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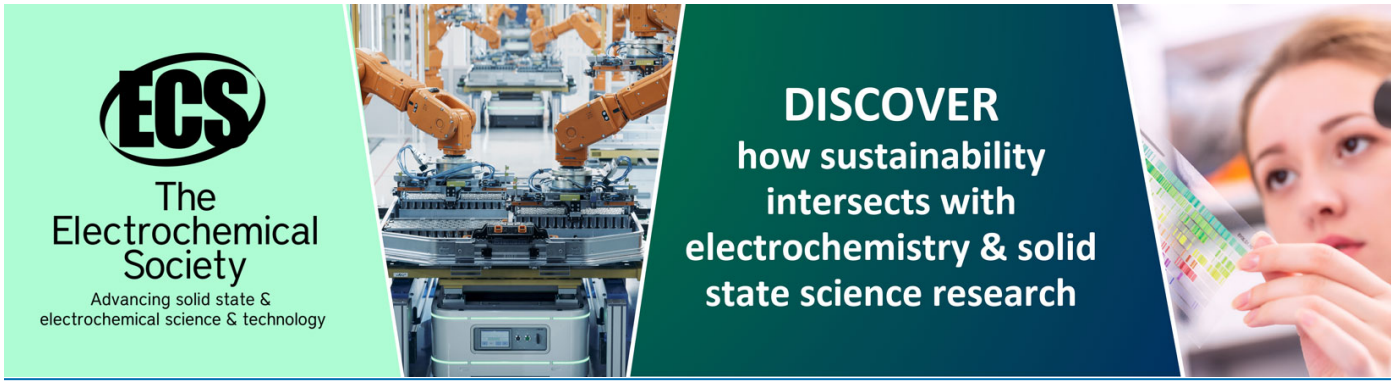

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The algorithm of numerical calculation of constraints reactions in a dynamic system of transport machine

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Abstract. The questions of construction and practical application of the automation system for the design of components and aggregates for the construction of transport vehicles are considered, taking into account their dynamic characteristics. Based on the results of the studies, a unified method for determining the reactions of bonds of a complex spatial structure is proposed. The technique, based on the method of substructures, allows us to determine the values of the transfer functions taking into account the reactions of the bonds. After the carried out researches it is necessary to note, that such approach gives the most satisfactory results and can be used for calculations of complex mechanical systems of machines and units of different purposes. The directions of increasing the degree of validity of technical decisions are shown, especially in the early stages of design, when the cost of errors is high, with careful thorough working out of all the elements of the design, which is really feasible only on the basis of automation of design and technological work.

Keywords: transport machine, dynamic system, complex structure, communication reaction, algorithm, design automation system

1. Introduction

In recent years, problems have arisen associated with the need to take into account the elastic properties of structures that perform spatial motion, which causes the emergence of a number of new situations that have a significant effect on the dynamic characteristics of the object.

The complexity of the objects created caused the expediency of using new computational models based on the development of modern scientifically grounded methods for studying structures in order to create efficient systems and programs for automating the design of new equipment objects that represent complex structures.

The motion of a medium of simple structure, used in the mechanics of a deformed body, is described by three functions - the projections of the displacement vector. The motion of the medium of a complex structure is characterized by the collection of a large number of functions. These functions are generalized coordinates of the individual elements of which the medium consists. Thus, the concept of a complex structure generalizes the classical concept used in the theory of elasticity. Historically, one of the first models of an elastic medium that can not be described within the framework of the classical theory of elasticity is the continuum introduced for the first time in 1909 by E.F. By Cosser [1]. Another paper [2] describes the vibration fields in the zone of the boundary of the contact of two bodies of a complex structure by the methods of the theory of vibration conductivity.

The purpose of this work is the development of models for determining the dynamic characteristics of transport machine designs as complex technical systems [3], taking into account the reactions of bonds and the creation on their basis of design techniques.

2. Formulation of the problem

The design of transport vehicles as a complex object from the point of view of its design, production and preparation of operation can be considered as a multi-level, hierarchical structure [4] The problem is [5] derivation of equations of reactions of the elements of the transport machine structure, with unrelated interaction with each other (Fig. 1).

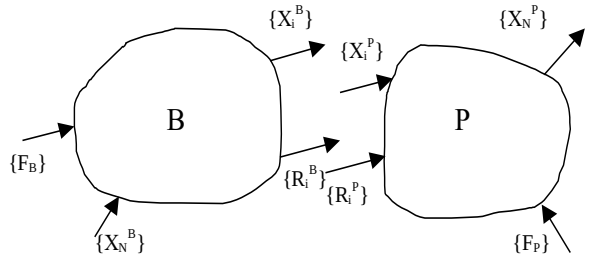

Figure 1. Scheme of interaction of unrelated components of the transport machine

Physically, this interaction occurs in a number of design points that have an indirect effect on both elements. Mathematically, this means that the vector of generalized displacements $\{x^B_i\}$ one element of the transport machine at these points should be equal to the displacement vector $\{x^P_i\}$ another, which can be written in the form

$$\{x^B_i\} = \{x^P_i\} \tag{1}$$

for any time t.. A similar expression can be written for the vectors of generalized reaction forces at common points of interaction

$$\{R^B_i\} = -\{R^P_i\} \tag{2}$$

for any time t.

