

PAPER • OPEN ACCESS

On The Dynamics and Design of a Two-body Wave Energy Converter

To cite this article: Changwei Liang and Lei Zuo 2016 *J. Phys.: Conf. Ser.* **744** 012074

View the [article online](#) for updates and enhancements.

You may also like

- [Isotope effects on energy, particle transport and turbulence in electron cyclotron resonant heating plasma of the Large Helical Device](#)
K. Tanaka, Y. Ohtani, M. Nakata et al.
- [Functionality-switchable terahertz metamaterial with perfect absorption and circular dichroism](#)
Shuang Liang, Zebin Zhu and Liyong Jiang
- [Hybrid model of radio-frequency low-pressure inductively coupled plasma discharge with self-consistent electron energy distribution and 2D electric field distribution](#)
Wei Yang and You-Nian Wang



ECS
The
Electrochemical
Society
Advancing solid state &
electrochemical science & technology

DISCOVER
how sustainability
intersects with
electrochemistry & solid
state science research

On The Dynamics and Design of a Two-body Wave Energy Converter

Changwei Liang¹, and Lei Zuo²

¹ Department of Mechanical Engineering, Stony Brook University, Stony Brook, NY 11794, USA

² Department of Mechanical Engineering, Virginia Tech, Blacksburg, VA, 24061, USA

Email: leizuo@vt.edu

Abstract. A two-body wave energy converter oscillating in heave is studied in this paper. The energy is extracted through the relative motion between the floating and submerged bodies. A linearized model in the frequency domain is adopted to study the dynamics of such a two-body system with consideration of both the viscous damping and the hydrodynamic damping. The closed form solution of the maximum absorption power and corresponding power take-off parameters are obtained. The suboptimal and optimal designs for a two-body system are proposed based on the closed form solution. The physical insight of the optimal design is to have one of the damped natural frequencies of the two body system the same as, or as close as possible to, the excitation frequency. A case study is conducted to investigate the influence of the submerged body on the absorption power of a two-body system subjected to suboptimal and optimal design under regular and irregular wave excitations. It is found that the absorption power of the two-body system can be significantly higher than that of the single body system with the same floating buoy in both regular and irregular waves. In regular waves, it is found that the mass of the submerged body should be designed with an optimal value in order to achieve the maximum absorption power for the given floating buoy. The viscous damping on the submerged body should be as small as possible for a given mass in both regular and irregular waves.

1. Introduction.

The utilization of wave energy has been actively explored for more than 200 years since the first patent was applied in Paris in 1799. Hundreds of wave energy converters have been developed in the past decades [1-11]. Among all the technologies that appeared in the past decades, point absorber is one of the most popular designs and has been considered one of the most promising and cost effective devices. The first generation of point absorber is a single body system oscillating in heave. Two characteristic devices of a single body wave energy converter are Lysekil [12] and CETO-6 [13].

The early theoretical study of single body wave energy converters by Budal [14], Evans [15] and Mei [16] show that the maximum power of an axisymmetric point absorber oscillating in the heave under regular wave excitation is $P_{max} = \frac{\rho g^3 A^2}{4\omega^3}$, where ρ is water density; g is standard gravity; A is wave amplitude and ω is wave frequency. In order to achieve this maximum absorption power, the natural frequency of the oscillating body needs to match with incident wave frequency. In real application, however, this condition is usually hard to realize since the typical incident wave frequency is usually very low (0.1Hz-0.2Hz). As a consequence, the dimension of the floating buoy needs to be impractically large in order to match its natural frequency with the incident wave frequency. Based on Falcao's calculation [17], the diameter of the submerged hemisphere needs to be 52.4m in order to match an incident wave frequency of 0.1Hz, too large to be practical. Therefore, people have developed different methods to approximate this frequency matching condition. In [18], an additional mass is attached under the floating buoy to decrease the natural frequency. Suboptimal control methods such as latching [19] and declutch control [20] were also developed based on this frequency match condition.

Another way to develop a resonant wave energy converter is to design an additional body under the floating buoy to create a two-body wave energy converter, where the energy is extracted through the



relative motion of the two bodies. There are two advantages to add an additional submerged body under the floating buoy. Firstly, the second body can act as a reaction body, which makes mooring easier in deep water, compared with directly connecting the floating buoy to the seabed. Secondly, the dynamics of second body can be utilized to improve the performance of the overall system. Two typical designs of two-body wave energy converters are the Powerbuoy (Figure 1 right), which was developed by Ocean Power Technology in the USA [21], and Wavebob (Figure 1 left), which was developed in Ireland [22]. Both devices use the relative motion between the floating buoy and submerged body to extract energy from ocean waves. However, it is found in Figure 1 that the design philosophy of the submerged body is quite different between these two devices. Wavebob used a second body with a streamline configuration and large mass while Powerbuoy used a resistive heave plate.

