

PAPER • OPEN ACCESS

A simulation of the performance of a self-tuning energy harvesting cantilever beam

To cite this article: J L Kaplan *et al* 2016 *J. Phys.: Conf. Ser.* **744** 012083

View the [article online](#) for updates and enhancements.

You may also like

- [Analysis of the power part architecture for short-medium-range aircraft control system with local hydraulic systems by reliability criterion](#)
S Postnikov, A Trofimov and D Smagin
- [Automatic control system for the technological process of chlorella cultivation](#)
Sh Rakhmanov, N Azizova, D Abdullaeva et al.
- [Performance of Control System Using Microcontroller for Sea Water Circulation](#)
A. Indriani, Y. Witanto, A.S. Pratama et al.



ECS
The
Electrochemical
Society
Advancing solid state &
electrochemical science & technology

DISCOVER
how sustainability
intersects with
electrochemistry & solid
state science research

A simulation of the performance of a self-tuning energy harvesting cantilever beam

J L Kaplan, P Bonello and M Alalwan

School of Mechanical, Aerospace and Civil Engineering, University of Manchester,
Sackville Street, Manchester M13 9PL

philip.bonello@manchester.ac.uk

Abstract. A vibration energy harvester is typically a cantilever beam made up of one or two layers of piezoelectric material that is clamped at one end to a vibrating host structure. The harvester is typically tuned to the frequency of the ambient vibration to ensure maximum power generation. One method to ensure that the system stays tuned in the presence of a varying frequency is to attach a mass to the cantilever and apply a control system to adjust its position along the cantilever according to the ambient frequency. This paper presents a simulation of the performance of such a system, based on a distributed parameter electromechanical model of the sliding-mass beam. A variety of control systems are used to adjust the position of the movable mass during operation and are compared for their efficacy in maintaining resonance over a varying excitation frequency. It was found that the resonance frequency of a bimorph cantilever VEH (Vibration Energy Harvester) could be successfully tuned over a wide frequency range. Moreover, it is also found that much of the voltage output reduction at higher frequencies could be compensated for by a separate control system used to adjust the capacitor load.

1. Introduction

Research on harnessing ambient vibrations for electrical energy generation largely focuses on three conversion methods: electrostatic, electromagnetic and piezoelectric [1]. Piezoelectric harvesting has received the most attention due to the large power density of devices based on this principle and their ease of application [2]. As observed in [3], most vibration energy harvesters (VEHs) are designed to be equivalent to spring-mass-damper systems (“linear resonators” [1]). In the case of the piezoelectric vibration energy harvesters (PVEHs), the linear resonator typically takes the form of a base excited cantilever unimorph or bimorph beam [4]. A linear harvester generates maximum power when its resonant frequency matches the frequency of the ambient vibration [1]. This “resonant frequency” is defined in the same way as the “tuned frequency” of the tuned vibration absorber (neutraliser) [5] i.e. the natural frequency of the device with its base (point of attachment to the host structure) blocked. Hence, it shall be referred to as such in the present paper. In the case of the VEH, the electrical output per unit base acceleration is a maximum at the tuned frequency and any mistuning (i.e. difference between excitation and tuned frequencies) will result in a significant reduction of the output. Hence, a practical linear harvester needs a tuning mechanism to increase its functionality [1]. This mechanism can involve either manual tuning or self-tuning. The self-tuning mechanism has been classified in [1] as either “active” or “passive”. Active tuning requires a continuous power input even when the tuning



condition is achieved. Passive tuning requires power only while the system is in the process of being tuned and no power when frequency matching is completed. Since both self-tuning approaches involve some form of active process, Zhu et al [3] classify self-tuning more precisely as either intermittent (formerly “passive”) or continuous (formerly “active”).

The method used for tuning can be either electrical or mechanical [3]. Electrical tuning of a PVEH changes its tuned frequency by adjusting the electrical load connected across it. Zhu et al [3] state that the most feasible electrical tuning approach is to adjust capacitive loads since resistive loads reduce the efficiency of power transfer and load inductances are difficult to adjust. Bonello and Rafique [4] studied a PVEH using a distributed parameter model and showed that its tuned frequency could be increased by around 6.6% as the load impedance increased from short to open circuit conditions. Such a narrow tuning frequency range is typical, making electrical tuning too restrictive for many applications.

A wider tuning range can be achieved by mechanical tuning of the PVEH, which alters its mass and/or stiffness properties [3]. The PVEH reported by Wu et al [6] consisted of a cantilever with a mass attachment comprising a proof mass fitted with a moveable screw. The proof mass was fixed to the cantilever but the centre of mass of the complete mass attachment could be shifted by turning the screw. This approach had a tuning range of 130-180 Hz but was only suitable for manual fine tuning. Self-tuning cantilever PVEH devices have been proposed based on stiffness adjustment. The device by Challa et al [7] enabled intermittent tuning through the application of a controllable negative stiffness in parallel with that of the cantilever. This controllable stiffness was derived from the field of two pairs of magnets, one pair at the free end of the cantilever and the other pair at an adjustable distance above and below the free end. A tuning frequency range of 13-22 Hz was achieved, but the energy consumed in the tuning process necessitated 72-88 minutes of recovery before the next tuning. The device proposed by Roundy and Zhang [8] had two separate piezo-electric sections – one acting as the energy harvester and the other acting as a piezo-actuator that maintained the desired stiffness through a net external *continuous* power input, which was clearly not viable.

As observed by Bonello and Rafique [4], in its tuned condition the VEH is also simultaneously functioning as a tuned vibration absorber (TVA) of the neutraliser variety [5], attenuating the vibration at its base. This duality between the VEH and TVA means that the well-documented concepts used in self-tuning or adaptive TVAs (ATVAs) [5] can be imported into *intermittently* tuned VEH technology.