Using the scheme of unbound bodies [6] (Figure 1), it is easy to derive the equations of motion for both elements

$$\begin{bmatrix} M_B & \\ & M_P \end{bmatrix} \begin{Bmatrix} \ddot{x}_B \\ \ddot{x}_P \end{Bmatrix} + \begin{bmatrix} K_B & \\ & K_P \end{bmatrix} \begin{Bmatrix} x_B \\ x_P \end{Bmatrix} = \begin{Bmatrix} F_B \\ F_P \end{Bmatrix} + \begin{Bmatrix} 0 \\ R_i^B \\ 0 \\ R_i^P \end{Bmatrix}, \tag{3}$$

where $\{x_B\}$ is the vector of generalized displacements, $[M_B]$ is the mass matrix, $[K_B]$ is the stiffness matrix, and $\{F_B\}$ is the vector of external forces for one of the elements of the transport machine. By analogy, we also introduce the notation for another element. Equations (3) are unrelated equations of motion for non-damped elements, i.e. at the moment, almost everywhere, damping is excluded from the equations. It is assumed that the damping can be taken into account by the generally accepted method of damping the vibration modes.

3. Theory

To obtain the equations of motion of the transport machine system, it is necessary to eliminate the previously unknown reaction-force vectors $\{R^B_i\}$ and $\{R^P_i\}$. Two most commonly used transformations are known to solve this problem. In the first one, [7] a non-free and free element of the transport machine is used, and in the second [8] both elements of the transport machine are not accepted when the interaction surface is free. In both approaches, the following separation of the vector $\{x_P\}$

$$\{x_P\} = \begin{Bmatrix} x_N^P \\ x_i^P \end{Bmatrix}. \tag{4}$$

where $\{x_N^P\}$ reflects the unlimited movement of the element, which can be represented as a sum

$$\{x^P_N\} = [S_P]\{x^P_i\} + \{\overline{x}^P_N\}, \tag{5}$$

where $[S_P] = -[K^P_{NN}]^{-1}[K^P_{Ni}]$ (6)

and the vector $\{\bar{x}^P_N\}$ is the vector of unbounded displacements of the element with respect to the interaction surface.

The equations of motion in the case of a free element [9] are derived by means of expressions (1) and (5) in the form of a coordinate transformation

$$\begin{Bmatrix} x_B \\ x_P \end{Bmatrix} = \begin{Bmatrix} x^B_N \\ x^B_i \\ x^P_N \\ x^P_i \end{Bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & S_P & I \\ 0 & I & 0 \end{bmatrix} \begin{Bmatrix} x^B_N \\ x^B_i \\ \bar{x}^P_N \end{Bmatrix}. \tag{7}$$

Using the notation, we can write in the form

$$\begin{Bmatrix} x_B \\ x_P \end{Bmatrix} = \begin{bmatrix} I_B & L_B & 0 \\ 0 & T_P & I_P \end{bmatrix} \begin{Bmatrix} x^B_N \\ x^B_i \\ \bar{x}_N \end{Bmatrix}, \tag{8}$$

As well as equations (2), (3) and (7), we obtain the following equations of motion for the case of a free element of the construction of a transport machine

$$\begin{bmatrix} I_B^T M_B I_B & I_B^T M_B L_B & 0 \\ L_B^T M_B I_B & L_B^T M_B L_B + T_P^T M_P T_P & T_P^T M_P I_P \\ 0 & I_P^T M_P T_P & I_P^T M_P I_P \end{bmatrix} \begin{Bmatrix} \ddot{x}^B_N \\ \ddot{x}^B_i \\ \ddot{\bar{x}}^P_N \end{Bmatrix} + \begin{bmatrix} I_B^T K_B I_B & 0 & 0 \\ L_B^T K_B I_B & L_B^T K_B L_B + T_P^T K_P T_P & 0 \\ 0 & 0 & I_P^T K_P I_P \end{bmatrix} \begin{Bmatrix} x^B_N \\ x^B_i \\ \bar{x}^P_N \end{Bmatrix} = \begin{Bmatrix} I_B^T F_B \\ L_B^T F_B \\ 0 \end{Bmatrix}. \tag{9}$$

If both elements are formally described identically, i.e.

$$\{x^B_N\} = [S_B]\{x^B_i\} + \{\bar{x}^B_N\}. \tag{10}$$

Then a suitable coordinate transformation is a transformation of the form

$$\begin{Bmatrix} x^B \\ x^P \end{Bmatrix} = \begin{Bmatrix} x^B_N \\ x^B_i \\ x^P_N \\ x^P_i \end{Bmatrix} = \begin{bmatrix} I & S_B & 0 \\ 0 & I & 0 \\ 0 & S_P & I \\ 0 & I & 0 \end{bmatrix} \begin{Bmatrix} \bar{x}^B_N \\ x^B_i \\ \bar{x}^P_N \end{Bmatrix}. \tag{11}$$