Early studies of two-body wave energy converters mainly focused on regular wave excitation. Falnes [23, 24] investigated the concept of using relative motion between a floating buoy and a submerged body to harvest energy. He found that it is possible to achieve the optimum power $P_{max} = \frac{\rho g^3 A^2}{4\omega^3}$ for a two-body wave energy converter oscillating in heave in regular waves. Korde [25] explored the concept of two-body wave energy converter and compared the performance of a submerged reaction mass with a reaction mass out of water under regular wave excitation. He found that the submerged mass appears to perform better overall. Bijun Wu et al. [26] also studied the response and conversion efficiency of a two-body wave energy converter under regular wave excitation and they found that the performance of a two-body wave energy converter highly depends on the system parameters, like the physical properties of the buoy and incident wave frequency. The results obtained by the previous literature helped people to understand that it is possible to design a submerged body to achieve the theoretical maximum absorption power under regular wave excitation by neglecting the viscous damping. In real application, however, it is necessary to investigate the design of a two-body wave energy converter under irregular wave excitation as well.

Recently, an experiment was conducted by Beatty et al. [27] to compare the hydrodynamic performance of the submerged body for Wavebob and Powerbuoy. The authors found that a streamline submerged body has a smaller added mass compared with a resistive heave plate. The experimental results of added mass and excitation force on the submerged body agreed fairly well with the results obtained using boundary element method (BEM) for both prototypes. However, the damping on the second body is much larger in comparison with radiation damping obtained with BEM code. Therefore, it is necessary to consider the viscous effect while studying the two-body wave energy converter, especially for the submerged body. Based on Beatty's experimental study, in this paper, linearized damping is considered to account for the viscous effect. In the case study, the viscous effect on the floating buoy was neglected [27].

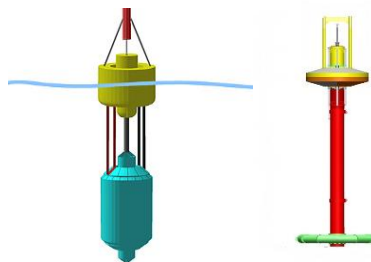


Figure 1. Model of Wavebob (left) and Powerbuoy (right)

Based on the aforementioned literature review, the first objective of this paper is to study the dynamics of the two-body wave energy converter, including the viscous effect. In addition to radiation damping, a linearized damping due to viscous effect was considered to build the frequency domain model. The closed form solution of the absorption power for a two-body wave energy converter was obtained in terms of system parameters and wave information under regular wave excitation. The suboptimal and optimal power take-off designs are defined based on the obtained closed form solutions.

The results presented in this paper are further used to study the influence of submerged body on the absorption power of the two-body system.

The second objective of this paper is to study the design of the submerged body on the overall performance of a two-body wave energy converter under regular and irregular wave excitations. When the submerged body is deployed far enough from the floating buoy and free surface, the hydrodynamic interaction between these two bodies, as well as the radiation damping on the submerged body, can be neglected. Numerical simulations were conducted to study the influence of the submerged body on the power take-off design and absorption power under regular wave excitation. The suboptimal and optimal designs are modified and investigated in irregular waves. The absorption power based on these two design principles are also compared with the maximum absorption power of a single body system with the same floating buoy. It is found that if the submerged body is designed properly, the absorption power of a two-body wave energy converter can be significantly higher than that of a single buoy system with the same floating buoy under regular and irregular wave excitation, even in the presence of a large viscous damping on the submerged body.

The remaining of this paper is organized as follows: In Section 2, the linearized model of a two-body wave energy converter under regular wave excitation is established and the closed form solution of absorption power and corresponding power take-off parameters are obtained. In Section 3, suboptimal and optimum power take-off design and the corresponding absorption power are studied in regular waves. In Section 4, the suboptimal and optimal design are implemented in irregular waves and compared with the single body system with the same floating buoy. The constrained simplex method is used to find the maximum absorption power of a two-body system under irregular wave excitation. Conclusions are given in Section 5.