The primary novel contribution of this paper is the simulation of a self-tuning PVEH cantilever with a sliding mass. This concept is based on the sliding-mass ATVA prototype developed by the co-author in [9], which achieved over 250% variation in the tuned frequency by sliding a mass along a cantilever using a motor-driven screw. The feedback control system tunes the proposed PVEH by positioning the mass along the beam according to the tuning criterion used in ATVA technology, hitherto unused in VEHs, that is based on the phase shift between the vibration signals at two reference points. Moreover, a separate control system is applied to the electrical load to ensure a consistent resonant output as the mass is shifted. Since lumped-parameter (single-degree-of-freedom approximations) of PVEHs may yield highly inaccurate results [10], the simulation shall use the analytical modal analysis method based on the distributed parameter Euler-Bernoulli model of the electrically coupled beam [4, 10]. In view of the sliding mass, the effective modal properties of the present system are time-varying. However, time domain analysis is only feasible if based on invariant modes. Hence, the secondary novel contribution of this paper is the modification of the current modal technique in [4, 10], which is restricted to a beam with a tip mass, to include the sliding mass while retaining the transformation based on invariant modes.

2. Theory

The theory is developed with reference to the system shown in Figure 1. It is assumed that the sliding (moveable) mass M_p is a point mass and that there is no rotation at the clamp. The beam itself is of uniform cross-section and has a mass per unit length of m .

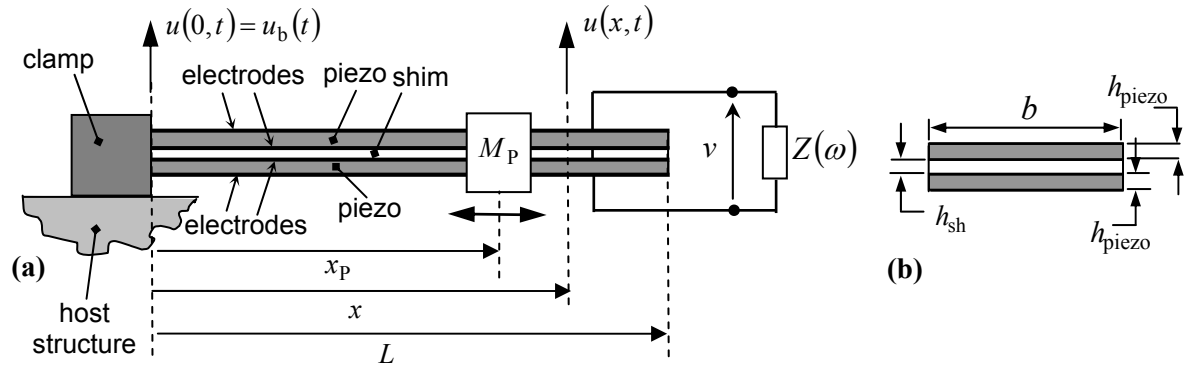


Figure 1. Cantilever harvester with sliding mass: (a) side view; (b) cross-section.

2.1. Modeling the beam with moveable mass

The time-domain analysis for real-time control requires a modal technique based on modes that are independent of the mass position. This can be achieved by considering the mass-beam system as the distributed parameter beam by itself acted upon by a concentrated “external” force at $x_p \in (0, L)$ which is the reaction force exerted by M_p on the beam. The electromechanical Euler-Bernoulli bending wave equation of [4] is then modified by adding the appropriate term on the right hand side:

$$B \frac{\partial^4 u}{\partial x^4} + A \frac{\partial^5 u}{\partial x^4 \partial t} + \vartheta v(t) [\delta'(x) - \delta'(x-L)] + c_a \frac{\partial u}{\partial t} + m \frac{\partial^2 u}{\partial t^2} = -\delta(x-x_p) M_p \ddot{u}(x_p, t), \quad 0 < x_p < L \quad (1)$$

where $(\dot{})$ denotes differentiation with respect to time t , $()'$ denotes differentiation with respect to x , $\delta(x)$ is the Dirac delta function, B is the bending stiffness of the composite section, A is its structural damping equivalent, ϑ is the electrical coupling factor, and c_a the viscous damping coefficient of the surrounding medium (air) per unit length. For the bimorph considered (Figure 1(b)) [4]:

$$\vartheta = -\frac{d_{31} Y_{\text{piezo}} b (h_{\text{piezo}} + h_{\text{sh}})}{2}, \quad B = b \left[\frac{Y_{\text{sh}} h_{\text{sh}}^3}{12} + Y_{\text{piezo}} \left(\frac{h_{\text{piezo}} h_{\text{sh}}^2}{2} + h_{\text{piezo}}^2 h_{\text{sh}} + \frac{2}{3} h_{\text{piezo}}^3 \right) \right] \quad (2, 3)$$

where Y_{piezo} , Y_{sh} are the Young's Moduli of the piezo and shim materials respectively. u is the absolute lateral displacement at location x , which can be expressed in terms of the base displacement u_b and the displacement at location x relative to the base (i.e. the flexural deformation), u_{rel} :

$$u(x, t) = u_b(t) + u_{\text{rel}}(x, t) \quad (4)$$

It is noted that in [4], the term on the right hand side of eq. (1) was not included since the point mass was located at the tip and the modal transformation of u_{rel} in [4] was based on the modes describing undamped free vibration of the *complete system* (clamped-free beam *with tip mass*) in the electrically uncoupled condition (i.e. zero electrical load – short circuit). In the present case, u_{rel} is expressed in terms of the electrically uncoupled undamped clamped-free modes of the beam *by itself* i.e. without the attached point mass:

$$u_{\text{rel}}(x, t) = \sum_{r=1}^{\infty} \phi_r(x) \eta_r(t) \quad (5)$$

where, omitting the tip mass from the mode-shapes in [4]:

$$\phi_r(x) = \sqrt{\frac{1}{mL}} \left[\cosh \frac{\lambda_r}{L} x - \cos \frac{\lambda_r}{L} x - \sigma_r \left(\sinh \frac{\lambda_r}{L} x - \sin \frac{\lambda_r}{L} x \right) \right], \quad \sigma_r = \frac{\sinh \lambda_r - \sin \lambda_r}{\cosh \lambda_r + \cos \lambda_r} \quad (6)$$

$$\omega_r = \left(\frac{\lambda_r}{L}\right)^2 \sqrt{\left(\frac{B}{m}\right)} \quad (7)$$

