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Noncommutative Black Holes

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Abstract. We study noncommutative black holes, by using a diffeomorphism between the Schwarzschild black hole and the Kantowski-Sachs cosmological model, which is generalized to noncommutative minisuperspace. Through the use of the Feynman-Hibbs procedure we are able to study the thermodynamics of the black hole, in particular, we calculate Hawking's temperature and entropy for the "noncommutative" Schwarzschild black hole.

1. Introduction

In the last years, noncommutativity (NC) has attracted a lot of attention [1, 2]. Although most of the work has been in the context of Yang-Mills theories, noncommutative deformations of gravity have been proposed (see for example [3] and references therein). If we attempt to write down the field equations and solve them, it turns to be technically very difficult, due to the highly non linear character of the theory. In [4], an alternative procedure to incorporate noncommutativity to cosmological models has been proposed, by performing a noncommutative deformation of the minisuperspace. Further, we know that from quantum mechanics we can get the thermodynamical properties of a system. This already has been used in connection with black holes [5]. In [6], the authors use the Feynman-Hibbs path integral procedure [7] to calculate the temperature and entropy of a black hole, in agreement with previous results [8].

In this paper we apply some of these ideas to obtain thermodynamical properties for a quantum black hole and its noncommutative counterpart. We propose a quantum equation for the Schwarzschild black hole, starting from the WDW equation for the Kantowski-Sachs cosmological model. We apply the Feynman-Hibbs method to calculate the thermodynamical properties. We extend this procedure to include noncommutativity by making the same kind of ansatz as in [4], namely, imposing that the minisuperspace variables do not commute; from this we are able to define the WDW equation for the noncommutative Schwarzschild black hole and following a similar procedure as in [6], we find the noncommutative wave function, the temperature and entropy of the "*noncommutative Schwarzschild black hole*".

This contribution is based on the paper [9] and a parallel talk given in the VII Mexican School on Gravitation and Mathematical Physics.

2. Non Commutative Quantum Cosmology and the Quantum Black Hole

Let us begin by reviewing the relationship between the cosmological Kantowski-Sachs metric and the Schwarzschild metric [10]. The Schwarzschild solution can be written as

$$ds^2 = - \left(1 - \frac{2m}{r}\right) dt^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2). \quad (1)$$

For the case $r < 2m$, the g_{tt} and g_{rr} components of the metric change in sign and ∂_t becomes a spacelike vector. If we make the coordinate transformation $t \leftrightarrow r$, and compare to the parametrization by Misner of the Kantowski-Sachs metric

$$ds^2 = -N^2 dt^2 + e^{(2\sqrt{3}\gamma)} dr^2 + e^{(-2\sqrt{3}\gamma)} e^{(-2\sqrt{3}\Omega)} (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (2)$$

we identify

$$N^2 = \left(\frac{2M}{t} - 1\right)^{-1}, \quad e^{2\sqrt{3}\gamma} = \frac{2M}{t} - 1, \quad e^{-2\sqrt{3}\gamma} e^{-2\sqrt{3}\Omega} = t^2, \quad (3)$$

where this metric with the identification of the N , γ , and Ω functions is also a classical solution for the Einstein equations. The metric (2) can be introduced into de ADM action and a consistent set of equations for N , γ , and Ω can be obtained, these equations are equivalent to Einstein equations. The corresponding Wheeler-DeWitt equation for the Kantowski-Sachs metric, with a particular factor ordering (see Ref.[4]), is

$$\left[-\frac{\partial^2}{\partial \Omega^2} + \frac{\partial^2}{\partial \gamma^2} + 48e^{-2\sqrt{3}\Omega} \right] \psi(\Omega, \gamma) = 0. \quad (4)$$

The solution of this equation is given by [11] $\psi_\nu = e^{\pm i\nu\sqrt{3}\gamma} K_{i\nu} \left(4e^{-\sqrt{3}\Omega}\right)$, where ν is the separation constant and $K_{i\nu}$ are the modified Bessel functions. Although the wave functions are not normalizable, by constructing a gaussian wave packet and analyzing the maximum of the probability density, the authors in [12] show that this wave function describes quantum planck size states.

The noncommutative deformation for this cosmological model, has been proposed in [4]. We begin by modifying the symplectic structure in minisuperspace, by assuming that the coordinates Ω and γ obey the commutation relation $[\Omega, \gamma] = i\theta$, in a similar fashion as in noncommutative quantum mechanics. As usual this deformation can be reformulated in terms of the Moyal product [4, 13], by replacing in the Wheeler-DeWitt equation in all products between functions, by Moyal products. It is possible to reformulate in terms of the commutative variables and the ordinary product of functions, if the new variables $\Omega \rightarrow \Omega + \frac{1}{2}\theta P_\gamma$ and $\gamma \rightarrow \gamma - \frac{1}{2}\theta P_\Omega$ are introduced. As a consequence, the original WDW equation changes, with a modified potential $V(\Omega, \gamma) * \psi(\Omega, \gamma) = V\left(\Omega - \frac{\theta}{2}P_\gamma, \gamma + \frac{\theta}{2}P_\Omega\right) \psi(\Omega, \gamma)$ as done in [13], so the NC-WDW equation takes the form

$$\left[-\frac{\partial^2}{\partial \Omega^2} + \frac{\partial^2}{\partial \gamma^2} + 48e^{(-2\sqrt{3}\Omega + \sqrt{3}\theta P_\gamma)} \right] \psi(\Omega, \gamma) = 0. \quad (5)$$