2. Dynamics Analysis and Design of Two-body Wave Energy Converter in Regular Waves

Figure 2 is the schematic of the two-body wave energy converter considered in this paper. A floating buoy is located on the water surface and is connected to a submerged body through a power take-off (PTO) system. The submerged body is suspended in the water. The energy is extracted through the relative motion of the floating and submerged bodies. One should notice that the floating and submerged bodies considered in this paper are axisymmetric but they do not need to be cylindrical as shown in Figure 2.

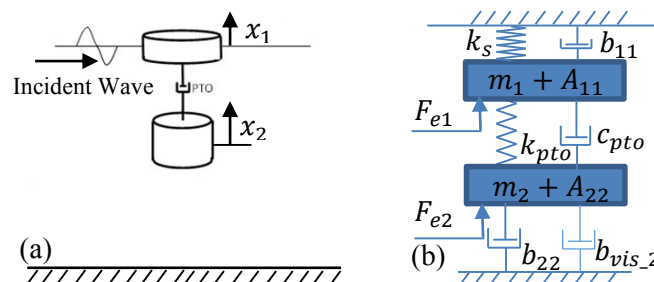


Figure 2. (a) Schematic of two-body wave energy converter; (b) Vibration model of a two-body wave energy converter

The power take-off system which is installed between the floating buoy and submerged body is assumed to be linear as in most literature, and the take-off force is:

$$F_{PTO} = -c_{pto}(\dot{x}_1 - \dot{x}_2) - k_{pto}(x_1 - x_2) \quad (1)$$

One should notice that the submerged body is assumed to be suspended in the water. Therefore, we assume $k_{pto} \geq 0$ since negative spring stiffness makes the system unstable. The equation of motion of a two-body wave energy converter shown in Figure 2 is:

$$m_1\ddot{x}_1 + A_{11}\dot{x}_1 + A_{12}\ddot{x}_2 + b_{11}\dot{x}_1 + b_{12}\dot{x}_2 + b_{vis_1}\dot{x}_1 + c_{pto}(\dot{x}_1 - \dot{x}_2) + k_{pto}(x_1 - x_2) + k_s x_1 = F_{e1} \quad (2)$$

$$m_2\ddot{x}_2 + A_{21}\dot{x}_1 + A_{22}\ddot{x}_2 + b_{21}\dot{x}_1 + b_{22}\dot{x}_2 + b_{vis_2}\dot{x}_2 + c_{pto}(\dot{x}_2 - \dot{x}_1) + k_{pto}(x_2 - x_1) = F_{e2} \quad (3)$$

where, x_1 and x_2 are the respective heave displacements of the floating and submerged bodies; m_1 and m_2 are the respective masses of the floating and submerged bodies; A_{ij} ($i, j = 1, 2$) is the frequency dependent added mass which accounts for the mass of the water moving with the i -th body, induced by the motion of j -th body (1 and 2 represent the floating and submerged bodies, respectively); b_{ij} ($i, j = 1, 2$) is the frequency dependent radiation damping coefficient, which accounts for the energy dissipation by the radiated wave due to the motion of the i -th body, induced by the motion of j -th body; b_{vis_i} ($i, j = 1, 2$) is the linearized viscous damping coefficients on the floating and submerged bodies; k_{pto} and c_{pto} are the stiffness and damping coefficient of the power take-off system; k_s is the hydrostatic stiffness of the floating buoy, which is the spring effect due to the difference of gravity and buoyancy; F_{e1} and F_{e2} are the exciting forces on the floating and submerged bodies, induced by the incident wave.

Under regular wave (sinusoidal) excitation, the exciting force can be expressed as a harmonic function with the following equation:

$$F_{e1} = F_1 e^{i\omega t}, \quad F_{e2} = F_2 e^{i\omega t} \quad (4)$$

The solutions of Eqs. (2) and (3) can be expressed as,

$$x_1 = X_1 e^{i\omega t}, \quad x_2 = X_2 e^{i\omega t} \quad (5)$$

Substitute Eqs. (4) and (5) into Eqs. (2) and (3), respectively.