Equation (1) is transformed into modal space by applying the following steps: (i) substituting (5) into (4); (ii) substituting the resulting expression for u into (1); (iii) multiplying both sides of (1) by $\phi_s(x)$ (s being an integer); (iv) integrating both sides of (1) with respect to x from 0 to L and applying the orthogonality conditions of modes [4]. The transformed equations are then:

$$\{1 + M_P[\phi_r(x_P)]^2\}\ddot{\eta}_r(t) + 2\zeta_r\omega_r\dot{\eta}_r(t) + \omega_r^2\eta_r(t) + \chi_r v(t) = -\ddot{u}_b(t) \left[m \int_0^L \phi_r(x) dx + M_P\phi_r(x_P) \right], r = 1, 2, \dots \quad (8)$$

where the electrical coupling ratio in the r th mode is:

$$\chi_r = \vartheta\phi_r'(L) \quad (9)$$

ζ_r is the viscous damping ratio of the clamped-free beam by itself (i.e. without attached mass), arising from structural and ambient damping. If the latter is neglected [4]:

$$\zeta_1 = \frac{A\omega_1}{2B}, \zeta_r = \zeta_1 \frac{\omega_r}{\omega_1}, r > 1 \quad (10a,b)$$

It is noted that, in the electrically uncoupled condition, the undamped natural frequency (tuned frequency) of the system for a given value of x_P is:

$$\omega_{\text{tuned}}|_{\text{short-circuit}} = \frac{\omega_r}{\sqrt{\{1 + M_P[\phi_r(x_P)]^2\}}} \quad (11)$$

As in previous work e.g. [4, 10] it is assumed that u_{rel} is adequately approximated by one term in the summation of eq. (5) (that corresponding to $r = 1$). The equations of the electromechanical system can then be expressed in state-space form as follows:

$$\frac{d}{dt} \begin{bmatrix} \eta_r \\ \dot{\eta}_r \\ v \end{bmatrix} = \begin{bmatrix} \dot{\eta}_r \\ \frac{-2\zeta_r\omega_r\dot{\eta}_r - \omega_r^2\eta_r - \chi_r v(t) - \ddot{u}_b(t) \left[m \int_0^L \phi_r(x) dx + M_P\phi_r(x_P) \right]}{1 + M_P[\phi_r(x_P)]^2} \\ f(t, \eta_r, \dot{\eta}_r, v) \end{bmatrix} \quad (12)$$

The third equation in (12) governs the electrical voltage output v and has mechanical coupling – its right hand side depends on the type of electrical impedance $Z(\omega)$ (Figure 1(b)). For a purely capacitive load [11]:

$$f(t, \eta_r, \dot{\eta}_r, v) = \left(\frac{1}{\frac{C_{\text{piezo}}}{2} + C} \right) [\sum_{r=1}^{\infty} (\alpha_r \dot{\eta}_r)] \approx \left(\frac{1}{\frac{C_{\text{piezo}}}{2} + C} \right) \alpha_r \dot{\eta}_r \quad (13)$$

where C is the external capacitance and C_{piezo} the capacitance of one layer of piezo [4, 11]:

$$C_{\text{piezo}} = \frac{\epsilon_{33}^S bL}{h_{\text{piezo}}} \quad (14)$$

ϵ_{33}^S being the permittivity at constant strain [4, 11]. The term α_r is defined as [11]:

$$\alpha_r = -Y_{\text{piezo}} d_{31} b z_c \phi_r'(L) \quad (15)$$

where d_{31} is the piezoelectric coefficient and z_c the distance from the mid-surface of one piezo layer to the neutral surface of the composite section. For a purely resistive load R :

$$f(t, \eta_r, \dot{\eta}_r, v) = \frac{2}{C_{\text{piezo}}} \left[\sum_{r=1}^{\infty} (\alpha_r \dot{\eta}_r) - \frac{v}{R} \right] \approx \frac{2}{C_{\text{piezo}}} \left[\alpha_r \dot{\eta}_r - \frac{v}{R} \right] \quad (16)$$

It is noted that, when a control system is applied, x_p in eq. (12) is a real-time varying quantity whose value is determined by the control system used (section 2.3) according to the error signal used for tuning (section 2.2). It is also noted that a separate control system (with separate error signal) can additionally be applied to vary the electrical load in real time (section 2.4) to further enhance the voltage output.

2.2. Tuning criterion

Just as in the ATVA under variable frequency harmonic excitation, the cosine of the phase shift between the absolute acceleration signals at the base A (Figure 1(a)) and any other arbitrary point B on the beam can be used as the error signal $e(t)$ of the feedback control system for tuning the VEH. In the tuned condition, these two signals are approximately in quadrature and therefore the cosine of the phase shift is approximately zero i.e. $|e(t)|$ is minimised. In the sliding mass ATVA of [9], the point B was taken to be on the moving mass. However, it can be observed from the results in [9] (Figure 7(b) in reference [9]) that, when the mass gets very close to the base, the tuning condition becomes badly conditioned (since A and B are then almost coincident). Hence, in the present work, the point B was fixed at the tip. The tuning error signal $e(t)$ was then continuously evaluated from $\ddot{u}_b$ and $\ddot{u}_{\text{tip}} = \ddot{u}(L, t)$ by integrating their normalised product over a sliding time interval of fixed short duration T_c , according to the following formula [5, 9]:

$$e(t) = \begin{cases} \frac{I_{AB}(t)}{[I_{AA}(t)]^{0.5}[I_{BB}(t)]^{0.5}} & (t \leq T_c) \\ \frac{I_{AB}(t) - I_{AB}(t - T_c)}{[I_{AA}(t) - I_{AA}(t - T_c)]^{0.5}[I_{BB}(t) - I_{BB}(t - T_c)]^{0.5}} & (t > T_c) \end{cases} \quad (17)$$

$$I_{AA}(t) = \int_0^t \ddot{u}_b(\tau) \ddot{u}_b(\tau) d\tau, \quad I_{AB}(t) = \int_0^t \ddot{u}_b(\tau) \ddot{u}_{\text{tip}}(\tau) d\tau, \quad I_{BB}(t) = \int_0^t \ddot{u}_{\text{tip}}(\tau) \ddot{u}_{\text{tip}}(\tau) d\tau \quad (18a-c)$$

where τ is a dummy time variable for integration between the limits of 0 and t .