The resulting equations of motion for non-free elements of the transport machine structure with a free interaction surface (boundary) can be written in the form

$$\begin{bmatrix} I_B^T M_B I_B & I_B^T M_B T_B & 0 \\ T_B^T M_B I_B & T_B^T M_B T_B + T_P^T M_P T_P & T_P^T M_P I_P \\ 0 & I_P^T M_P T_P & I_P^T M_P I_P \end{bmatrix} \cdot \begin{Bmatrix} \ddot{\bar{x}}^B_N \\ \ddot{x}^B_i \\ \ddot{\bar{x}}^P_N \end{Bmatrix} + \begin{bmatrix} I_B^T K_B I_B & 0 & 0 \\ 0 & T_B^T K_B T_B + T_P^T K_P T_P & 0 \\ 0 & 0 & I_P^T K_P I_P \end{bmatrix} \cdot \begin{Bmatrix} \bar{x}^B_N \\ x^B_i \\ \bar{x}^P_N \end{Bmatrix} = \begin{Bmatrix} I_B^T F_B \\ T_B^T F_B \\ 0 \end{Bmatrix}. \tag{12}$$

Matrices $[S_B]$ and $[T_B]$ are defined similarly to $[S_p]$ and $[T_p]$. The main difference between equations (9) and (12) is the presence in the last additional term due to the rigidity of the unbound system.

The columns of the matrices $[T_p]$ and $[T_B]$ will be referred to as coupling modes, which allows us to represent the motion of structural elements using a combination of coupling modes, rigid-body motion modes, and modes of a fixed boundary. The introduction of such a simplification allows the modes of motion of a rigid body not to be considered separately from the bond modes when using the transformation (5). This is precisely the transformation of the coordinates, which leads to the appearance in (9) and (12) of a term due to the rigidity of the unbound system.

The forces acting on the structural element can be assigned to two categories [8]: the forces arising in the bonds and the forces applied by means of sources external to the system.

The first category includes forces acting through links to other elements, as well as reactions acting on the system with fixed connections. The forces arising in the bonds work on the movements of the element, while the reactions in the fixed bonds do not work.

The forces acting in the bonds between the elements are further subdivided into two categories: R_s^R is the force applied to the statically determined connection and R_r^C is the force applied to the redundant link.

External forces with respect to the system can be distributed or concentrated. Let f be the intensity of the distributed load at an arbitrary point, and let it be the concentrated force at some point with displacement.

The virtual work [8], produced by these forces on the virtual displacement of the element, is given by the expression

$$\delta W = \sum_s R_s^R \delta\tilde{u}_s^R + \sum_r R_r^C (\delta\tilde{u}_r^R + \delta\tilde{u}_r^C) + \int \bar{f} \cdot \delta\bar{u} dV + \sum_i \overline{F}_i \cdot \delta\overline{u}_i \,. \tag{13}$$

Where $\delta\tilde{u}_s^R$ is the virtual displacement of the *s*-th statically determinable connection during a hard transfer; $\delta\tilde{u}_r^R$ is the virtual movement of *r*-th redundant communication during hard transfer; $\delta\tilde{u}_r^C$ is the virtual movement of the *r*-th redundant communication at offset.

Note that the movements of the links are distinguished by signs in the form of a wavy line above the letter. These displacements are assumed to be scalar quantities, since the bonds themselves determine the direction of displacements [6], which can be determined through typical constraints that limit the movement of elements and their reactions

$$\left.\begin{aligned} \tilde{u}_s^R &= \sum_j \tilde{\phi}_{sj}^{RR} \rho_j^R \\ \tilde{u}_r^R &= \sum_j \tilde{\phi}_{rj}^{CR} \rho_j^R \\ \tilde{u}_r^C &= \sum_j \tilde{\phi}_{rj}^{CC} \rho_j^C \end{aligned}\right\}, \tag{14}$$

where $\tilde{\phi}_{sj}^{RR}$ is the typical displacement of the *s*-th statically determined connection for the *j*-th rigid form; $\tilde{\phi}_{rj}^{CR}$ is the typical movement of the rth redundant link at the *j*-th rigid form; $\tilde{\phi}_{rj}^{CC}$ is the typical movement of the r-th redundant link at the *j*-th form of displacement due to the coupling.