$$-\omega^2(m_1 + A_{11})X_1 - \omega^2 A_{12}X_2 + i\omega b_{11}X_1 + i\omega b_{12}X_2 + i\omega b_{vis_1}X_1 + i\omega c_{pto}(X_1 - X_2) + k_{pto}(X_1 - X_2) + k_s X_1 = F_1 \quad (6)$$

$$-\omega^2(m_2 + A_{22})X_2 - \omega^2 A_{21}X_1 + i\omega b_{21}X_1 + i\omega b_{22}X_2 + i\omega b_{vis_2}X_2 + i\omega c_{pto}(X_2 - X_1) + k_{pto}(X_2 - X_1) = F_2 \quad (7)$$

Denote the matrices

$$M = \begin{bmatrix} (m_1 + A_{11}) & A_{12} \\ A_{21} & (m_2 + A_{22}) \end{bmatrix}, \quad C = \begin{bmatrix} (b_{11} + b_{vis_1} + c_{pto}) & (b_{12} - c_{pto}) \\ (b_{21} - c_{pto}) & (b_{22} + b_{vis_2} + c_{pto}) \end{bmatrix}$$

$$K = \begin{bmatrix} k_s + k_{pto} & -k_{pto} \\ -k_{pto} & k_{pto} \end{bmatrix}, \quad X = [X_1, X_2]^T, \quad F = [F_1, F_2]^T$$

Eqs. (6) and (7) can be rewritten as:

$$(-\omega^2 M + i\omega C + K)X = F \quad (8)$$

Define

$$Z(i\omega) = -\omega^2 M + i\omega C + K = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \quad (9)$$

where $Z(i\omega)$ is the impedance matrix. The elements in $Z(i\omega)$ can be written as the following equations:

$$\begin{cases} Z_{11} = -\omega^2(m_1 + A_{11}) + i\omega(b_{11} + b_{vis_1} + c_{pto}) + k_s + k_{pto} \\ Z_{12} = -\omega^2 A_{12} + i\omega(b_{12} - c_{pto}) - k_{pto} \\ Z_{21} = -\omega^2 A_{21} + i\omega(b_{21} - c_{pto}) - k_{pto} \\ Z_{22} = -\omega^2(m_2 + A_{22}) + i\omega(b_{22} + b_{vis_2} + c_{pto}) + k_{pto} \end{cases} \quad (10)$$

The solution of Eq. (8) can be expressed as,

$$X = Z(i\omega)^{-1}F \quad (11)$$

where,

$$\begin{cases} Z(i\omega)^{-1} = \frac{1}{\det(Z(i\omega))} \begin{bmatrix} Z_{22} & -Z_{21} \\ -Z_{12} & Z_{11} \end{bmatrix} \\ \det(Z(i\omega)) = Z_{11}Z_{22} - Z_{12}Z_{21} \end{cases} \quad (12)$$

The solution of Eqs. (6) and (7) can be obtained as,

$$X_1 = \frac{Z_{22}F_1 - Z_{21}F_2}{\det(Z(i\omega))}, \quad X_2 = \frac{Z_{11}F_2 - Z_{12}F_1}{\det(Z(i\omega))} \quad (13)$$

The average absorption power of such a two-body wave energy converter is:

$$P_{ave} = \frac{1}{T} \int_0^T c_{pto} (\dot{x}_1 - \dot{x}_2)^2 dt = \frac{1}{2} \omega^2 c_{pto} |X_1 - X_2|^2 \quad (14)$$

By combining Eqs. (10), (12), (13) and (14), the closed-form solution of average absorption power P_{ave} can be expressed as:

$$P_{ave} = \frac{1}{2} \omega^2 c_{pto} \left| \frac{p+iq}{(a+ib)c_{pto} + (c+id)k_{pto} + e+if} \right|^2 \quad (15)$$

where,

$$\begin{cases} p = k_s F_2 - \omega^2(m_2 + A_{22} + A_{21})F_1 - \omega^2(m_1 + A_{11} + A_{12})F_2 \\ q = \omega(b_{22} + b_{vis_2} + b_{21})F_1 + \omega(b_{11} + b_{vis_1} + b_{12})F_2 \\ a = -\omega^2(b_{11} + b_{vis_1} + b_{22} + b_{vis_2} + b_{12} + b_{21}) \\ b = \omega k_s - \omega^3[(m_1 + A_{11}) + (m_2 + A_{22}) + A_{12} + A_{21}] \\ c = b/\omega \\ d = -a/\omega \\ e = \omega^4[(m_1 + A_{11})(m_2 + A_{22}) - A_{12}A_{21}] - \omega^2[(m_2 + A_{22})k_s + (b_{11} + b_{vis_1})(b_{22} + b_{vis_2}) - b_{12}b_{21}] \\ f = -\omega^3[(m_1 + A_{11})(b_{22} + b_{vis_2}) + (m_2 + A_{22})(b_{11} + b_{vis_1}) - A_{12}b_{12} - A_{21}b_{21}] + \omega k_s(b_{22} + b_{vis_2}) \end{cases} \quad (16)$$