This choice for the shift of the variables is the same used in reference [4], and has the advantage that the classical solutions obtained through a WKB type method are the same that the ones we find by modifying the Poisson brackets to include noncommutativity [14]. We solve this equation by separation of variables with the

ansatz $\psi(\Omega, \gamma) = e^{i\sqrt{3}\nu\gamma}\chi(\Omega)$, where $\sqrt{3}\nu$ is the eigenvalue of P_γ . Thus $\chi(\Omega)$ satisfies the equation,

$$\left[-\frac{d^2}{d\Omega^2} + 48e^{(-2\sqrt{3}\Omega+3\nu\theta)} - 3\nu^2 \right] \chi(\Omega) = 0. \quad (6)$$

The resulting wave functions are not normalizable, but as in the commutative case a normalizable gaussian wave packet can be constructed. As a result of noncommutativity the probability density has several maxima [4], which correspond to new stable states of the Universe, opposite to the commutative case where only one stable state exists [4].

Following the previous discussion we consider Eq.(4). This equation depends on two variables (Ω, γ) , but after the ansatz $\psi(\Omega, \gamma) = e^{i\sqrt{3}\nu\gamma}\chi(\Omega)$, the dependence on γ is the one of a plane wave and is eliminated when computing the thermodynamical observables. Therefore, we could consider this as a suitable approach for the black hole, described by the following equation:

$$\left[-\frac{d^2}{d\Omega^2} + 48e^{-2\sqrt{3}\Omega} \right] \chi(\Omega) = 3\nu^2 \chi(\Omega). \quad (7)$$

From this quantum equation, we use the Feynman-Hibbs procedure to compute the partition function of the black hole. This has the advantage that the relevant information is contained in the potential function $V(\Omega) = 48e^{-2\sqrt{3}\Omega}$. This exponential potential can always be expanded, in particular, the calculations are simplified for small Ω . After expanding to second order in Ω , we make the change of variable $\alpha = \sqrt{6}\Omega - 1/\sqrt{2}$, multiply Eq. (7) by $\frac{E_p}{l_p^2}$, and finally rename $\alpha = \frac{x}{l_p}$ to arrive to,

$$\left[-\frac{1}{2}l_p^2 E_p \frac{d^2}{dx^2} + 4\frac{E_p}{l_p^2} x^2 \right] \chi(x) = E_p \left[\frac{\nu^2}{4} - 2 \right] \chi(x). \quad (8)$$

When comparing with the usual harmonic oscillator we identify $\hbar\omega = \sqrt{\frac{3}{2\pi}}E_p$ in order to obtain the correct Hawking temperature for the black hole. The factor corresponds to an ambiguity in the value of the lowest eigenvalue of the spectrum [15].

Further, the Feynman-Hibbs procedure allows to incorporate the quantum corrections to the partition function through the “corrected” potential [7], and “corrected” partition function

$$U(x) = \frac{3E_p}{4\pi l_p^2} \left[x^2 + \frac{\beta l_p^2 E_p}{12} \right], \quad Z_Q = \sqrt{\frac{3}{2\pi}} \frac{e^{-\frac{\beta^2 E_p^2}{16\pi}}}{\beta E_p}. \quad (9)$$

The internal energy of the black hole is $\bar{E} = -\frac{\partial}{\partial\beta} \ln Z_Q = \frac{1}{8\pi}\beta E_p^2 + \frac{1}{\beta} = Mc^2$. Solving for β in terms of the Hawking temperature $\beta_H = \frac{8\pi Mc^2}{E_p^2}$, the corrected temperature is

$$\beta = \frac{8\pi Mc^2}{E_p^2} \left[1 - \frac{1}{8\pi} \left(\frac{E_p}{Mc^2} \right)^2 \right] = \beta_H \left[1 - \frac{1}{\beta_H} \frac{1}{Mc^2} \right]. \quad (10)$$

In order to calculate the entropy we use the known relationship $\frac{S}{k} = \ln Z_Q + \beta\bar{E}$, from which we arrive to the corrected entropy. In terms of the Bekenstein-Hawking entropy [16], $\frac{S_{BH}}{k} = 4\pi \left(\frac{Mc^2}{E_p} \right)^2 = \frac{A_s}{4l_p^2}$, the black hole entropy takes the simple form,

$$\frac{S}{k} = \frac{S_{BH}}{k} - \frac{1}{2} \ln \left[\frac{S_{BH}}{k} \right] + \mathcal{O}(S_{BH}^{-1}). \quad (11)$$

This result has the interesting feature that the coefficient of the first correction, the logarithmic one, agrees with the one obtained in string theory [17], as well as in loop quantum gravity [18].