Substituting expression (14) into (13), we obtain the following equation for virtual work:

$$\begin{aligned} \delta W = \sum_j \delta\rho_j^R \left[\sum_s R_s^R \tilde{\phi}_{sj}^{RR} + \sum_r R_r^C \tilde{\phi}_{rj}^{CR} + \int \tilde{f} \cdot \tilde{\phi}_j^R dV + \sum_i \overline{F}_i \cdot \tilde{\phi}_{ij}^R \right] + \sum_j \delta\rho_j^C \times \\ \times \left[\sum_r R_r^C \tilde{\phi}_{rj}^{CC} + \int \bar{f} \cdot \tilde{\phi}_j^C dV + \sum_i \overline{F}_i \cdot \tilde{\phi}_{ij}^C \right] + \sum_j \delta\rho_j^N \times \left[\int \bar{f} \cdot \tilde{\phi}_j^N dV + \sum_i \overline{F}_i \cdot \tilde{\phi}_{ij}^N \right]. \end{aligned} \tag{15}$$

Virtual work can also be written through generalized forces and movements

$$\delta W = \sum_j P_j^R \delta\rho_j^R + \sum_j P_j^C \delta\rho_j^C + \sum_j P_j^N \delta\rho_j^N . \tag{16}$$

Comparing relations (15) and (16) and taking into account that the generalized virtual displacements $\delta\rho_j (j = 1,2,3,...)$ are independent, we obtain the following equations for generalized forces:

$$\left.\begin{aligned} P_j^R &= \sum_s R_s^R \tilde{\phi}_{sj}^{RR} + \sum_r R_r^C \tilde{\phi}_{rj}^{CR} + \int \bar{f} \cdot \bar{\phi}_j^R dV + \sum_i \bar{F}_i \cdot \bar{\phi}_{ij}^R \\ P_j^C &= \sum_r R_r^C \tilde{\phi}_{rj}^{CC} + \int \bar{f} \cdot \bar{\phi}_j^C dV + \sum_i \bar{F}_i \cdot \bar{\phi}_{ij}^C \\ P_j^N &= \int \bar{f} \cdot \bar{\phi}_i^N dV + \sum_i \bar{F}_i \cdot \bar{\phi}_{ij}^N \end{aligned}\right\}. \tag{17}$$

It is convenient to write the system of generalized external forces separately in this form:

$$\left.\begin{aligned} F_j^R &= \int \bar{f} \cdot \bar{\phi}_j^R dV + \sum_i \bar{F}_i \cdot \bar{\phi}_{ij}^R \\ F_j^C &= \int \bar{f} \cdot \bar{\phi}_j^C dV + \sum_i \bar{F}_i \cdot \bar{\phi}_{ij}^C \\ F_j^N &= \int \bar{f} \cdot \bar{\phi}_j^N dV + \sum_i \bar{F}_i \cdot \bar{\phi}_{ij}^N \end{aligned}\right\}. \tag{18}$$

Taking expression (18) into account for generalized forces, we can rewrite equation (17) in the matrix form

$$\{ \mathrm{P} \} = \left[\tilde{\phi}\right]^T \{ \mathrm{R} \} + \{ \mathrm{F} \}, \tag{19}$$

Where $\{P\} = \begin{Bmatrix} P^R \\ P^C \\ P^N \end{Bmatrix}$, $\{R\} = \begin{Bmatrix} R^R \\ R^C \end{Bmatrix}$, $\{F\} = \begin{Bmatrix} F^R \\ F^C \\ F^N \end{Bmatrix}$, $\left[\tilde{\phi}\right] = \begin{bmatrix} \tilde{\phi}^{RR} & 0 & 0 \\ \tilde{\phi}^{CR} & \tilde{\phi}^{CC} & 0 \end{bmatrix}$.

Here it should be noted that the translations of the links are transformed into generalized displacements by means of a matrix $\left[\tilde{\phi}\right]$ in this way:

$$\begin{Bmatrix} \tilde{u}_R \\ \tilde{u}_C \end{Bmatrix} = \begin{bmatrix} \tilde{\phi}^{RR} & 0 & 0 \\ \tilde{\phi}^{CR} & \tilde{\phi}^{CC} & 0 \end{bmatrix} \begin{Bmatrix} \rho^R \\ \rho^C \\ \rho^N \end{Bmatrix}, \tag{20}$$

where $\tilde{u}_R$ is the complete movement of the statically determinate connection; $\tilde{u}_C$ is the complete redundancy transfer.

Using the continuity condition [10] in the relations of the elements, it can be shown that the equilibrium of the interacting forces in these bonds is automatically satisfied. In order to show the validity of this statement, it is necessary to clearly formulate and track the steps leading to the definition of a system of generalized forces. Substitution of relation (19) into (2) shows that the force vector for the system has the form

$$\{\mathrm{Q}(t)\} = [\,\beta\,]\mathrm{T}\,\left[\tilde{\phi}\right]^T \{\mathrm{R}\} + [\,\beta\,]\mathrm{T}\,\{\mathrm{F}\}. \tag{21}$$

In this equality, the matrix is $\left[\tilde{\phi}\right]$ composed of matrix components arranged as follows:

$$\left[\tilde{\phi}\right] = \begin{bmatrix} \left[\tilde{\phi}\right]_1 & & & & & & \\ & \left[\tilde{\phi}\right]_2 & & & & & \\ & & \ddots & & & & \\ & & & \left[\tilde{\phi}\right]_r & & & \\ & & & & \left[\tilde{\phi}\right]_s & & \\ & & & & & \ddots & \\ & & & & & & \ddots \end{bmatrix}. \tag{22}$$

