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# A stochastic aerodynamic model for stationary blades in unsteady 3D wind fields

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#### Abstract.

Dynamic loads play an important roll in the design of wind turbines, but establishing the life-time aerodynamic loads (e.g. extreme and fatigue loads) is a computationally expensive task. Conventional (deterministic) methods to analyze long term loads, which rely on the repeated analysis of multiple different wind samples, are usually too expensive to be included in optimization routines. We present a new stochastic approach, which solves the aerodynamic system equations (Lagrangian vortex model) in the stochastic space, and thus arrive directly at a stochastic description of the coupled loads along a turbine blade. This new approach removes the requirement of analyzing multiple different realizations. Instead, long term loads can be extracted from a single stochastic solution, a procedure that is obviously significantly faster. Despite the reduced analysis time, results obtained from the stochastic approach match deterministic result well for a simple test-case (a stationary blade). In future work, the stochastic method will be extended to rotating blades, thus opening up new avenues to include long term loads into turbine optimization.

#### 1. Introduction

Today, aerodynamic optimization of wind turbines faces two major challenges. First, the computational tools commonly at hand (BEM solvers) are inherently limited to planar rotor designs [1], while more flexible simulations, based on the full time-domain solution of the discretized Reynolds Averaged Navier Stokes equations (CFD methods), are too expensive to be viable for optimization [2]. Second, it is computationally difficult to include long term (fatigue and extreme) loads into the optimization loop [3], because this kind of analysis requires the simulation of long load samples in order to be able to extract consistent statistics and thus reliably extrapolate to long term loads. IEC 61400-1, Ed. 3 [4], for example, bases the long term analysis of every load case on ten minute simulations, run at different mean wind speeds, each repeated six times with different random seeds for the turbulent inflow field. Obviously this results in a large

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number of ten-minute simulations. Still, it has been shown[5, 1] that even with this many simulations, extrapolation to extreme loads is a delicate exercise and results may vary significantly. Zwick and Muskulus [6] for example showed recently that when basing a wind turbine analysis on six ten-minute wind speed realizations a difference of up to 34% occurs in the ultimate load results for the most extreme 1% of seed combinations. Other recent research [7] indicates that turbine loads extracted even from 20 different ten minute wind fields generated from 20 different random seeds for each wind speed vary greatly. Moreover, often load variations from different random seeds dominate effects from design parameter changes, obviously a severe problem, especially when concerned with gradient-based optimization where obtaining reliable design variable gradients is vital.

As a result of these two challenges, current wind turbine optimization is inherently limited, as it a) cannot explore unconventional, but potentially beneficial designs such as winglets, swept or downwind coning rotors, etc., and b) is blind to the important[8] cost savings of modified long term loads and power production from different blade designs operating in unsteady conditions. For example, Zhou et al. [9] show that the peak power fluctuation for wind turbines can reach 22% of its average. A steady state optimization potentially fails to find a solution close to the 'real' (unsteady) optimum.

Lagrangian vortex models (LVM)[10, 11, 12, 13] based on Prandtl's lifting line theory [14, 15] are an attractive solution to the first challenge. They are relatively fast to solve, but flexible enough for unconventional geometries, see e.g. [16]. However, time stepping through multiple LVM solutions to obtain data for extrapolation to long term loads is not an option within the time budget of optimization. To tackle this dilemma, the available model has to be extended, such that the full set of unsteady inflow conditions with all extreme loads can be modeled, while keeping the computational cost down. To do so, we present a stochastic lifting line model to obtain long term loads and power output relatively quickly from a single stochastic solution. Besides a faster solution, the stochastic formulation potentially captures true extreme loads better than relying on the extrapolation of limited short term loads from short term time stepping simulations.

In this paper we present a stochastic blade load model, resolved with several spanwise lifting line elements in a stochastic unsteady (coherent) three dimensional wind field. Section 2 will give an introduction to the approach. Starting from a basic aerodynamic LVM blade model in its deterministic formulation (2.1) and the stochastic inflow formulation (section 2.2), the transition to the stochastic blade model with correlated wing blade sections will be presented (section 2.3). For clarity of the method and its presentation we direct our focus to a stationary blade. In section 3 we will compare the load time series as well as the statistics from a deterministic lifting line solution to the results from the proposed stochastic method. Moreover, the reduction of the computational cost will be discussed. Future steps will extended the model to stochastic unsteady aerodynamics simulations of a full wind turbine in operation.