If $k_{pto} = 0$, by taking the derivative of Eq. (15) with respect to c_{pto} , the corresponding suboptimal damping is obtained as,

$$(c_{pto})_{subopt} = \sqrt{\frac{e^2 + f^2}{a^2 + b^2}} \quad (17)$$

The corresponding maximum absorption power is:

$$(P_{ave})_{subopt} = \frac{1}{2} \omega^2 \frac{p^2 + q^2}{2\sqrt{(a^2 + b^2)(e^2 + f^2)} + 2(ae + bf)} \quad (18)$$

Eq.(17) and (18) are called suboptimal condition of a two body wave energy converter in this paper.

If $k_{pto} > 0$, by taking the partial derivative of Eq. (15) with respect to k_{pto} and c_{pto} , respectively, the optimal conditions of $(k_{pto})_{opt}$ and $(c_{pto})_{opt}$ are obtained as,

$$\begin{cases} (k_{pto})_{opt} = -\frac{ce+df}{c^2+d^2} \\ (c_{pto})_{opt} = \sqrt{\frac{e^2+f^2+(c^2+d^2)k_{pto}^2+(2ce+2df)k_{pto}}{\omega^2(c^2+d^2)}} \end{cases} \quad (19)$$

By substituting $(k_{pto})_{opt}$ into $(c_{pto})_{opt}$, one can get,

$$(c_{pto})_{opt} = \frac{1}{\omega} \frac{|cf-de|}{c^2+d^2} \quad (20)$$

By substituting $(k_{pto})_{opt}$ of Eq. (19) and $(c_{pto})_{opt}$ of Eq. (20) into Eq. (15), the maximum absorption power for such a two-body wave energy converter can be found as:

$$(P_{ave})_{opt} = \frac{1}{2} \omega^2 \frac{p^2+q^2}{2|ae+bf|+2(ae+bf)} \quad (21)$$

Eqs. (19), (20) and (21) are called optimal condition of a two body wave energy converter in this paper. They are identical to the results obtained by Fanles [23] and Recci [28]. The physical meaning of $(k_{pto})_{opt}$ in Eq. (19) is to match one of the damped natural frequency with the incident wave frequency, in a similar way as the resonance condition for single body system. Based on the closed form solutions obtained above, one can find that the power take-off parameters are determined by the system parameter for the maximum power absorption. Therefore, the design principles of a two-body wave energy converter are proposed based on the optimal and suboptimal conditions as follows;

Suboptimal design: For a given two-body system and incident wave, the PTO stiffness is zero while the damping coefficient is defined as Eq.(17), which is determined by buoy and incident wave parameters. The maximum power is defined as Eq. (18) for suboptimal design.

Optimal design: For a given two-body system and incident wave, the spring stiffness and damping of the power take-off system are defined as Eq. (19) for $(k_{pto})_{opt} \geq 0$. If $(k_{pto})_{opt} < 0$ for the given system, k_{pto} is set to zero and the damping is defined as Eq. (17). The maximum power is defined as Eq. (21) for $(k_{pto})_{opt} \geq 0$ and Eq. (18) for $k_{pto} = 0$.

3. Simulation Results in Regular Waves

In this section, the closed-form solutions of suboptimal and optimal conditions obtained in Section 2 are numerically studied first. The influence of the submerged body on the absorption power of a two-body system is also investigated. Since this part is mainly focused on the design of the submerged body under regular wave excitation, the floating buoy is chosen as a cylindrical buoy with a diameter of 10m and draft of 3.5m. The submerged body is assumed to be axisymmetric with an undefined shape. The added mass and radiation damping of the floating buoy were calculated with matched eigenfunction method [31-34] and are shown in Figure 3. For an axisymmetric body, based on Haskind relation [24], the exciting force F_{ei} can be written as:

$$F_{ei} = \sqrt{\frac{2\rho g^3 b_{ii}}{\omega^3}} A \quad (i = 1,2) \quad (22)$$

where, ρ is water density; g is the standard gravity; ω is the wave frequency; b_{ii} is the radiation damping of the floating and submerged bodies; A is the wave amplitude, which is chosen as 1m.