Similarly, the components of the vectors $\{\mathrm{R}\}$ and $\{\mathrm{F}\}$ are the corresponding vectors for the individual elements of the construction [11], arranged in this order:

$$\{R\} = \begin{Bmatrix} \{R\}_1 \\ \{R\}_2 \\ . \\ . \\ . \\ \{R\}_r \\ \{R\}_s \\ . \\ . \\ . \end{Bmatrix} \quad \{F\} = \begin{Bmatrix} \{F\}_1 \\ \{F\}_2 \\ . \\ . \\ . \\ \{F\}_r \\ \{F\}_s \\ . \\ . \\ . \end{Bmatrix}. \tag{23}$$

Since the components of the vector of the system $\{R\}$ are the forces acting in the bonds of the elements, i.e. Are internal with respect to the entire structure under consideration, they do not affect the generalized forces of the system {Q (t)}. Therefore, the first term on the right-hand side of (21) disappears, and the system is simplified.

$$\{Q(t)\} = [\beta]T\{F\}. \tag{24}$$

Thus, the assertion concerning the self-equilibrium of internal forces can be carried out by showing that the first term on the right-hand side of (21) actually vanishes, that is

$$[\beta]^T \left[\tilde{\phi}\right]^T \{R\} = \{0\}. \tag{25}$$

The proof can be carried out by applying the principle of a minimum of virtual work [3] to two interrelated elements of the system. It is not difficult to show that two reactions in the general connection are equal in magnitude and opposite in direction with continuity of displacements, which is the proof of the validity of (24).

Consequently, the coupling reactions of a system of several bodies consisting of p rigid bodies with q bonds can be described by a linear overdetermined system of equations:

$$\overline{Q} \cdot g = \overline{q}^z \tag{26}$$

with $6_{p \times q}$ – distributed matrix $\overline{Q}$; q – a vector of the generalized coupling reaction; q and $6p$ are the vectors denoting the coupling reactions $\overline{q}^z$ at the center of mass.

A numerical solution can be a numerical standard approach, for example, the Gauss elimination method [5]. In this paper it is shown that it is possible to use a block matrix structure $\overline{Q}$ as the basis for the algorithm.

Thus, the reactions of the positions of L_k, $k = 1$ (1) l of the bodies K_i and K_j under consideration; $i, j = 1$ (1) p are connected (Fig. 2) with the help of the radius vector r_{Gk} and the rotation matrix G_k determine the stability of the coordinate system [4].

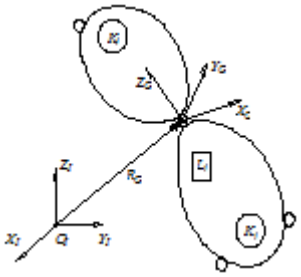

Figure 2. Geometrical description of the position of the reaction of bonds

Next, the scalar factor gm, $m = I$ (1) q is carried out for each direction of the position, which corresponds to the magnitude of the generalized coupling reaction in this direction and still retains a visual significance.

If we describe the law of conservation of momentum and the law of angular momentum for bodies with identical vector starts, we obtain the equation of the bond reaction [12].

$$\begin{bmatrix} I_E|0 \\ \ldots\ldots \\ \underbrace{0|I_E}_{\tilde{I}_E} \end{bmatrix} \text{RGTg} = \text{qz0} \tag{27}$$

With a block matrix $\overline{T} = Diag\{T_{T1} \ldots T_{Ti}; T_{R1} \ldots T_{Ri}\}$, $\overline{G} = Diag\{G_1 \ldots G_i; G_1 \ldots G_i\}$,

$$\overline{R} = \begin{bmatrix} E & & & 0 \\ \ldots & \ddots E & \ldots & \ldots \\ \tilde{r}_{G1} & & \ddots E & \\ & \ddots r_{Gi} & & \ddots E \end{bmatrix},$$

while the symbol is a cross product. The constant matrix IE is the matrix of the incidence of a system of several bodies, where, as necessary, the unit elements are replaced 3×3 by the unit matrix and the zero elements by means 3×3 of the zero matrix.

To solve equation (27), first calculate the *G*-vector $y = \overline{R}\overline{G}\overline{T}g$ from (27). Then, for systems with a complex structure, it is possible to find a solution, for example, by direct use of the Gauss algorithm [5].

In systems with S kinematic nodes, a similar solution for y is not possible, since the matrix IE reflects the slope 6_S [9]. Therefore, we obtain a 6_S-parametric solution $y = \hat{y} + Y\lambda$ with a particular solution vector $\hat{y}$, a constant $6l\times6s$-matrix of a homogeneous solution of Y and a 6_s vector of an arbitrary parameter λ.

For the vector g, only equation:

$$\begin{aligned} &\overline{T}g = \overline{G}^T R^{-1}(\hat{y} + Y\lambda) \\ &\overline{G}^T = \overline{G}^{-1} \end{aligned} \tag{28}$$

And

$$\overline{R}^{-1} = \begin{bmatrix} E & & & 0 \\ & \ddots E & & \\ -\tilde{r}_{G1} & & E & \\ & \ddots -\tilde{r}_{Gl} & & \ddots E \end{bmatrix}.$$

What corresponds to the vector component for all positions of the multiple-body system.