# 2. Approach and methods

When dealing with wind turbine optimization, the driving wind inflow is usually represented by a stochastic model, typically specified in the frequency domain. However, instead of dealing with the notion of stochasticity during system modeling, several realizations of the stochastic input wind are usually generated prior to the simulation procedure [17]. For wind turbine analysis several ten minutes wind speed samples (deterministic blocks of frozen wind) are generated and then each analyzed consecutively [1]. This means that instead of using the abstract stochastic equations, several deterministic time-series are obtained (see Figure 1, upper path). While the benefit of this approach is its ability to use standard (and more intuitive) deterministic, generally nonlinear, time-marching simulation codes, it discards the advantages of a direct stochastic description during the system modeling phase.



Figure 1: Alternate solution methods. Upper path: conventional with realizations generated before solution; Lower path: the new process with a stochastic solution.

In [18, 19] we demonstrated two ways of conducting the system model analysis directly in the stochastic space, thus eliminating the need for multiple realizations of the inflow. Instead of the common practice of repeatedly solving the aerodynamic equations for multiple deterministic realizations (Figure 1, upper path) one single stochastic solution is sought (Figure 1, lower path). This stochastic solution will contain all possible realizations. Thus long term loads can either be extracted quickly from several realizations of the stochastic solution, or directly from the statistics of the solution's random variables. For the case of a very simple horseshoe vortex lifting line model of a stationary rectangular wing, resolved with a single spanwise lifting line element, we were able to retain stochastic properties and thus the effects captured with different misses significant effects from the spatial variability and cross-correlation of the wind field and hence it omits the spacial coupling of the blade loads. In the remainder of this section we will introduce the necessary model components to arrive at a stochastic lifting line model for the coupled loads along wind turbine blades.

# 2.1. Aerodynamic model

For the blade load calculations, a LVM in unsteady lifting line formulation [14, 15] will be used.<sup>1</sup> The vortex system is set up as indicated in Figure 2 with an arbitrary number  $N_b$  of bound (linearly spaced) elements. Trailing and shed elements of variable strength are included to capture shed vorticity and the resulting time lagged variation of the

<sup>1</sup> For a detailed introduction to lifting line models see Prandtl's original publications [14, 15] or one of the many more recent works, e.g. [20, 21, 22].

induced velocity from the inflow changes at the wing as well as the delayed vortex shedding downstream. The wake length  $N_t$  can be chosen freely. For the time being, wake element positions are assumed fixed and in the x-y-plane.

At each station I along the blade we assume small angles of attack  $\alpha_{g,I}$ , small induced velocities  $w_I$ , and thus use linear airfoil properties  $(c_{l,I} = 2\pi(\alpha_{g,I} + w_I/u_\infty))$  to arrive at the well known unsteady lifting line equation:

$$\Gamma_I(t_n) = A_I \cdot u_{\infty,I}(t_n) + \sum_i \sum_j H_{I,ij} \cdot \Gamma_i(t_{n-j})$$
(1)

With this equation we calculate the bound circulation (and thus load)  $\Gamma_I(t_n)$  at each station I for each time step  $t_n = n \cdot \Delta t$  implicitly from the current free stream wind at the respective station  $u_{\infty,I}(t_n)$  and the previous blade loads  $\Gamma_i(t_{n-j})$  at all wake stations i; here  $A_I = c_I \pi \alpha_{g,I}$ . The tensor  $H_{I,ij}$  contains the Biot-Savart influence from each wake element ij onto the blade element I, as well as the coupling of the current wake strength and blade load history:  $\Gamma_{t,ij} = \gamma_{ij} - \gamma_{(i-1)j}$  and  $\Gamma_{s,ij} = \gamma_{ij} - \gamma_{i(j-1)}$  respectively, with  $\gamma_{ij} = \Gamma_i(t_{n-j})$  (see Figure 2).