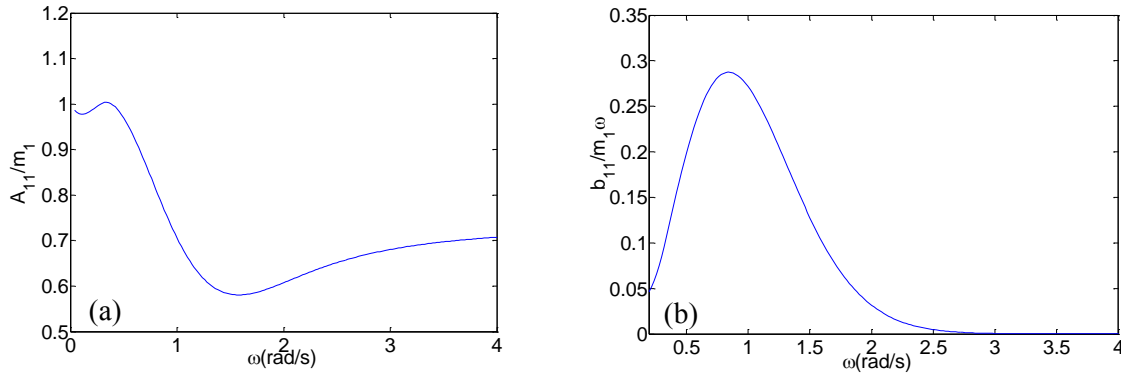


Figure 3. Added mass (a) and radiation damping (b) of the cylindrical floating buoy with a diameter of 10m and draft of 3.5m

In addition, the submerged body is assumed to be deployed far enough from the floating buoy and free surface (this assumption is widely used for different devices [27, 29, and 30]). As a result, the cross coupling added mass (A_{12} , A_{21}) and radiation damping (b_{12} , b_{21}) as well as radiation damping (b_{22}) on the submerged body can be neglected. With this simplification, the submerged body can be characterized with a total mass ($m_2 + A_{22}$) and a linearized damping ($b_{vis,2}$) in Eqs. (2) and (3). The suboptimal and optimal design discussed in Section 2 under the influence of the submerged body are numerically studied based on this simplified model.

Based on the experiment results and analysis in [27], it is believed that the damping on the floating buoy is dominated by radiation damping. Therefore, the viscous damping on the floating buoy is neglected here, i.e. $b_{vis,1} = 0$. Moreover, since the floating buoy is the same for all the systems considered in this paper, neglecting the viscous damping on the floating buoy does not have any influence on the results. The viscous damping on the submerged body $b_{vis,2}$ is dimensionless with the following equation,

$$\zeta_2 = \frac{b_{vis,2}}{2m_1\omega_f} \quad (23)$$

where, ζ_2 is the dimensionless viscous damping on the submerged body; $b_{vis,2}$ is the linearized viscous damping coefficient on the submerged body; m_1 is the mass of the floating buoy; $\omega_f = \sqrt{k_s/m_1}$ is the natural frequency of the floating buoy in still water. One should be aware that $\zeta_2 = 0.1$ means the damping on the submerged body is in the same order of the largest radiation damping on the floating buoy in Figure 3(b), which is a large damping.

Figure 4 shows the optimal and suboptimal designs described in Section 2 with a different total mass ($m_2 + A_{22}$) of the submerged body for $\zeta_2 = 0$. The excitation wave periods T are chosen as 8s, 10s and 12s. In Figure 4(a), the optimum PTO stiffness becomes *negative* (and the system will be unstable) when the total mass of the submerged body is larger than a critical mass m_c . This critical mass can be obtained by setting $(k_{pto})_{opt} = 0$ in Eq. (19), which yields,

$$ce + df = 0 \quad (24)$$

Substitute c , e , d and f from Eq. (16) into Eq. (24) and notice that $A_{12}=A_{21} = b_{12} = b_{21} = b_{22} = 0$ for the system considered here. Eq. (24) can be written as,

$$-\omega^4(M_1 + M_2)^2 + \omega^2[k_s M_2^2 + k_s M_1 M_2 + k_s(M_1 + M_2) - k_s^2 M_2 - M_2 b_{11}^2 - M_1 b_{vis,2}^2] - k_s b_{vis,2}^2 = 0 \quad (25)$$

where, $m_1 + A_{11} = M_1$, $m_2 + A_{22} = M_2$. The critical mass, $m_c = M_2/m_1$, can be found by solving Eq. (25) for the floating buoy described in this Section. When $m_2 + A_{22} > m_c$, the PTO stiffness is set

to zero in Figure 4(a) as described in Section 2. The damping coefficients of the suboptimal and optimal designs are shown in Figure 4(b), the damping of the optimal design is the same with the suboptimal design when the mass is larger than the critical mass.