Each q component, which corresponds to the closed position of the direction, contains, depending on the circumstances, the components of the vector q. For each node, 6 equations are obtained for the calculation of the vector. The further course of action corresponds to a system with a building structure.

4. Results

On the basis of the obtained algorithm, a computer program is developed that leads the numerical calculation of the communication reactions, according to which the calculation time is much less than required for the Gauss algorithm.

In Fig. 3 represents the object and its typical element e.

$$\{M_p\}\{x_p\} + \{K_p\}\{x_p\} = \{F_p\}. \tag{29}$$

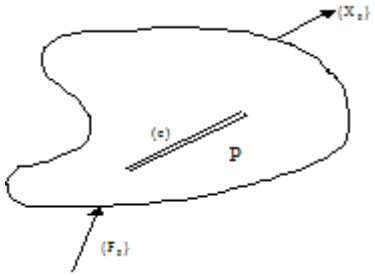

Figure 3. The object and the definition of its element e

It is required to determine the load or the vector of internal forces acting on this element. The force vector can be represented in the form

$$\{F_e^P\} = [k_e][T_e]\{x_P\}, \quad (30)$$

where $[x_p]$ is the displacement vector of the payload, $[k_e]$ is the stiffness matrix of the element e and $[T_e]$ is the transformation matrix reflecting the displacements of the element e through the displacements of the payload, which are determined from the equation of the form (28).

We assume that the approach based on the formulation of Newton's equations for each mode [7] makes it possible to more accurately determine the vector $\{X_p\}$ and, consequently, the internal force vector $\{F_e^p\}$. The use of such an approach leads to the need to solve the equation relative to $\{x_p\}$ and the modal decomposition $\{x_p\}$, that is,

$$\{x_p\} = \{K_p\}^{-1}[\{F_p\} - [M_p]\{x_p\}]. \quad (31)$$

Assuming that *M* the mod of the free element and the N modes of the cantilevered second element are stored in $[\phi_B]$.

First N + M related equations in the system are replaced by N M systems of two equations, each of which reflects the interaction of one mode of one element with one mode of another, namely:

$$\begin{bmatrix} \boldsymbol{I} + \{\boldsymbol{\phi}_i^B\}_i^T \boldsymbol{T}_P^T \boldsymbol{M}_P \boldsymbol{T}_P \{\boldsymbol{\phi}_i^B\}_i & \{\boldsymbol{\phi}_i^B\}_i^T \boldsymbol{T}_P^T \boldsymbol{M}_P \boldsymbol{I}_P \{\overline{\boldsymbol{\phi}}_N^P\}_j \\ \{\overline{\boldsymbol{\phi}}_N^P\}_j^T \boldsymbol{I}_P^T \boldsymbol{M}_P \boldsymbol{T}_P \{\boldsymbol{\phi}_i^B\}_i & \mathbf{1} \end{bmatrix} \cdot \begin{Bmatrix} \ddot{\boldsymbol{q}}_{Bi} \\ \ddot{\overline{\boldsymbol{q}}}_{Nj} \end{Bmatrix} + \quad (32)$$

$$+ \begin{bmatrix} \boldsymbol{\omega}_{Bi}^2 & \mathbf{0} \\ \mathbf{0} & \overline{\boldsymbol{\omega}}_{Pj}^2 \end{bmatrix} \cdot \begin{Bmatrix} \boldsymbol{q}_{Bi} \\ \overline{\boldsymbol{q}}_{Nj}^P \end{Bmatrix} = \begin{Bmatrix} \{\boldsymbol{\phi}_N^B\}_i^T \boldsymbol{F}_N^B \\ \mathbf{0} \end{Bmatrix} \begin{matrix} i = 1,2,\ldots, M \\ j = 1,2,\ldots, N \end{matrix}$$

In equation (30), it was assumed that the interaction surface was statically determined, ie. $[T_P^T K_P T_P] = [0]$.

Then, for each of the $N \times M$-modal reactions of the element, the limiting value q_{BP} is introduced. This is realized by compiling a new model for the input action (forcing function [8]) in equation (30). A rather complex forcing function is replaced by a much simpler (for example, a force pulse), which generates the same maximum reaction value as the original force. Thus, from the equation with some subsequent simplifications it turns out to be possible to obtain an analytical expression of the reaction, which makes it possible to determine its maximum value. Then the limit value of the payload reaction qP is determined by summing the modal limit values q_{BP} ((in absolute units or in the rms sense, which allows the introduction of weighting coefficients)). The responses of the payload element are determined by summing the contributions of all the modal components.

As noted above, the forcing function in equation (30) is replaced by a modal δ-function of a given amplitude FB. This amplitude is determined by analyzing the transient processes of the transport machine in the presence or absence of an artificial load. Advantages of the method of analyzing the spectrum of maximum loads are the small amount of computation and the brevity of each computational cycle.