With a known, deterministic inflow  $u_{\infty}$  equation 1 can readily be solved with the common methods.

2.2. Stochastic wind model



system used to model blades loads.

To calculate blade loads, deterministic models use random, but correlated wind speed data  $u_I(t_n, \boldsymbol{\xi}_I)$ generated in advance at many locations  $P_I$  over the rotor disc from adequately correlated realizations of the random vectors  $\boldsymbol{\xi}_I$ . In Veers' model [23], which is widely used in industry (e.g. NREL's *TurbSim* [24] tool), the wind speed time series at each location  $P_I$  is generated though an inverse Fourier transform

$$u_I(t_n) = \sum_m V_{mI} e^{i(\omega_m t_n)}$$
(2)

Here each (complex) Fourier coefficients  $V_{mI}$  for each frequency  $\omega_m$  at each location  $P_I$  is initially generated from the wind speed spectrum and an independent random variable  $\xi_{mI}$  (i.e. a random phase angle), uniformly distributed in [0, 1]. To

correctly model the coherence of the random wind field correlation of the Fourier coefficients at different locations is subsequently enforced via the coherence matrix (for details see e.g. Veers original publication [23]). With this procedure Veers' approach requires  $N_R = N_I \cdot N_f$  independent random variables for  $N_I$  locations and  $N_f$  frequencies. Since the computational cost of our stochastic method is highly sensitive to the number

of random variables  $N_R$ , we first split Veers' complex Fourier coefficients into amplitude and phase,  $V_{mI} = \sqrt{S_m} e^{i\theta_{mI}}$ , with  $\theta_{mI} = 2\pi\xi_{mI}$  the (correlated) phases at each point and each frequency and the amplitude prescribed by the discrete wind speed power spectrum  $S_m$ . Next we split the phase angles into the phases at one arbitrary base point  $P_0$  and a set of correlated phase increments:  $\theta_{mI} = \theta_{m0} + \Delta\theta_{mI}$ . Thus equation 2 becomes:

$$u_I(t_n) = \sum_m \sqrt{S_m} \, e^{i(2\pi\xi_m + \Delta\theta_{mI})} \, e^{i\omega_m t_n} \tag{3}$$

Now  $\boldsymbol{\xi} = [\xi_m]$  is a single vector of uncorrelated random variables specified at one (arbitrary) base point  $P_0$ . The phase (and thus wind speed) correlation between the points  $P_I$  and  $P_0$  is contained in the correlated phase increment vectors  $\Delta \boldsymbol{\theta}_I = [\Delta \theta_m]_I$ . Details of this reduced order wind model will be given in a forthcoming publication. In the following, we focus on the aerodynamic blade model and its stochastic solution.

With Veers' model one wind field realization is obtained from first generating independent phase vector realizations for all points and subsequently correlating the phase angles. Equation 3, on the other hand, finds one wind field realization with one random phase vector  $\boldsymbol{\xi}$  at one base point  $P_0$  only and a set of random phase increments  $\Delta \boldsymbol{\theta}_I$  for each other point. For a stochastic analysis equation 3 has a major advantage: the generation of random phases angles and the correlation between two points is now separated into  $\boldsymbol{\xi}$  and  $\Delta \boldsymbol{\theta}_I$  respectively. While all  $\boldsymbol{\xi}_m$  are independent,  $\Delta \boldsymbol{\theta}_I$  are correlated, which provides the opportunity to establish a reduced order model. If we assume that the random dimension of the wind field is sufficiently approximated by  $\boldsymbol{\xi}$  only, than  $\Delta \boldsymbol{\theta}_I$  can be considered a deterministic variable. This reduces the number of necessary random variables by several orders of magnitude. A detailed discussion of the consequences of this split between stochastic phases and deterministic phase increments is beyond the scope of this paper. A comprehensive study is currently underway and will be available shortly.