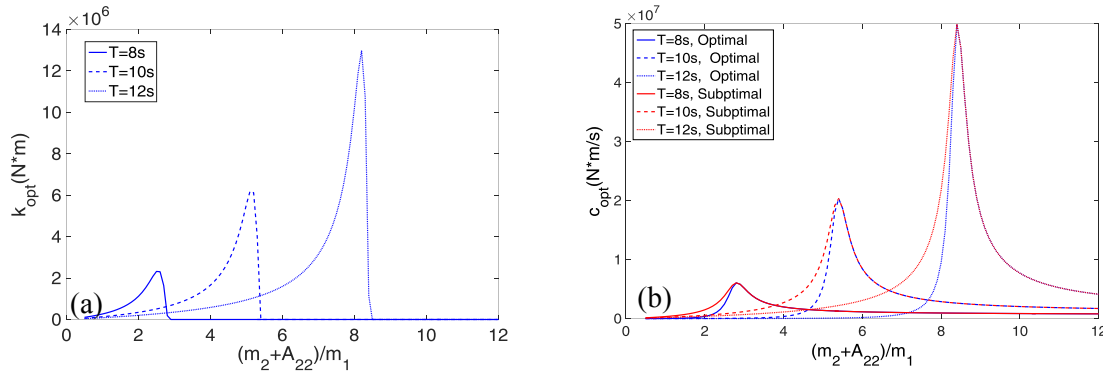


Figure 4. Spring stiffness (a) and damping coefficient (b) of optimum and suboptimum PTO design with different total mass of the submerged body. For the results in this figure, the floating buoy is a cylinder with a diameter of 10m and draft of 3.5m. The regular wave periods considered here are 8s, 10s, and 12s with an amplitude of 1m. The viscous damping ζ_2 is 0.

Figure 5 considers the influence of various damping ζ_2 on the suboptimal and optimal design when the wave period $T = 10s$. In Figure 5 (a) and (b), with the increase of the viscous damping ζ_2 , the overall trend of the power take-off system (stiffness and damping) stays the same, i.e. $(k_{pto})_{opt}$ and $(c_{pto})_{opt}$ first increase, then decrease with the increase of $m_2 + A_{22}$. The only difference between different lines in Figure 5(a) and (b) is the magnitudes of the curves. The magnitudes of k_{pto} and c_{pto} decrease for both optimal and suboptimal designs when ζ_2 increases for the same $m_2 + A_{22}$. Since the absorption power is proportional to c_{pto} , the decrease of c_{pto} leads to the decrease of absorption power.

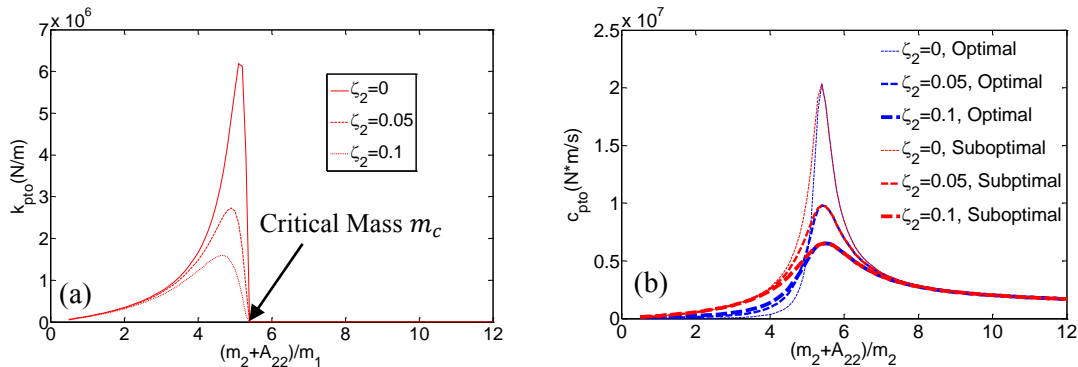


Figure 5. Spring stiffness (a) and damping coefficient (b) of optimum and suboptimum PTO design with different $m_2 + A_{22}$ and ζ_2 . The floating buoy is a cylinder with a diameter of 10m and draft of 3.5m. The regular wave period considered here is 10s with an amplitude of 1m.