It can be noted that this method can be effectively used in cases where the mass of the structure is not the main factor or when preliminary estimates of the dynamic responses at the initial stages of the payload design process are preliminary calculated.

Therefore, in principle, it is necessary to consider only methods that take into account only free vibrations. The advantage of this approach is to simplify the process of obtaining and analyzing data. In this case, for free vibrations, the equation of motion, neglecting damping, is written in a simplified form:

$$[M]\{\ddot{q}\} + [K]\{q\} = 0. \quad (33)$$

The solution of this equation, using the Euler substitution, can be represented in the following form:

$$\{q\} = \{q_1, q_2, \ldots, q_N\} = \{D_1, D_2, \ldots, D_N\} \cdot e^{\omega_C \cdot t} = \{D\} \cdot e^{\omega_C \cdot t}, \quad (34)$$

Where {D} is a column vector of the oscillation amplitudes; ω_C is the circular frequency of its own undamped oscillations.

Substituting (32) into the equation of free oscillations, we obtain a system of linear homogeneous algebraic equations with respect to the coefficient Dj [G]{D} = 0,

$$\{D\} = \begin{Vmatrix} D_1 \\ : \\ D_N \end{Vmatrix}; \qquad [G] = \begin{Vmatrix} G_{11} & ... & G_{1N} \\ : & ... & : \\ G_{N1} & ... & G_{NN} \end{Vmatrix} \tag{35}$$

where $G_{ij} = -m_{ij} \cdot \omega_C^2 + K_{ij}$.

From this we obtain the condition for oscillations of the construction at natural frequencies

$$\begin{vmatrix} (-m_{11}\omega_C^2 + K_{11}) & ... & (-m_{1N}\omega_C^2 + K_{1N}) \\ ... & ... & ... \\ (-m_{N1}\omega_C^2 + K_{N1}) & ... & (-m_{NN}\omega_C^2 + K_{NN}) \end{vmatrix}. \tag{36}$$

From (33) we find N roots $(\omega_C^2)_1 \le (\omega_C^2)_2 \le ... \le (\omega_C^2)_N$.

(37)

After finding the roots, we find the coefficients D_1, ..., D_N. Each root of equation (36) has its own system of coefficients. Most often, not the coefficients themselves, reduced to a given natural frequency of oscillations, are used, but their ratio:

$$\Phi_i = \frac{D_i}{D_1}, \ (i=1, 2, 3, ..., N), \tag{38}$$

Called the eigenmodes of the system's vibrations (modes).

When solving the characteristic equation (36) with allowance for damping, we obtain N roots,

$$(\lambda_C^2)_1 \le (\lambda_C^2)_2 \le ... \le (\lambda_C^2)_N, \tag{39}$$

Where $\lambda_C^2 = \omega_C^2(1-\xi_c^2)$ is the circular frequency of the self-damped oscillations; ξ_C – damping parameter of natural oscillations.

Thus, the equation has 2N roots, which determine 2N values of λ, from which, on the basis of the Laplace transform [10] with zero initial position and velocity {D}={Q*} $[B(\lambda) = (\lambda^2[M] + \lambda[C] + [K])^{-1}$, where $[B(\lambda) = (\lambda^2[M] + \lambda[C] + [K])^{-1}$ is the matrix of transfer functions.

And then, the transfer function $B_{ij}^J(\lambda)$ for the characteristic points *i* and *j* of the subsystem *J* can be determined analytically [4] or experimentally [12]. Due to the presence of damping, there is a lag in phase between the disturbance and the reaction. Consequently, in the general case, they are complex numbers with real and imaginary part, or modulus and phase angle, as a function of frequency λ.

In the analytic determination of the transfer function, it is necessary to know the values of the normal forms V_{ik}, representing the reaction of the *k*-th tone at the point i, as well as the frequencies of the *k*-th tone ω_k and the generalized masses m_k. For example, the transfer function transfer, or conductivity, with small or proportional damping is defined as

$$B_{ij}^J(\lambda) = \sum \frac{V_{ik} \cdot V_{jk}}{m_k \left[\left(\lambda_k^2 - \lambda^2\right) + i2\eta_k \lambda_k \lambda \right]}, \tag{40}$$

where η_k is the calculated or measured value of the damping coefficient (generally depending on the frequency).

For the movements and rotations of the transport machine as a solid, the corresponding eigenfrequencies in equation (38) are zero. From equation (38) it is possible to obtain other expressions for the transfer functions. For example, the transfer function in terms of speed or acceleration is obtained by multiplying equation (38) by $i\omega$ and $-\omega^2$, respectively.

5. Discussion of results

In the experimental determination of the transfer function (Fig. 4), the structure is excited by a vibrator creating a perturbing force that is close in shape to a sinusoid with a slowly varying frequency; The reaction of the system is recorded. After appropriate filtering, the analog data processing system allocates a phase shift between them as a function of frequency; Then these data are digitized. In the analytical determination of transfer functions, the calculation of vibration modes is performed taking into account the boundary conditions imposed by the characteristics of the chosen model. However, in

experimental studies it is expedient to determine the transfer functions for free, non-attached subsystems. For this purpose, the "soft suspension" method is often used, when the frequencies of the solid body on the suspension are removed from the fundamental disturbance frequency.