Figure 3 shows a comparison of the input wind obtained from *TurbSim* and two realizations of the reduced order model for the wind speed at four points with the coordinates (horizontal, vertical):  $P_1$  (1,0) m;  $P_5$  (1,90) m;  $P_6$  (0,0) m;  $P_{10}$  (0,90) m. By inspection, it can be seen that the properties of the wind seem to be reproduced well. Note that the reduced order model of equation 3 does not result in a complete determination of the spatial relation between wind speeds at different points, as the samples still contain different constructive/ destructive phase interference and thus gusts/ lulls at different instances in time. Both the theoretical details of this model as well as a comprehensive discussion of the resulting wind speed time series are beyond the scope of the present paper and will be discussed in a separate publication. Here, Figure 3 and the wing load results presented later shall suffice as justification to use this model.

#### 2.3. Stochastic blade load model

For the stochastic solution we follow the process of stochastic projection (Fourier-Galerkin method) presented in [19], expanded to the multi-element lifting line equation 1. We directly use equation 3 in the reduced order stochastic formulation with  $\Delta \theta_I$ 



Figure 3: Samples of wind speed time series at four points generated from *TurbSim* and two different realizations of the reduced order model.

understood as deterministic phase increments and  $\boldsymbol{\xi}$  a random phase angle vector to represent the correlated random wind field. Similar as in [19] the wing load can be expressed in terms of the same complex Fourier series as the inflow:

$$\Gamma_I(t_n) = \sum_l g_{lI} \Phi_l(t_n) e^{i \cdot 2\pi\xi_l} \tag{4}$$

with  $\Phi_l(t_n) = e^{i\omega_l t_n}$ . Recognizing that l = m, equations 3 and 4 can be inserted into equation 1. With  $t_n = n\Delta t$  and basic algebra we can write  $\Phi_l(t_{n-j}) = \Phi_l(t_n)\Phi_l(-j\Delta t)$ . Thus, after truncating the series to a reasonable number of frequencies  $N_f$  and projecting onto the basis functions  $\Phi_l(t_n)$ , we arrive at a system of equations in the stochastic space  $I = 1...N_f$  (for a detailed derivation and a discussion of this method refer to [19]):

$$g_{Il} = \sqrt{S_l} \cdot A_I e^{i\Delta\theta_{Il}} + \sum_i \sum_j H_{I,ij} g_{Il} \Phi_l(-j\Delta t)$$
(5)

Note that although equation 5 yields the coefficients  $g_{Il}$  of the stochastic solution (equation 4), it is a deterministic equation itself. Hence it can be solved with common (deterministic) approaches in the same way as equation 1. However, since the stochastic solution (equation 4) still contains the random vector  $\boldsymbol{\xi}$ , it contains all possible realizations. Hence this one solution contains all possible long term blade loads that can possibly result from any phase combination in the inflow wind (lower path in Figure 1).

At the moment linear airfoil properties are assumed. This results in equation 5 being conveniently decoupled for each frequency. Moving forward to the optimization of wind turbine blades in turbulent wind conditions two more steps are necessary. First, the equations need to be applied to a rotating blade. This will be straight forward through the introduction of a rotating coordinate system. Secondly, nonlinear airfoil properties, such as (dynamic) stall, need to be included. This will turn equation 5 into a more complicated set of coupled equations. To proceed we currently see three options:

- (i) Deal with the nonlinear, coupled equations in the current framework;
- (ii) Find and include a stochastic surrogate model of the nonlinear on-blade effects;
- (iii) Move from a Fourier expansion of Eqs. 3 and 4 to a polynomial chaos expansion (PCE) [19]. Since the PCE retains the time domain through the solution it will likely be easier to handle non-linear effects in the equations.

An investigation of these options and advancing to wind turbine blade optimization will be the subject of future work.

## 3. Results

In this section we compare lifting line results for blade loads from the stochastic model (equation 5) to deterministic results (equation 1). We consider a blade of b = 60 m span with constant chord c = 4 m, stationary 100 m above ground in  $\bar{u}_{\infty} = 10$  m/s mean wind (IEC normal turbulence model, class A[4]). The blade and wake are resolved with  $N_b = 6$  spanwise and  $N_t = 5$  trailing elements. Loads are calculated at  $\Delta t = 1$  s intervals. Wind is calculated from  $N_f = 20$  frequencies logarithmically spaced at  $f_m = \omega_m/(2\pi) \in [1/600, 0.5]$  Hz. For the deterministic case the usual process was followed: one wind speed realization was calculated first using Veers' model [23], then this wind was fed into equation 1 as inflow  $u_{\infty,I}(t_n)$ .

