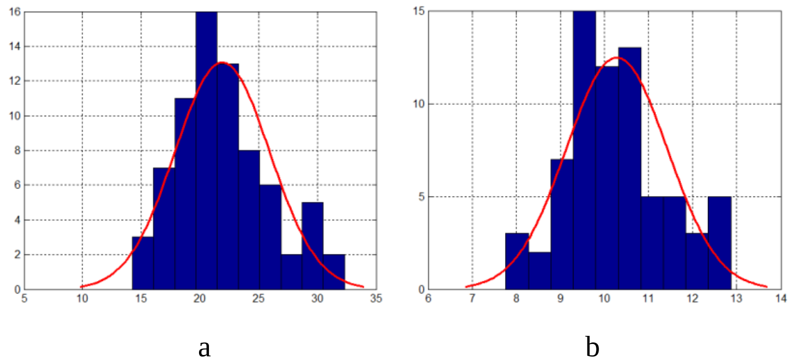

Figure 6. Histogram of the measurement error: a – flatness, b – cylindricity.

Therefore, the comparison of options was carried out on the arithmetic mean value and the standard deviation of the measurement error. Also, the tolerances for the corresponding deviations of the form were conventionally given and calculated the values of the defective workpieces in percent and measuring I and II errors. The results of the calculations are given in Fig. 7 for the flatness, Fig. 8 – for the cylinder, Fig. 9 for the sphere. Shown: I – the arithmetic mean, µm, II – the standard deviation, µm.

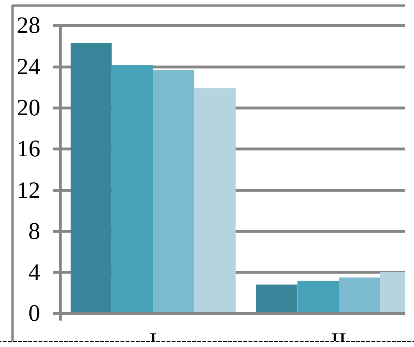

Figure 7. Measuring error for the flatness for option 1–4.

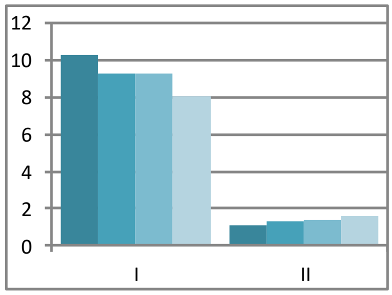

Figure 8. Measuring error for the cylindricity for option 1–4.

It is established that for the considered variants only β-error is characteristic. The results of calculating the β-error in percent are shown in Fig. 10: I – flatness, II – cylinder, III – sphere.

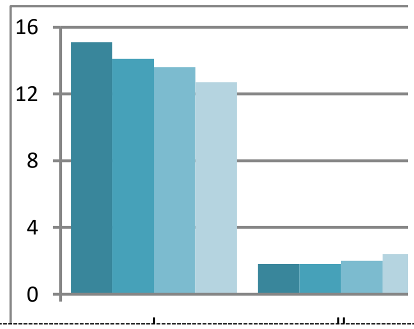

Figure 9. Measuring error for the sphericity for option 1–4.

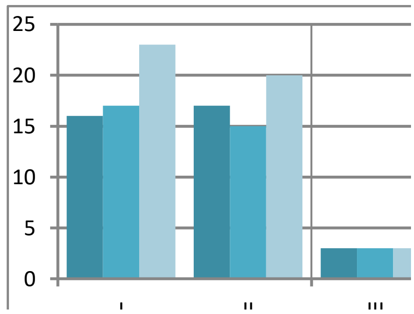

Figure 10. Measuring β-error error for option 2–4: I – flatness, II – cylindricity, III – sphericity.

Analysis of the obtained modeling data allows us to draw the following conclusions:

1) when using a smaller number of control points, the measured value of the surface error always decreases, the largest variation with the base option 1 is given by option 4;

2) for all options only the measurement β-error is characteristic, that is, the probability increases that the defective workpieces is fit for measurement;

3) when measuring the plane, a satisfactory result gives a decrease in control points from 121 to 66, while their location is not of fundamental importance (options 2, 3);

4) when measuring the cylinder, a satisfactory result corresponds to a non-uniform arrangement of control points along the angular step, with a decrease in their number from 110 to 55 (option 3);

5) when measuring the sphere, a satisfactory result was obtained for all variants; therefore, a preference should be given to increase the productivity of variant 4 with the number of points 31, and to ensure accuracy – to the variant with a decrease in the number of control points along the zenith angle to 61 (option 2).

5. Conclusion

The method of optimization of the number of control points with the use of Monte Carlo simulation is considered in the article. Based on the measurement of a small sample from batches of workpieces, statistical modeling is performed, which allows one to obtain interval estimates of the measurement error. The conducted researches established that in the measurement and analysis of flatness, cylindricity and sphericity, the number and location of the control points significantly affects the magnitude of the error. It is revealed that when the number of control points decreases, the arithmetic mean decreases, the standard deviation of the measurement error increases and the probability of the measurement β-error increases. In general, it has been established that it is possible to repeatedly reduce the number of control points while maintaining the required measurement accuracy.

Acknowledgments
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