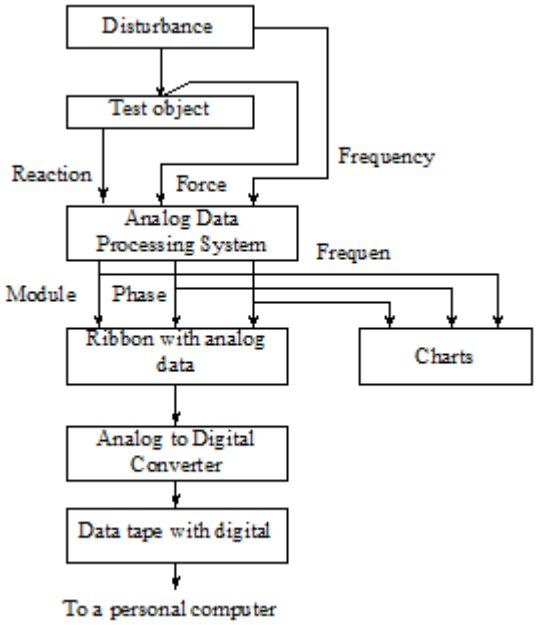

Figure 4. Block diagram of the transfer function determination system

On the other hand, it is often possible to use the symmetry of subsystems with respect to axes or planes, when the support devices can be installed so that they have no effect on the reaction being investigated. The most widely used is the elastic fixing of the subsystem at the points under study with measurements for each level of disturbance of all forces and displacements of the support (Fig. 5), the main components of the signal,

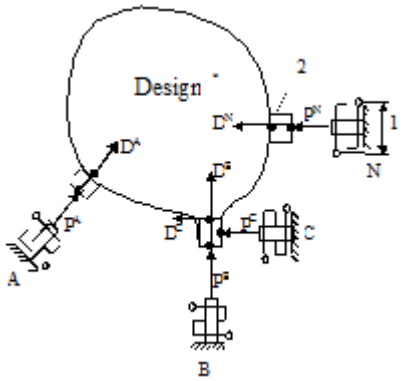

Figure 5. Excitation of structures by several vibrators with simultaneous measurement of all supporting reactions: DB – typical movement; PB is a typical support force; 1 is the excitation voltage; 2 – displacement sensor and dynamometer

those. Module of the ratio of the reaction to the disturbance and, as will be shown below, the influence of the test equipment can be eliminated by computation, resulting in the transfer functions of an unattached system.

As will be shown below, the influence of the test equipment can be eliminated by computation, resulting in the transfer functions of an unattached system.

We represent through the real [H] and imaginary [J] matrices in the form:

$$([M]+j[J])\cdot[B(\lambda)]=[E], \tag{41}$$

Where [E] is the identity matrix.

On the basis of the solutions obtained, the vector-column of the accelerations of the system can be represented in the form

$$\{\ddot{q}\} = \sum_{j=1}^{2N} \lambda_{cj}^2 \{\Phi\}_j \cdot e^{\lambda_j t}, \tag{42}$$

Where N is the number of measurement points.
Displacements at any point in the system are obtained from their own forms by modal overlap

$$q_i = \sum_{k=1}^{L} \Phi_{ik} \cdot q_k \cdot \tag{43}$$

Then, taking into account (37) and (38) from expression (36), we can write:

$$B(\omega)_{ik} = \frac{\ddot{q}_{ik}}{\ddot{x}_c \cdot M \cdot \omega_c} = \frac{k_{gik}}{\bar{k}_k} \omega_C, \tag{44}$$

Where i = 1, 2, 3, ..., N is the number of measurement points; K = 1, 2, 3, ..., L is the number of proper forms.
The ratio of the acceleration at the output to the input frequency is usually denoted by the acceleration transfer coefficient of the construction [11]

$$k_{gik} = \frac{\ddot{q}_{ik}}{\ddot{x}_C} \cdot$$

Then we get

$$k_{gik} = \frac{\bar{k}_k \cdot B(\omega)_{ik}}{\omega_C} \cdot \tag{45}$$

6. Conclusion

The dynamic properties of linear systems in solving they described problem are usually characterized by the ratio of the reaction to the external action in the frequency function, that is, the transfer functions. It is also established that the nature of the signal passing through a complex system determined by its structure (stiffness and inertial mass characteristics), as well as the type of the input action, have, in the main, nonlinear dependences on the magnitude of the input action.
Thus, with the linearization of the construction of the transport machine as a complex mechanical system [11], the proposed method of substructures allows us to determine both its own forms and frequencies of oscillations, and its individual parts, using solutions for individual elements and aggregates. In this case, the dynamic model of the individual elements and aggregates of the transport machine can be reduced to a spatial system of solids modeled by finite element methods [14, 15], connected together and the base by elastic-inertial elements representing the bonds.

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