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# Nonlinear waves in a falling film with phase transition

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Abstract. Nonlinear wave formation and heat transfer in wavy film flowing over the isothermal wall in the present of phase transition are studied numerically. The integralboundary-layer model, modified with account of the phase change at the interface has been used to describe the wave motion. For the first time the nonlinear evolution of forced twodimensional waves was investigated, and wave effect on heat transfer was determined. It is shown that forced waves essentially intensify heat transfer within a certain range of frequencies as compared to the case of naturally occurring waves. Heat transfer enhancement by waves due to the predominant contribution of the thin residual layer between the peaks was demonstrated. It is shown that by applying the superimposed periodic oscillations, one can intensify heat transfer within a certain range of frequencies as compared to the case of naturally occurring waves.

#### 1. Introduction

Intensive investigation of thin liquid films is caused by their wide application in engineering. Film condensation on the cooling wall and film evaporation on the heated wall are ones of the most important applications. The theoretical study of this problem began with the Nusselt's [1] pioneering work where the stationary laminar film condensation has been considered. In most practically important cases, the film flow is unstable and the liquid surface is covered by developing waves. It is known that the waves in liquid film intensify processes of heat and mass transfer even for laminar flow. Thus, in [2, 3] heat transfer coefficients at evaporation and condensation have been experimentally measured for a wave flow of laminar liquid film. The measurement results are 40-80% higher than the theoretical values for a smooth film. Wave generation and transport processes in falling liquid films without the phase transition were investigated in many studies; the results of these studies are summarized in [4]. The wave dynamics and heat transfer in non-isothermal films are much less studied. In [5] the linear stability of the condensate film is considered in the presence of vapor flow over the film surface. Dispersion dependences, curves of neutral stability, and characteristics of maximum growth waves were obtained.

The purpose of the presented research is to study the nonlinear evolution of waves in the film with a phase transition, and to determine the effect of waves on heat transfer. An integral-boundary-layer model [6] which takes into consideration phase transition is applied to describing nonlinear waves in a liquid film.

#### 2. Theoretical model

Let us consider a vertical laminar film flow on a uniformly heated or cooled plate. Let us introduce Cartesian coordinate system Oxy with Ox axis directed downward, and Oy axis normal to the plate. Let us assume the following basic simplifications, acceptable for a wide range of practically important flow conditions. 1) Plate temperature  $T_w = const$ , the liquid surface is in contact with stagnant

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saturated vapor with temperature  $T_s = const \cdot 2$ ) The perturbation of the film surface is considered to be longwave (the typical length of the perturbation is much greater than the film thickness). 3) Density  $\rho$ , dynamic viscosity  $\mu$ , liquid conductivity  $\lambda$ , surface tension  $\sigma$ , and latent heat of evaporation r are assumed constant. Let us introduce the distance scale l along the Ox axis and take film thickness  $h_m$  at x = 0 as the distance scale along the Oy axis. We introduce velocity scale  $u_m = gh_m^2/3v$ , time scale  $t_m = l/u_m$ , flow rate scale  $q_m = u_m h_m$ , temperature scale  $\Delta T = |T_s - T_w|$ , and move on to the dimensionless variables x/l,  $y/h_m$ ,  $h/h_m$ ,  $q/q_m$ ,  $t/t_m$ ,  $u/u_m$ ,  $v/u_m$  keeping for them the same notations. In dimensionless variables the film flow with phase transition is described by the set of equations [7, 8] with respect to film thickness h(x,t), flow rate q(x,t) and temperature  $\theta(x,\eta,t) = \pm (T - T_w)/\Delta T$ :

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{6q^2}{5h} \right) = \frac{3}{\chi \operatorname{Re}_m} \left( h - \frac{q}{h^2} \right) + \chi^2 Weh \frac{\partial^3 h}{\partial x^3},$$

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = \pm \frac{A}{\chi \operatorname{Re}_m h},$$
(1)
$$\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + \frac{W}{h} \frac{\partial \theta}{\partial \eta} = \frac{1}{\chi \operatorname{Re}_m \operatorname{Pr} h^2} \frac{\partial^2 \theta}{\partial \eta^2}.$$

Here and in other formulae sign "plus" corresponds to condensation and sign "minus" corresponds to evaporation,  $A = \varepsilon \cdot \partial \theta / \partial \eta \Big|_{\eta=1}$ ,  $\varepsilon = c_p \Delta T / (r Pr)$  is a parameter characterizing intensity of phase transition,  $Re_m = gh_m^3 / 3v^2$  is the Reynolds number at the inlet,  $\chi = h_m / l$  is the ratio of scales,  $We = (3Fi / Re_m^5)^{1/3}$ ,  $Fi = \sigma^3 / \rho^3 g v^4$  is the Kapitsa number, Pr is the Prandtl number,

$$W = \left(\eta - \frac{3\eta^2}{2} + \frac{\eta^3}{2}\right) \frac{\partial q}{\partial x} - \frac{3(\eta^2 - \eta^3)}{2} \frac{\partial}{\partial x} \left(\frac{qA}{4+A}\right) - \frac{A\eta}{\chi Re_m h}.$$

