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## The calculation of rectangular plates on elastic foundation the finite difference method

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Abstract. The article describes the main advantages and disadvantages existing in the present time of calculation methods for plates on elastic Foundation. Consider automation of the calculation of rectangular plates on elastic basis by finite difference method, on the basis of which received automatic design algorithms. Conducted research of discretization on the accuracy of the calculations. The comparison of the results of strain and effort obtained by the finite element method and the proposed method.

#### **1. Introduction**

Despite the fact that currently there are a large number of software complexes for analysis of structures (beams, plates, shells), the bulk calculations are performed manually or by software based on the finite element method (FEM). However, it is not necessary to forget and about other methods of calculation. One such method is the finite differences method (MD).

This method allows to solve a wide range of tasks in the field of construction:

- allows to count the rods [1], rectangular [2 -5], round [6] of the plate as static loads and dynamic loads [7, 8];

- allows to define the stability of the plates [9] and cores;

allows you to set the vibration plates [10, 11];

-allows to make calculations taking into account the nonlinear characteristics of the Foundation [12].

As to the advantages of the method include its high accuracy [13]. However, the MD has its drawbacks, chief among which is the low automation of calculations. In contrast to the FEM software systems implemented in MD just yet. Therefore, all calculations are performed manually. However, it is worth noting that in recent years began to carried out the work on the automation of MD [14]. In these works were developed automated complex for calculation of rectangular plates implemented in a publicly accessible electronic mathematical environment of MS Excel.

This article considers the issue of automation of calculation of rectangular plates on elastic Foundation

#### 2. Theory

The equation of the middle surface of a plate on elastic Foundation, loaded with distributed area load can be described by the differential equation of the following form [15]:

$$\nabla^{2}\nabla^{2}v_{(x:y)} - \frac{C_{2}}{D}\nabla^{2}v_{(x:y)} + \frac{C_{1}}{D}v_{(x:y)} = \frac{q}{D}$$
(1)

where v is the deflection of an arbitrary point of the middle surface of the plate;  $C_1$  – the coefficient of compression (kN/m<sup>3</sup>);

 $C_2$  – coefficient shear (kN/m);

q – distributed load perpendicular to the middle plane of the plate (kg/m<sup>2</sup>);

D – cylindrical stiffness of the plate;

x and y coordinates of the plates in the plan;

$$\nabla^2 \nabla^2 v_{(x,y)} = \frac{\partial^4 v}{\partial x^4} + 2 \frac{\partial^4 v}{\partial x^2 \partial y^2} + \frac{\partial^4 v}{\partial y^4}$$
(2)

- biharmonic operator;

$$\nabla^2 v_{(x,y)} = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}$$
(3)



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Figure 1. Numbering of the grid points of the plate

For further calculations equation (4) should be submitted in differential form (5), for this we use difference schemes of [16]:

$$\begin{bmatrix} 6(\alpha + \frac{1}{\alpha}) + 8 \end{bmatrix} v_{(x;y)} - 4 \begin{bmatrix} (1+\alpha)(v_{(x-1;y)} + v_{(x+1;y)}) + (1+\frac{1}{\alpha})(v_{(x;y+1)} + v_{(x;y-1)}) \end{bmatrix} + \\ + 2(v_{(x-1;y+1)} + v_{(x+1;y-1)} + v_{(x-1;y-1)} + v_{(x+1;y+1)}) + \alpha(v_{(x-2;y)} + v_{(x+2;y)}) + \frac{1}{\alpha}(v_{(x;y+2)} + v_{(x;y-2)}) + \\ + \frac{2C_2v_{(x;y)}(\Delta x^2 + \Delta y^2)}{D} - \frac{C_2\Delta y^2(v_{(x-1;y)} + v_{(x+1;y)})}{D} - \frac{C_2\Delta x^2(v_{(x;y-1)} + v_{(x;y+1)})}{D} + \frac{C_1v_{(x;y)}\Delta x^2\Delta y^2}{D} = \frac{\Delta x^2\Delta y^2}{D}q$$

$$(5)$$

where  $\alpha = \Delta y^2 / \Delta x^2$ 

Displacements v at the edges of the plate (contour points) and edges (edge points) is determined by the conditions of support according to [17].

The resulting equation is cumbersome, so when manually calculating the designers go on a number of simplifications, most of which is the decision to make payments of a square grid when  $\Delta x=\Delta y$ . And the second plate is applied to a very large grid (in first approximation) when  $\Delta x=\Delta y=L/4$ . In the third plate count only two mutually perpendicular cross-sections. Only in this case the system of algebraic equations is easy to calculate. But in this case, neither of which the accuracy of the calculation of the speech cannot go.

The situation began to change with the advent of computers. When using the need for manual calculation anymore and the designers were able to increase the degree of sampling, thereby improving the accuracy of the calculations. However, to date, even when using a computer, on the plates break the mesh not more frequently than  $\Delta x = \Delta y = L/8$  [18].

So FDM is currently used only for educational purposes.

Everything changes, however, if the calculations Platin FDM to available mathematical environment of MS Excel. A built-in feature of the iterative calculus allows to solve the system (5) with a substantial degree of sampling.