2.3. Tuning via real time control of mass position

The feedback control system for the mass position, in order to maintain the tuned condition, comprises the controller and the actuator. The input to the controller is $e(t)$ and its output is $n(t)$, the nominal (desired) velocity of the mass driven by the actuator, which is given by:

$$n(t) = F(e(t)) \quad (19)$$

where F depends on the type of controller used (section 2.3.1). The actuator used in this study is a simulated generic device. Its input is the signal $n(t)$ and its output is the mass position $x_p(t)$, determined as follows:

$$x_p(t) = \int_0^t \dot{x}_p(\tau) d\tau + x_p(0) \quad (20)$$

The velocity of the mass $\dot{x}_p$ in (20) is determined from $n(t)$ and the current value of $x_p(t)$ as follows:

$$\dot{x}_p(t) = \begin{cases} n(t) & (0 < x_p < L) \text{ \& } (|n(t)| \leq |\dot{x}_p|_{\text{lim}}) \\ +|\dot{x}_p|_{\text{lim}} & (0 < x_p < L) \text{ \& } (n(t) > |\dot{x}_p|_{\text{lim}}) \\ -|\dot{x}_p|_{\text{lim}} & (0 < x_p < L) \text{ \& } (n(t) < -|\dot{x}_p|_{\text{lim}}) \\ 0 & (x_p = L) \text{ or } (x_p = 0) \end{cases} \quad (21)$$

From eq. (21) it is seen that $\dot{x}_p$ is capped at $\pm|\dot{x}_p|_{\text{lim}}$ and reset to 0 if the mass is at the extremities of the beam. In all control systems apart from the bang-bang controller (which had a prescribed fixed speed), $|\dot{x}_p|_{\text{lim}}$ was set to 0.05L m/s (i.e. 5% of the beam length in one second).

2.3.1. Controllers. Four types of controllers were investigated: (i) Bang-Bang (on/off); (ii) Proportional-Integral-Derivative (PID); (iii) Polynomial; (iv) Fuzzy logic [12]. The Bang-Bang controller provides the most basic form of control, where:

$$n(t) = |\dot{x}_p|_{\text{bang-bang}}(e(t) > 0), n(t) = -|\dot{x}_p|_{\text{bang-bang}}(e(t) < 0), n(t) = 0 (e(t) = 0) \quad (22)$$

since, $e(t) > 0$ implies that $\omega_{\text{tuned}} > \omega$, so the device needs to be tuned down by moving the mass outwards (and vice-versa). The fixed speed used in the bang-bang controller, $|\dot{x}_p|_{\text{bang-bang}}$, was set to 0.006L m/s. In the case of PID control:

$$n(t) = k_p e(t) + k_i \left\{ \begin{array}{ll} \int_0^t e(\tau) d\tau & (t \leq T_i) \\ \int_{t-T_i}^t e(\tau) d\tau & (t > T_i) \end{array} \right\} + k_d \dot{e}(t) \quad (23)$$

where k_p , k_i and k_d are constants and T_i a sliding time interval of fixed short duration. ‘‘P control’’ refers to specific case where $k_i = k_d = 0$ and ‘‘PI control’’ refers to the specific case where $k_d = 0$. The PID is a linear controller. However, as observed in the ATVA works e.g. [9], the difference between the forcing frequency and the tuned frequency, $\omega_{\text{tuned}} - \omega$ is non-linearly related to $e(t)$ and therefore, non-linear controllers (e.g. based on a polynomial relationship or fuzzy logic) can be more appropriate to minimize $|e(t)|$. In the case of polynomial control:

$$n(t) = k_1 e(t) + k_2 [e(t)]^3 + k_3 [e(t)]^5 \quad (24)$$

where k_1 , k_2 and k_3 are constants. In the case of fuzzy logic, the input $e(t)$ is fuzzified i.e. converted to input membership functions that are weighted according to $e(t)$ and the design of the specific fuzzy logic system used. Logical rules are defined to map these weighted inputs to weighted output membership functions, which are then defuzzified to provide the discrete output $n(t)$. The fuzzy logic system used in this work is the Mamdani system [12].

2.4. Real time control of electrical load for fixed peak voltage output at all mass positions

As will be shown in the subsequent sections, the peak voltage output in the tuned condition ($\omega_{\text{tuned}} = \omega$) decreases as the mass is moved towards the base (i.e. as ω is increased with time). However, by separately adjusting the electrical load in real time, it is possible to maintain a reasonably constant peak voltage as the mass is shifted. The control system for the electrical load aims to keep the integral of the square of the instantaneous voltage v over a sliding time interval of fixed short duration T_v as close as possible to a set fixed value ψ . The voltage control error signal to be minimized is therefore:

$$e_v(t) = \begin{cases} \int_0^t \{v(\tau)\}^2 d\tau - \psi & (t \leq T_v) \\ \int_{t-T_v}^t \{v(\tau)\}^2 d\tau - \psi & (t > T_v) \end{cases} \quad (25)$$

In this work, the control system is applied to a capacitive load C . The separate feedback control system comprises a controller and actuator and follows a similar approach to that already presented in section 2.3, where e_v replaces e and C replaces x_p (limits may be similarly applied to C and $\dot{C}$).

3. Simulations and discussion of results

With the exception of section 3.1, the results in this paper refer to the bimorph used by Dalzell and Bonello [11], to which, in the present study, a point mass $M_p = 0.5m_{\text{beam}}$ is attached, where m_{beam} is the mass of the beam on its own (see top row of Table 1).