Figure 4 shows the wind speed realization used for the deterministic solution at the six blade elements together with the resulting deterministic bound circulation and one realization of the stochastic solution (bottom), as well as the steady state solution for u = 10 m/s with five snapshots of the stochastic solution (top). Note the transient start-up period in the deterministic blade loads for t < 5 s (bottom part for Figure 4). This is due to the wake initialization with  $\Gamma_{I,ij}(t_{n=0}) = 0$ . As a result of the periodic nature of  $\Phi_l(t_n)$  this transient phase does not exist in the stochastic solution.

The realization of the stochastic solution is generated from the same random seed as was used to generate the deterministic wind. Hence the stochastic solution perfectly reproduces that one particular deterministic solution (after the initial transient phase t > 5 s). Note, however, that the stochastic solution also contains all other possible realizations. Hence all other load time series can be calculated directly via equation 4 from only one stochastic solution.

Next we will compare cross-correlation, covariance, and cross-spectrum of the dynamic blade loads from our stochastic method to the deterministic results. For the deterministic analysis, blade loads were calculated for 100 realizations of 600 s samples of turbulent wind synthesized with the commonly used wind simulator *TurbSim* [24]. For better comparison the spectrum S in equation 3 and 5 was replaced with the (slightly different) spectrum extracted from the *TurbSim* wind data set via Welch's periodogram method as implemented in Matlab's *cpsd* function [25]. Similarly, phase increments  $\Delta \Theta_I$  were extracted from the Fourier transform of the *TurbSim* data. To achieve smoother cross-correlation, covariance, and cross-spectrum curves we used 100 different sets of phase increments  $\Delta \theta_I$ . As discussed in section 2.2, most of the stochasticity is contained in the uncorrelated random variables  $\boldsymbol{\xi}$ . The influence of random or deterministic  $\Delta \theta_I$  is hence a second order effect. Assessing the implications of only using one (or a few) set(s) of phase increments remains the subject of further study. As before, we use  $N_b = 6$ ,  $N_t = 5$ ,  $N_f = 20$  with  $f_m \in [1/600, 5]$  Hz, and  $\Delta t = 1$  s.



Figure 4: Wing load (circulation) and wind speed time series.

#### 3.1. Blade loads cross-correlation

Figure 5 shows the cross-correlation function of the circulation on the bound element pairs  $\Gamma_1$ - $\Gamma_1$ ,  $\Gamma_1$ - $\Gamma_3$ , and  $\Gamma_1$ - $\Gamma_6$ . It can be seen that the stochastic model results in a very similar cross-correlation as the computationally much more expensive deterministic model. The obvious difference – a much smoother curve in the deterministic cases – is due to the difference in the number of frequencies contained in the data. While the deterministic data (provided through *TurbSim*) is synthesized with well over ten thousand frequencies, the stochastic data is based on the reduced order model with only 20 frequencies.

#### 3.2. Blade loads covariance

Table 3.2 shows the covariance of the blade loads for selected bound vortex elements (cf. figure 2) for the stochastic and deterministic results, and the error between the two results. This error is small, especially considering that (assuming normal distributed blade loads) the 90%  $\mathcal{X}^2$  confidence bounds of the deterministic variance Var[ $\Gamma_1$ ,  $\Gamma_1$ ] are [-0.94, 0.96]%. Thus it can be seen that the stochastic and deterministic results are in very good agreement.