If the calculated plate spacing grid (Fig.1), expressing a from equation (5) the value of the deflection at point (x:y) and applying this expression to all points of the wafer, it is possible to obtain a system of interrelated equations.

The result is a field where each cell is interconnected with neighboring ones. And thus, formed between them a circular reference. Then, after you activate "Enable iterative calculation" movement in the plate are calculated automatically.

After calculating the movements of effort are calculated by the formulas given in [19]. However, the question of discretization remains open.

#### **3.** The results of the calculations

To determine the impact of discretization on the accuracy of the calculations the calculations of the plate lying on the elastic basis for the action of concentrated load FDM and MKE (Lira) with the step of:

-  $\Delta x = Lx/10$  and  $\Delta y = Ly/10$ ;

-  $\Delta x = Lx/15$  and  $\Delta y = Ly/15$ ;

-  $\Delta x = Lx/20$  and  $\Delta y = Ly/20$ .

Table 1 shows the comparative analysis of the results of calculation of reinforced concrete plate with dimensions in plan  $6 \times 3$  m on an elastic Foundation with a thickness of 200 mm at a point load 1000 kN in the middle of the span in the mathematical environment of MS Excel, and finite element method in Lira. Picture of plate deflections and the bending moments shown in Fig. 2, 3, 4.

	FDM - Excel			FEM - Lira CAD		
	10x10	15x15	20x20	10x10	15x15	20x20
V <sub>max</sub> , mm	19.1	18.9	18.6	19.2	18.8	19.0
* <i>M</i> <sub>x</sub> , кN*m/m	308.9	225.3	228.2	184.7	267.7	226.9
* <i>M<sub>y</sub></i> , кN*m/m	273.3	182.2	181.5	136.5	205.4	174.7
* <i>M<sub>xy</sub></i> , кN*m/m	18.53	18.91	18.56	22.30	24.20	23.50

Table 1. The results of the calculation

\*calculated values in the table are modulo



Figure 2. Plot vertical displacement of plate V

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Figure 3. Plot the bending moments Mx



Figure 4. Plot the bending moments My

From table 1 it follows that in the calculation of MCR under discretization element more than L/15 the values of the deflections are comparable, and the difference between values of effort does not exceed 2%. In the calculation of the FEM the convergence of the internal forces are not observed. This picture for FEM due to the fact that in him the value of effort is determined by the center of gravity of the elements. And therefore, FEM is more dependent on sampling than FDM.

Next, we conducted a series of comparative calculations in the FDM and FEM plates with various geometry parameters, the reasons and types of loads. Table 2 shows comparative analysis of the results of calculating the maximum deflection for various plates on elastic Foundation. Graphical results of calculated deflections of plates is presented in Fig. 5

N⁰	Deflection	Lira CAD	MS Excel	%	Options
1	V <sub>max</sub> , mm	10	10	0	Concrete slab 3x3 m with a thickness of 250 mm C1=5000 kN/m3 C2=500 kN/m The uniformly distributed load q=50 kN/m <sup>2</sup>
2	V <sub>max</sub> , mm	12.1	11.94	1.3	Concrete slab 4 x 2 m thickness 100 mm C1=10000 kN/m <sup>3</sup>

Table 2. Comparative analysis the maximum deflections in plates on elastic foundation

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					C2=1000 kN/m	
					Load strip in the middle of the	
					span along the long side of	
					q=200 kN/m	
3	V <sub>max</sub> , mm	9.24	9.43	2	Concrete slab 5x5 m with a	
					thickness of 150 mm	
					S1=9000 kN/m <sup>3</sup>	
					C2=900 kN/m	
					Load in the form of four	
					concentrated forces at the	
					corners of the plate R=150x4 kN	
4	V <sub>max</sub> , mm	3.8	3.91	2.8	Concrete slab 6x2 m and a	
					thickness of 200 mm	
					S1=8000 kN/m <sup>3</sup>	
					C2=800 kN/m	
					Load strip in the middle of the	
					span along the short side q=100	
					kN/m	
5	V <sub>max</sub> , mm	6.76	6.51	1.7	Concrete slab 5x1 m with a	
					thickness of 50 mm	
					C1=7000 kN/m <sup>3</sup>	
					C2=700 kN/m	
					Load in the form of three	
					concentrated forces along the	
					length of the plate P=50x3 kN	

A comparative analysis of the calculations, the discrepancy of the values of the maximum vertical deflection obtained by FEM and MD are no different. The difference between values of bending moments does not exceed 3%.

#### 4. Conclusion

Calculation of plates on elastic Foundation MD in the electronic environment MS Excel showed his versatility. It allows the calculation of rectangular plates of arbitrary size with the various soil parameters for different types of loads. Also this method is easy to implement the calculation of plates with variable rigidity on elastic basis. MD showed the best convergence in comparison with FEM. However, MD has weaknesses. The studies revealed that MD considers incorrect movements in plates under load located on the edge. This issue requires further study.

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Figure 5. Plot vertical displacement V

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