For steady flow the effect of surface tension can be neglected. Under a condition  $\varepsilon \ll 1$  the temperature profile is closed to linear  $\theta = \eta$ , and the solution to the first two equations (1) will be as follows [7, 8]:

$$h_{steady} = \left(1 \pm \frac{4\varepsilon \cdot x}{3\chi \, Re_m}\right)^{1/4}, \quad q_{steady} = h^3.$$
<sup>(2)</sup>

Wave flow regimes are obtained by numerical solution to equations (1) using the finite-difference method described in [7]. The waves were generated by a small artificial perturbation of the flow rate at the inlet  $q(0,t) = q_0(1 + Q_a \sin 2\pi f t)$ . Here  $q_0$  is the undisturbed flow rate,  $Q_a$  is small amplitude, and f is flow rate oscillation frequency. The constant film thickness h(0,t) = 1 and the linear temperature profile  $\theta(0,\eta,t) = \eta$  were set at the inlet. Linear temperature profile was set at time t = 0 for the all region of calculation; distributions of h(x,0) and q(x,0) were set by formulas (2). Boundary conditions for the energy equation are  $\theta|_{\eta=1} = 1$ ,  $\theta|_{\eta=0} = 0$ . Region of calculations was also chosen sufficiently large to assure that the wave regime is developed over the all computation region.

For the computation of heat transfer from the liquid to the isothermal plate, time averaged local Nusselt number was calculated as  $\langle Nu_x \rangle = \frac{1}{t} \int_0^t Nu(x,t) dt$ . Time averaged local Reynolds number was

calculated at the same time:  $\langle Re_x \rangle = \frac{1}{t} \int_0^t Re(x,t) dt$ . Here  $Nu(x,t) = \frac{1}{(3Re_m)^{1/3}} \frac{\partial \theta}{\partial \eta} \Big|_{\eta=0}$  and

 $Re(x,t) = Re_m q(x,t)$  are instantaneous values of the local Nusselt and Reynolds numbers.

#### 3. Simulation results

All calculations were carried out for water at the temperature of 373 K (Pr = 1.75,  $Fi^{1/3} = 14700$ ) for  $Q_a = 0.01$ ,  $\varepsilon = 0.005$ . Reynolds number at the inlet  $Re_m = 1$  in a case of condensation and  $Re_m = 40$  in a case of evaporation. The natural waves caused by instability of film flow appear in the absence of artificial perturbations at the inlet. In a case of condensation the perturbations of the film surface become obvious in the narrow zone near x = 0.4 m; above this zone the film remains smooth. At distance 0.4 - 0.45 m from the inlet perturbations develop fast into the nonlinear waves of high amplitude. Similar natural waves were observed in [8] at numerical simulation of the R11 condensate film flow.

#### 3.1. Forced waves

The forced waves imply the waves, which develop in the liquid film due to periodic external action. In this case the wave pattern over a certain frequency range takes the regular 2D character. This allows the detailed investigation of wave dynamics and identification of the mechanism of wave effect on transfer processes that is the main goal of the current research. Usually, in experiments with forced waves the pulsations of liquid flow rate are used. As applied to the isothermal films, the forced waves are described in detail in [4]. The wave surface of condensate film at forcing frequency of 18 Hz, which slightly differs from frequency of developed natural waves, is shown in Fig 1. The forced waves have the regular character and develop at a distance of 0.25-0.3 m from the inlet, i.e. earlier than in case of natural waves. Upstream the wave amplitude is so small that perturbations are not visible in Fig 1. Time period 1/f corresponds to the distance between the peaks indicated in Fig. 1. We should note that the term "wavelength" here is conventional. Since the thickness of undisturbed film and the average film flow velocity increase downstream, there is no strict spatial periodicity of the waves. We note also that despite the amplitude of developed wave increases downstream, the residual layer thickness between the peaks does not depend on coordinate. The film surface at lower frequency of 6 Hz is shown in Fig. 2. Here, in contrast to the case with f = 18 Hz, we can observe spatial irregularity, i.e. existence of lower peaks between the high peaks. Appearance of additional peaks caused by the growth of higher harmonics in the region of nonlinear wave development occurs at frequencies significantly lesser than 18 Hz.

On Fig. 3 the wave surface of an evaporating film for frequency of 18 Hz is shown. Like the condensation case, the forced waves are regular and develop much earlier, than natural ones (on







Figure 2. Forced waves in a condensate film; f = 6 Hz. The length with the arrows shows the "wave length" corresponding to 6 Hz.