In the case of section 3.1, the parameters of the system were taken from the work by Erturk and Inman [10] who considered a bimorph with a tip mass (see bottom row of Table 1). It is noted that, in the present analysis, ζ_1 is the damping ratio of the beam without any attached mass. Hence, the second entry for ζ_1 in Table 1 is not the one used in [10] (which was 2.7%), but a value that was deduced from the data in [10] using eqs. (10a,b) as follows:

$$\frac{\zeta_1}{\omega_1} = \frac{\zeta_1|_{\text{tip mass}}}{\omega_1|_{\text{tip mass}}} = \frac{A}{2B} \Rightarrow \zeta_1 = \omega_1 \times \frac{\zeta_1|_{\text{tip mass}}}{\omega_1|_{\text{tip mass}}} \quad (26)$$

where $\zeta_1|_{\text{tip mass}}$ is the damping ratio quoted in [10] (2.7%) and $\omega_1|_{\text{tip mass}}$ the associated undamped natural frequency (measured at 45.6 Hz in [10]).

Table 1. Parameters used in the simulations.

	Y_{piezo} GPa	Y_{sh} GPa	h_{piezo} mm	h_{sh} mm	ρ_{piezo} kg/m ³	ρ_{sh} kg/m ³	L mm	b mm	d_{31} pm/V	ϵ_{33}^s nF/m	ζ_1	$\frac{M_p}{m_{\text{beam}}}$
section 3.2 onward	66	72	0.267	0.300	7800	2700	58	25	-190	13.555	0.79%	0.5
section 3.1 only	66	105	0.260	0.140	7800	9000	50.8	31.8	-190	13.281	7.02%	1.3974

The mathematical model of the previous section was implemented in *Simulink*® which used a solver (integrator) with variable time-step size to maintain the error within a prescribed tolerance.

3.1. Validation of mass-beam model

The moveable mass beam model was validated by considering the case where $x_p = L$ and the system parameters in the bottom row of Table 1, which correspond to the experimental set-up of [10]. It is noted that, as $x_p \rightarrow 0$, the mass-beam system converges to a beam without attached mass (this is evident from eq. (11) since $\phi_r(0) = 0$). Hence, the assumption made in eq. (5) (i.e. use of the modes of the beam without attached mass to represent the vibrating shape of the complete system) is least accurate when the mass M_p is at the tip of the beam. This fact, in addition to the substantial value of the ratio M_p/m_{beam} ($= 1.3974$), makes this case a reliable validation test. The first row of Figure 2 shows the magnitude of the voltage frequency response functions (FRFs – the voltage output per harmonic base acceleration of amplitude 9.81 m/s²), measured in [10] at three different resistor loads. Overlaid on the same axes in the first row of Figure 2 are the predictions from the *specific* tip-mass model developed in [10]. The second row of Figure 2 shows the corresponding voltage FRFs obtained from the present model. These FRFs were constructed from the amplitudes of the steady-state voltage responses obtained by integrating eqs. (12) for harmonic inputs $\ddot{u}_b(t)$ of discrete fixed frequencies covering the frequency range. It is seen that the resonance frequency predicted by the present mass-beam model does not deviate by more than 1.75% from the experimental measurements. Moreover, the levels of the voltage output are also reasonably close. As reasoned above, the accuracy of the model is expected to be higher for cases where $x_p < L$ and/or the ratio M_p/m_{beam} is lower. Having thus validated the model, the simulation results in the remainder of this paper shall refer to linear chirp excitation $\ddot{u}_b(t)$ of the system with the parameters in the top row of Table 1.

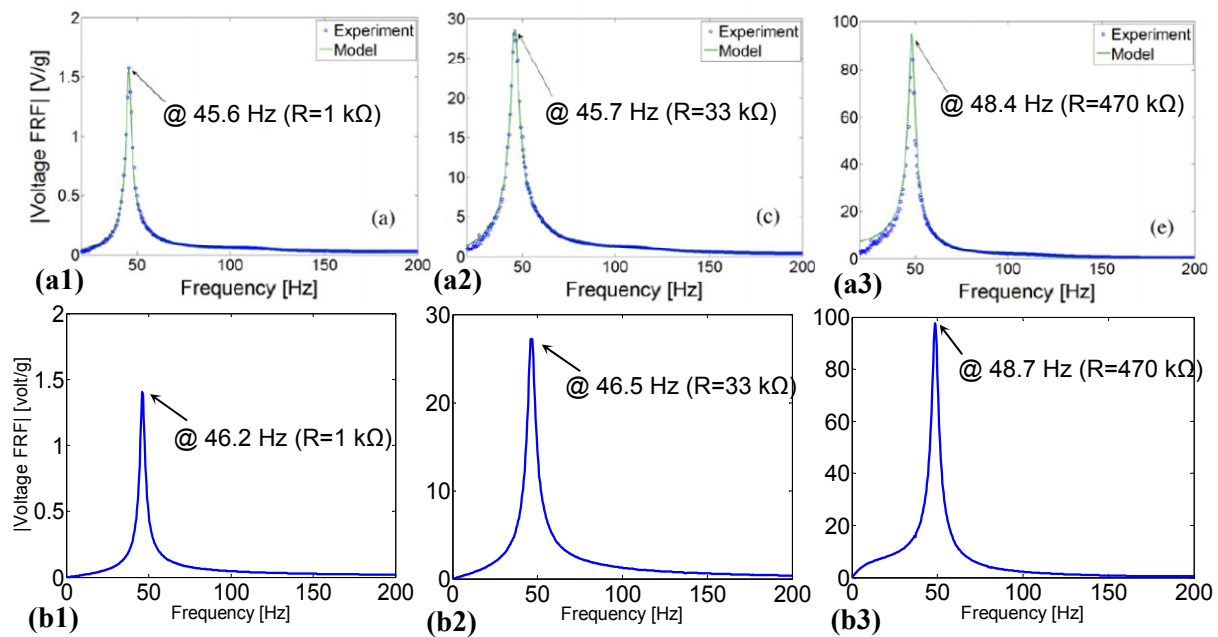


Figure 2. Comparison of experimental and predicted voltage FRFs for beam with tip mass: (a1-a3) measured/predicted according to [10] (© IOP Publishing. Reproduced with permission. All rights reserved); (b1-b3) predicted according to the moveable mass model.