#### 3.3. Blade loads cross-spectrum

Figure 6 shows the cross power spectral density functions of the circulation for the same bound element pairs as before. Both the deterministic and stochastic results were again obtained from Welch's spectral estimation from 100 realizations of the resulting blade loads, binned to the frequencies  $\omega_m$  as used in the reduced model. Again, the stochastic

	$\operatorname{Var}[\Gamma_1,\Gamma_1]$	$\operatorname{Var}[\Gamma_1,\Gamma_2]$	$\operatorname{Var}[\Gamma_1,\Gamma_3]$	$\operatorname{Var}[\Gamma_1,\Gamma_5]$	$\operatorname{Var}[\Gamma_4,\Gamma_5]$
$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$4.06 \\ 4.20$	$3.76 \\ 3.77$	$2.60 \\ 2.56$	$\begin{array}{c} 4.17\\ 4.20\end{array}$	4.82 4.91
error [%]	3.27	0.20	-1.37	0.63	1.81

Table 1: Covariance of blade loads.

results are very similar to the computationally much more expensive deterministic model.

Note the two zero-power frequencies  $\omega_2$  and  $\omega_3$  result from the different frequency spacing in the *TurbSim* data (linear spacing) versus the reduced model (logarithmic spacing). The *TurbSim* data does not contain frequencies between  $\omega_1$  and  $\omega_4$ , and hence there is zero power in the spectrum there. Although these frequencies exist in the stochastic data, they do not contain any power because we use the *TurbSim* spectrum  $\boldsymbol{S}$  in equation 5.



Figure 5: Cross-correlation function for circulation of selected bound element pairs.



Figure 6: Discrete cross-spectrum for circulation of selected bound element pairs.

# 3.4. Numerical Cost

At this point we use a crude Matlab implementation of both the deterministic and stochastic models. This implementation was designed for experimenting with the new model, rather than for performance. Hence, only a very rough estimate of the computational cost can be given. However, the superiority of the stochastic model becomes apparent nonetheless. On an Intel i5 quad core processor solving 100 realizations of 600 s of the deterministic model takes roughly 10 minutes (upper path in figure 1). On the other hand, calculating one stochastic solution with one set of

deterministic phase increments takes less than one second, and realizing 100 samples at the end (lower path in figure 1) is done in about three seconds, yielding 100 realizations from the stochastic model in under five seconds. This is less than 1% of the time originally used for the deterministic solution.

As shown in the preceding subsections, the stochastic solution reproduces the statistics of the (correlated) blade loads well. Through postponing the generation of various different realizations from before calculating a solution (deterministic approach, upper path in Figure 1) to generating realizations of a stochastic solution at the end (stochastic approach, lower path in Figure 1) long term loads can be estimated very quickly and thus become accessible to be included into an optimization routine.

Besides generating individual solution realizations, a stochastic expression for the blade loads also holds the possibility of directly extracting load statistics (mean, variance, possibly peak return periods). Details of such a stochastic load analysis from a stochastic load solution will presented in a future publication.

# 4. Conclusions

We extended a stochastic model previously introduced for a single element blade to model correlated aerodynamic loads on a stationary wind turbine blade in turbulent atmospheric wind. The blade is now resolved with several spanwise and wake lifting line elements. A reduced order model for the (stochastic) wind inflow fields, based on phase angle increments between the wind speed at a base point and any other data point in the wind field is employed to limit the number of random variables necessary.

Through a comparison to deterministic results (based on several wind field realizations generated from TurbSim) it was shown that the stochastic model conserves the stochastic properties of the dynamic blade loads, including coupling of loads on different stations along the blade (cross-correlation, covariance, cross-spectrum). For a simple Lagrangian vortex model test case (lifting line model), the stochastic formulation yielded results for 100 blade load samples in less than 1% of the computation time needed by the deterministic model. This indicates that long term (extreme and fatigue) blade loads, commonly extracted from the tails of a load probability distribution, can now be assessed quickly.

Future work will take the currently stationary blade into a rotating reference frame to model a wind turbine rotor and subsequently include the stochastic formulation into a multidisciplinary design optimization framework. While the Lagrangian vortex model allows extension of the design space to unconventional blade geometries, the stochastic aerodynamic model limits the computational effort required to extract reliable blade load statistics. The new stochastic model also makes long term loads directly accessible to optimization. Hence the combination of a Lagrangian vortex model with a stochastic aerodynamic model might open up new avenues to pursue improved and novel wind turbine blade designs.

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