For the thermodynamics of the noncommutative black hole, we proceed as before, but instead we consider Eq. (6). A straightforward calculation, using the same steps as in the commutative case, gives a modified version of Eq. (8),

$$\left(-\frac{1}{2}l_p^2 E_p \frac{d^2}{dx^2} + 4\frac{E_p}{l_p^2} e^{3\nu\theta} x^2\right) \chi(x) = E_p \left(\frac{\nu^2}{4} - 2e^{3\nu\theta}\right) \chi(x), \quad (12)$$

with the potential $V_{NC}(x) = 4\frac{E_p}{l_p^2} e^{3\nu\theta} x^2$, and a “frequency”, $\hbar\omega_{NC} = \sqrt{\frac{3}{2\pi}} E_p e^{\frac{3\nu\theta}{2}}$, which coincides with the commutative case for $\theta = 0$. By applying the Feynman-Hibbs method to Eq. (12) we find the corrected partition function, from which we calculate the temperature of the noncommutative black hole in terms of the commutative Hawking temperature. If we define the noncommutative Hawking temperature $\beta_H^{NC} = \beta_H e^{-3\nu\theta}$, the black hole temperature takes the same form as in the commutative case,

$$\beta = \beta_H^{NC} \left[1 - \frac{1}{\beta_H^{NC}} \frac{1}{Mc^2}\right]. \quad (13)$$

The entropy is calculated as before, and defining the noncommutative Hawking-Bekenstein entropy as $S_{NCBH} = S_{BH} e^{-3\nu\theta}$, we get

$$\frac{S_{NC}}{k} = \frac{S_{NCBH}}{k} - \frac{1}{2} \ln \left[\frac{S_{NCBH}}{k} \right] + \mathcal{O}(S_{NCBH}^{-1}). \quad (14)$$

It has the same form as the commutative case; again the logarithmic correction to the entropy appears with a $-\frac{1}{2}$ factor. Also it is clear that we get the commutative entropy in the limit $\theta \rightarrow 0$. As we can see, these thermodynamic quantities are modified due to the presence of the noncommutative parameter. In particular noncommutativity decreases the value of the entropy, which can be understood from the fact that noncommutativity decreases the available physical states.

In this paper we have extended the proposal of noncommutativity in [4] to the Schwarzschild black hole by using the diffeomorphism between the Kantowski-Sachs and Schwarzschild metrics. The corresponding NC-WDW equation is used to describe the noncommutative black hole. Its entropy and temperature are obtained by means of the Feynman-Hibbs formalism on NC-WDW equation, the thermodynamic quantities calculated are modified due to the presence of the noncommutative parameter.

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References

- [1] A. Connes, M. R. Douglas, and A. Schwarz, JHEP **02**, 003 (1998).
- [2] N. Seiberg and E. Witten, JHEP **09**, 032 (1999).
- [3] H. García-Compeán, O. Obregón, C. Ramírez and M. Sabido, Phys. Rev. D **68**, 044015 (2003).
- [4] H. García-Compeán, O. Obregón and C. Ramírez, Phys. Rev. Lett. **88**, 161301 (2002).
- [5] J. W. York, Phys. Rev. D **33**, 2092 (1986).
- [6] O. Obregón, M. Sabido and V. I. Tkach, Gen. Rel. Grav. **33**, 913 (2001).
- [7] R. P. Feynman and A. R. Hibbs, “*Quantum Mechanics and Path Integrals*”, McGraw-Hill, New York, 1965.
- [8] S. Hod, Phys. Rev. Lett. **81**, 4293 (1998); C. Vaz, Phys. Rev. D **61**, 064017 (2000).
- [9] J. C. Lopez-Dominguez, O. Obregon, M. Sabido and C. Ramirez, Phys. Rev. D **74**, 084024 (2006).
- [10] R. Kantowski and R. K. Sachs, J. Math. Phys. **7**, 443 (1966).
- [11] C. Misner, “Minisuperspace”, in *Magic without Magic: John Archibald Wheeler* (Freeman, 1972).
- [12] O. Obregón and M. P. Ryan. Mod. Phys. Lett. A **13**, 3251 (1998).
- [13] L. Mezincescu, “Star Operation in Quantum Mechanics,” arXiv:hep-th/0007046.
- [14] E. Mena, O. Obregón and M. Sabido, “On the WKB Approximation of Noncommutative Quantum Cosmology” to appear in Rev. Mex. Fis; G. D. Barbosa and N. Pinto-Neto, Phys. Rev. D **70**, 103512 (2004).
- [15] H. A. Kastrup, Phys. Lett. B **385**, 75 (1996); H. A. Kastrup, Phys. Lett. B **413**, 267 (1997).
- [16] J. D. Bekenstein, Phys. Rev. D **7**, 2333 (1973); J. D. Bekenstein, Phys. Rev. D **9**, 3292 (1974).
- [17] S. Mukherji and S. S. Pal, JHEP **05**, 026 (2002).
- [18] M. Domagala and J. Lewandowski, Class. Quant. Grav. **21**, 5233 (2004).