3.2. Validation of tuning criterion

In all simulations in this paper, the duration of the sliding interval T_c for computation of the tuning criterion error $e(t)$ (eq. (12)) was $T_c = 100\Delta$ where Δ is the maximum step size applied by the integrator. The chirp base excitation was applied with the mass M_p at a fixed position and the beam shunted across a fixed capacitance load of $1.2 \mu\text{F}$. The excitation frequency was swept from 70 Hz to 140 Hz in 400 s, covering the tuneable frequency range of the harvester (77.1 Hz to 133.5 Hz).

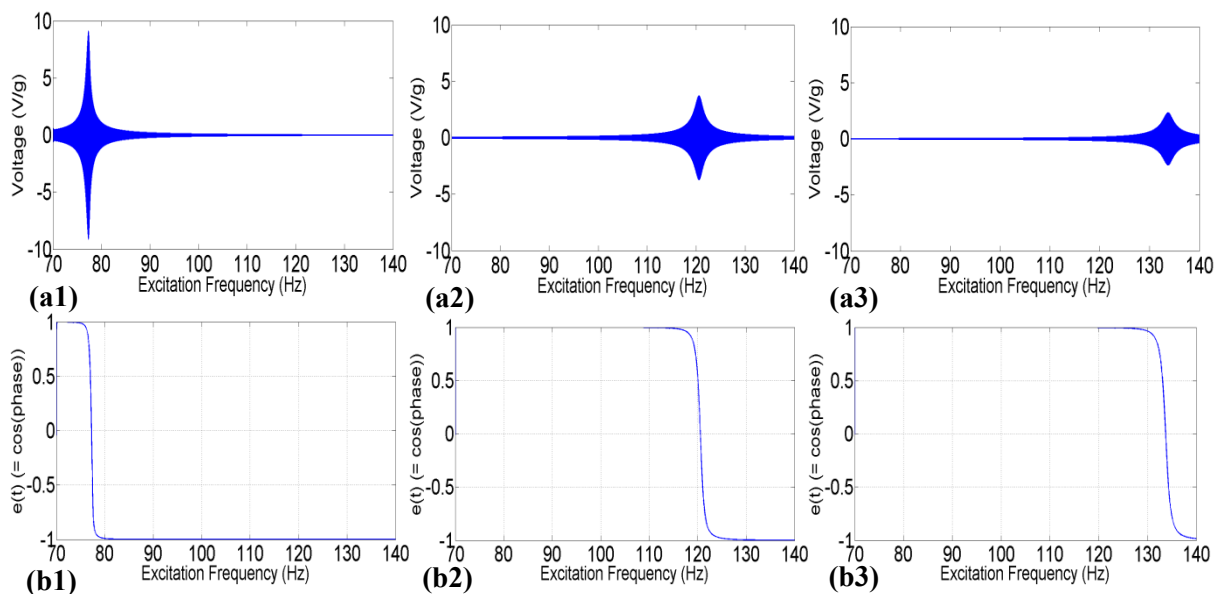


Figure 3. Voltage output and error signal response to linear chirp base excitation for three different fixed positions of M_p : $x_p = L$ (a1, b1); $x_p = L/2$ (a2, b2); $x_p = 0$ (a3, b3).

Figure 3 shows the variation of the voltage output and tuning error signal $e(t)$ with the drifting excitation frequency for the mass fixed at three different positions: $x_p = L$ (a1, b1), $x_p = L/2$ (a2,

b2), $x_p = 0$ (a3, b3). In all three cases, the voltage output reached a maximum when the excitation frequency passed through the electrically coupled resonance and, at this instant, the error signal was approximately zero. The latter observation validates the criterion to be used for tuning the harvester. Another important observation is the significant decline in the resonant voltage as the mass was shifted closer to the base. In the limit ($x_p \rightarrow 0$), the voltage output per unit base acceleration converges to that of the beam without the attached mass.

3.3. Simulation of tuning via real time control of mass position

Figure 4 shows the simulated response to linear chirp base excitation when the mass is repositioned in real time using PID control for a fixed capacitance load of $1.2 \mu\text{F}$. The control parameters are: $k_p = 0.75L \times 10^{-1}$, $k_i = 0.75L \times 10^{-4}$, $T_i = 10^4 T_c$, $k_d = 0.75L \times 10^{-3}$. The excitation frequency is swept from 70 Hz to 140 Hz in 400 s. The mass is initially at the tip and starts to move inward when the excitation drifts into the tuning range (~ 77 Hz to ~ 133 Hz). Beyond 133 Hz, the mass stays fixed at the base. While the excitation frequency is within the tuning range, the control system is seen to satisfactorily maintain the tuned condition, as evident from the error signal and the voltage output. Figure 4(d) shows that the mass velocity $\dot{x}_p$, although volatile, is oscillating about a value slightly less than 0, which is to be expected given that the general motion of the mass is towards the base.