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distance of 4–5 wavelengths from an inlet). Apparently from a figure, the amplitude of the developed waves in evaporating film smoothly decreases with distance, unlike a condensate film for which the amplitude of the forced waves increases downwards on a stream. As well as in case of condensation, the time period 1/f corresponds to distance between adjacent peaks. As an undisturbed film thickness decreases with distance, here there is no strict space periodicity of a wave. So for frequency of 18 Hz the space interval between peaks smoothly decreases from 1,5 cm on an input to 0,9 cm on distance of 0,4 m owing to decreasing of movement velocity of peaks. The wavy film surface for frequency of 5 Hz is shown on Fig. 4. Here intermediate peaks like on Fig. 2, appear already at a linear stage simultaneously with the main peaks. On an interval in one "wavelength" are formed two additional peaks. As a result of merging of intermediate peaks the amplitude of the main peaks considerably grows. As a whole, for an evaporating film as well as for a condensate film the same features of evolution of the forced waves, are observed. At frequency comparable to ones of natural waves the steady wave mode takes place with monotonously varying amplitude downwards on a stream. At frequency essentially smaller, than ones of natural waves, intermediate peaks are formed already at a linear stage. Unlike a condensation case, the wave amplitude decreases with distance, but thickness of a residual layer increases downstream a little.

#### 3.2. The effect of waves on heat transfer

The temperature profiles in a condensate film are shown in Fig. 5 for forcing frequency of 9 Hz in different film cross-sections. According to the figure, in the region of residual layer (cross-section 1) the temperature profile does not differ from the linear one, and in the peak area (cross-sections 2, 3, 4) it differs significantly from linear trend. This proves a considerable contribution of convection into heat transfer in the peak area. This fact is also proved by Fig. 6, where the streamlines are shown in a





**Figure 5.** Temperature profiles in the crosssections 1-5; condensate film; forced waves.



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reference frame moving with the speed of the wave. As it follows from the figure, there is a zone of circulation near the maximum, where convection should enhance heat transfer in transverse direction. The zone of circulation was observed repeatedly at numerical simulation of large waves.

Dependences of time-averaged local Nusselt number  $\langle Nu_x \rangle$  on coordinate are shown in Figures 7 and 8 for different values of forcing frequency. In the region of smooth film calculation coincides with theoretical dependence, but in the region of nonlinear wave development the value of  $\langle Nu_{1} \rangle$  increases rapidly. The coordinate, at which the nonlinear wave starts to develop, depends on frequency, but in all cases the forced waves develop earlier than the natural ones. Calculations show that heat transfer enhancement by the waves occurs mainly because of a decrease in the film thickness between the peaks. The local Nusselt number is determined by the length of the region between the peaks and depends on the coordinate and the forcing frequency. The distance between the peaks increases because of wave merging, and value of  $\langle Nu_x \rangle$  increases significantly owing to absorbing of lower peaks by the higher ones.





**Figure 7**. The Nusselt number versus coordinate for evaporating film; 1 - natural waves; forced waves: condensate film; 1 - natural waves; forced waves: 2 - 30Hz, 3 - 10Hz, 4 - 8Hz, 5 - 5Hz, 6 - 18Hz; dashed line: the theoretical value for smooth film.

Figure 8. Nusselt number versus coordinate for 2 - 25Hz, 3 - 3Hz, 4 - 5Hz, 5 - 18Hz, 6 - 9Hz; dashed line: the theoretical value for smooth film.

#### 4. Conclusions

In the presented paper, a numerical method is used to study nonlinear wave formation and heat transfer in the film of water flowing down a vertical wall in the presence of a phase transition. For description of the wave motion of the liquid film the integral-boundary-layer model modified with considering the phase transition is applied. The equations were solved using a finite difference method for the spatial statement of the problem. The evolution of both natural waves arising due to the instability of the flow, and forced waves generated by small periodic perturbation of flow rate at the inlet, was investigated. Natural waves appear at some distance from the inlet and very quickly transform into large-amplitude nonlinear waves with a thin residual layer between them. Because of stochastic behavior of the natural waves, their parameters at the nonlinear development stage change randomly, that leads, in particular, to the merging of the individual waves.

The application of small periodic forced perturbation within a certain frequency range results in appearance of regular waves with stable characteristics. For the first time the evolution of forced waves in a film with phase transition is studied in a wide range of frequencies. The coordinate, at which the nonlinear wave starts to develop, depends on frequency, but in all cases the forced waves develop earlier than the natural ones. At sufficiently low forcing frequency, the intermediate peaks (generation of higher harmonics) are observed. This results in the wave interaction with the subsequent merge of the individual peaks. Sufficiently high forcing frequency results in appearance of subharmonic instability, which also leads to the interaction and merging of the waves.

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Heat transfer calculations show that at the appearance of waves (both natural and forced) heat transfer coefficient abruptly increases up to a certain level, depending on the frequency of the waves. The analysis of the calculation results shows that the heat transfer coefficient in the wavy film is described rather accurately in the approximation of a linear temperature profile, i.e. the local Nusselt number is proportional to the value of 1/h, where h is local film thickness. Thin residual layer between peaks gives main contribution to the heat transfer intensification. The vortex zone observed in nonlinear waves also leads to the heat transfer intensification, though this enhancement is insignificant as compared to the contribution of the residual layer. Wave interaction plays an important role in heat transfer; it manifests itself at low and high forcing frequencies, as well as in case of natural waves. This leads to merging of the waves and increasing the zone of residual layer, i.e. results in heat transfer enhancement. From a practical point of view, the results presented can be used to select the operation modes of industrial condensers and evaporators.

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