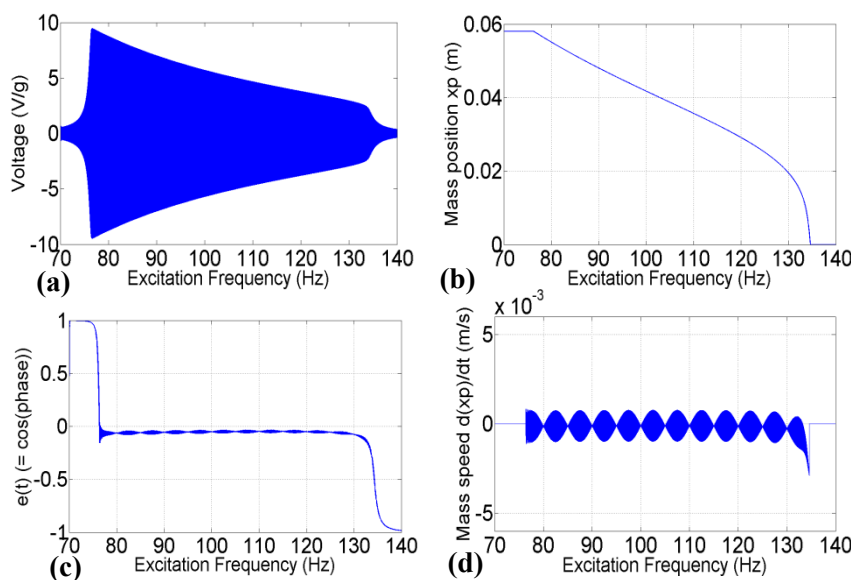


Figure 4. Simulated response to linear chirp base excitation with PID control of mass position and fixed electrical load: (a) voltage output; (b) mass position; (c) tuning error signal; (d) mass velocity.

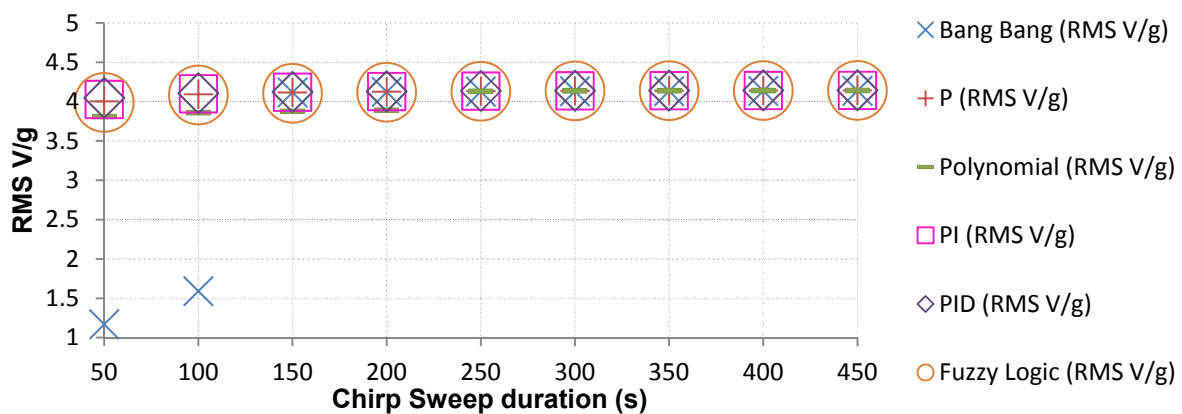


Figure 5. Comparison of the rms voltage output across a fixed electrical load using alternative mass position control systems for different chirp sweep durations.

Figure 5 compares the voltage output across the fixed capacitor load of 1.2 μF obtained using alternative control systems for different chirp sweep durations ranging from 50 s to 450 s, the frequency being swept from 77 Hz to 133 Hz in all cases. The performances of the different controllers are seen to be not significantly different, except for very poor performance of the bang-bang controller at the two shortest chirp lengths. This can be explained by the fact that the mass speed $|\dot{x}_P|_{\text{bang-bang}}$ (eq. (22)) was the same for all chirp durations and evidently too low for the two shortest chirp lengths. The visibly inferior performance of the polynomial controller at the four lowest chirp durations is somewhat surprising. However, it should be noted that the controller constants were prescribed on a trial and error basis rather than using a formal optimisation procedure. It should also be noted that the vertical axis of Figure 5 shows root-mean-square (rms) voltage, in view of the gradual reduction in the resonant voltage as the mass moves towards the base (see Figure 4(a)).

3.4. Simulation of real time control of electrical load for fixed peak voltage at all mass positions

By enabling real time control of the electrical load, in parallel with the tuning process, it is possible to obtain an approximately constant peak resonant voltage at all mass positions. This is illustrated in Figure 6, where the excitation is swept from 75 Hz to 135 Hz over 450 s. The control parameters are as follows.

- Mass movement control: $k_p = 0.75L \times 10^{-1}$, $k_i = 0.75L \times 10^{-1}$, $T_i = 10T_c$, $k_d = 0.75L \times 10^{-3}$.
- Capacitance load control: $k_p = 12$, $k_i = 6$, $T_i = 10T_c$, $k_d = 0$, $T_v = 0.007$ and $\psi = 0.24$.

It is noted that for the mass movement control, relative to the case in section 3.3, k_i was increased by a factor of 10^3 and T_i reduced by the same factor – this adjustment was found to produce the best results. The initial value of the capacitance load C was 3 μF . By comparing Figure 6(a) to Figure 4(a), it is seen that a consistent output voltage is maintained over the tuning range. A comparison of Figure 6(b, c) with Figure 4(b, c) shows that the adjustment of the capacitor load (Figure 6(d)) has not detrimentally interfered with the separate mass movement / resonance tuning process. Figure 7 shows the standard deviation of the voltage peaks as the frequency is swept from 77 Hz to 133 Hz at different uniform rates (according to the chirp duration) using P or PI control for the electrical load (capacitance) and PID control for the mass position. It is seen that the standard deviation in voltage peaks ranges from less than 0.2 V (i.e. $\sim 2.5\%$ of the average voltage peak of 8.1 V) at the slowest sweep rate, to no more than 0.55 V (i.e. $\sim 6.8\%$ of the average peak) at the fastest sweep rate.

4. Conclusions

This paper has simulated the performance of a self-tuning sliding-mass piezoelectric vibration energy harvesting (PVEH) cantilever device, based on the concept of the moveable-mass adaptive tuned vibration absorber (ATVA) previously developed by the co-author. The modal transformation of the distributed parameter beam model was developed to enable time-domain analysis for arbitrary (time varying) position of the attached mass. The modified model was successfully validated for a specific mass position (the beam tip) against experimental results from the literature. Simulations for the dynamic response to chirp excitation of the base showed that the mass position can be controlled in real-time, using the same criterion used to tune ATVAs, to ensure that the resonant voltage output condition is maintained over a wide frequency range. Moreover, a separate control system can be applied simultaneously to the external load to ensure a consistent voltage output as the mass is repositioned. Current efforts are directed at building and testing a prototype sliding-mass PVEH.

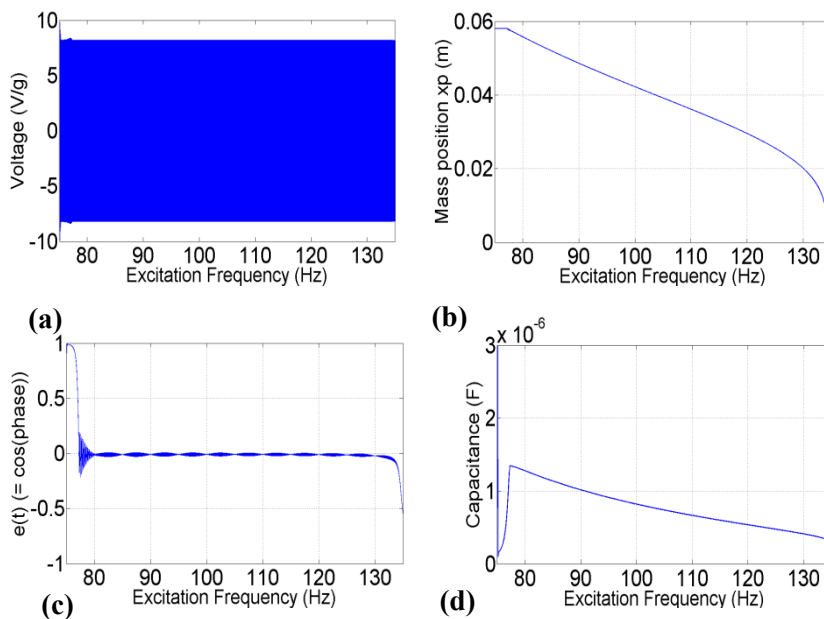


Figure 6. Simulated response to linear chirp base excitation with PID control of mass position and PI control of the electrical load: (a) voltage output; (b) mass position; (c) error signal; (d) mass speed.

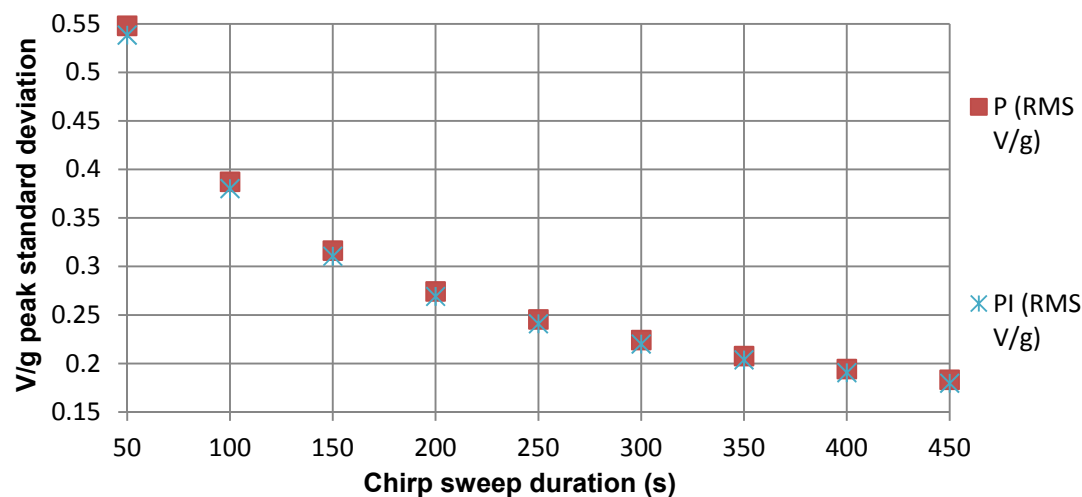


Figure 7. Variation of the standard deviation of the voltage peaks with different chirp sweep durations sweep using P/PI control for the electrical load and PID control for the mass position.

5. References

- [1] Tang L, Yang Y and Soh C K 2013 Broadband Vibration Energy Harvesting Techniques *Advances in Energy Harvesting Methods* ed N Elvin and A Erturk (New York: Springer Science) chapter 2 pp 17-61
- [2] Elvin N and Erturk A 2013 *Advances in Energy Harvesting Methods* ed N Elvin and A Erturk (New York: Springer Science) preface p vi
- [3] Zhu D, Tudor M J and Beeby S P 2010 *Meas. Sci. Technol.* **18** 022001
- [4] Bonello P and Rafique S *Trans ASME, J. Vib. Acoust.* **133**(1) 011009-011009-11
- [5] Bonello P 2011 Adaptive Tuned Vibration Absorbers: Design Principles, Concepts and Physical Implementation *Vibration Analysis and Control - New Trends and Developments* ed F Beltran-Carbajal (Rijeka: InTech) chapter 1 pp 1-26
- [6] Wu X, Lin J, Kato S, Zhang K, Ren T and Liu L 2008 A frequency adjustable vibration energy harvester *Proc. PowerMEMS 2008+ microEMS2008 (9-12 November 2008, Sendai, Japan)* pp 245-8
- [7] Challa V R, Prasad M G, Fisher F T 2011 *Smart Mater. Struct.* **20** 025004

- [8] Roundy S and Zhang Y 2005 Toward self-tuning adaptive vibration based micro-generators *Proc SPIE* **5649** pp. 373–384
- [9] Bonello P and Groves K 2009 *Proc. IMechE Part C: J. Mechanical Engineering Science* **223** pp. 1555-1567
- [10] Erturk A and Inman, D J 2009 *Smart Mater. Struct.* **18** 025009
- [11] Dalzell P and Bonello P 2012 *Smart Mater. and Struct.* **21**(10) 105029
- [12] Fuzzy Logic Toolbox™ MathWorks® <http://uk.mathworks.com/help/fuzzy/index.html>