Figure 6 (a) shows the damped natural frequency of the optimal design and suboptimal design for different choices of $m_2 + A_{22}$ when the wave period is 10s and viscous damping ζ_2 is 0.05. Since the power take-off stiffness is zero for the suboptimal design, there is only one vibration frequency $(\omega_d)_{sub}$ for this design. This damped natural frequency $(\omega_d)_{sub}$ decreases with the increase of $m_2 + A_{22}$, and when $m_2 + A_{22}$ is around 6 in Figure 6(a), this damped natural frequency is very close to the wave excitation frequency. As a result, the maximum absorption power of the suboptimal design is achieved, as one can find in Figure 6(b). The corresponding optimal mass m_o can be obtained by take the derivative of Eq. (18) with $m_2 + A_{22}$ and set it to zero,

$$\frac{d((P_{ave})_{subopt})}{d(m_2 + A_{22})} = 0 \quad (26)$$

where, $(P_{ave})_{subopt}$ is expressed in Eq. (18); For a given wave condition and buoy information, one can solve the optimal mass m_o from Eq. (26). One should notice that optimal mass $m_o >$ critical mass m_c when $\zeta_2 > 0$ and $m_o = m_c$ when $\zeta_2 = 0$.

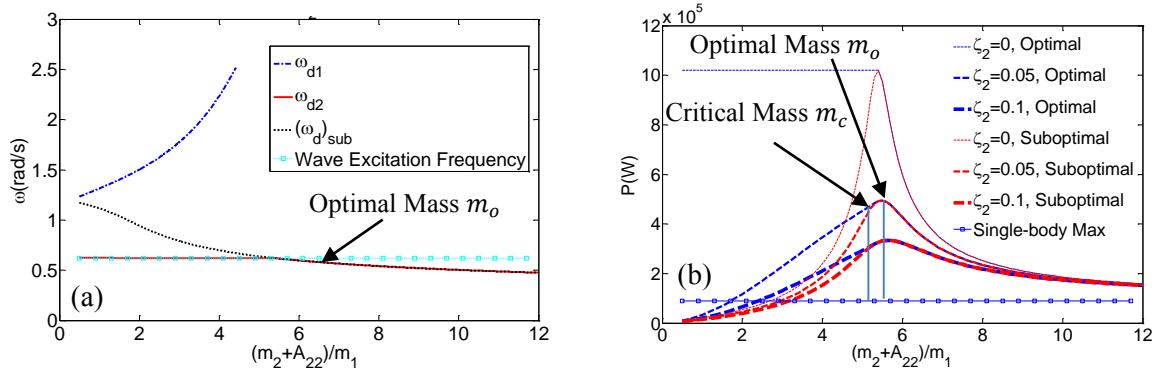


Figure 6. (a) Damped natural frequencies of the two-body system subjected with optimal and suboptimal design. The excitation wave frequency is also plotted as a comparison. (b): Maximum absorption power for optimum and suboptimum design with different $m_2 + A_{22}$ and ζ_2 . For the results presented here, the floating buoy is a cylinder with a diameter of 10m and draft of 3.5m. The regular wave period considered here is 10s with an amplitude of 1m.

For the optimal design, there are two natural frequencies (ω_{d1} and ω_{d2}) when $(k_{pto})_{opt} > 0$, as one can find in Figure 6(a). With the increase of $m_2 + A_{22}$, one of the natural frequencies (ω_{d1}) increases while the other one (ω_{d2}) matches the excitation frequency when $m_2 + A_{22} \leq m_o$. When $m_2 + A_{22} > m_o$, the spring stiffness is zero and the damped natural frequency of this system reduces from two to one. This damped natural frequency becomes smaller than the excitation frequency with the increase of $m_2 + A_{22}$, which is the same as the suboptimal design. These results show that the optimal condition in Eq. (19) is the resonance condition. In Figure 6(b), when $\zeta_2 = 0$, the maximum absorption power of the optimal design is a constant if $m_2 + A_{22} \leq m_o$. This means that one can always achieve an optimal absorption power when $m_2 + A_{22} \leq m_o$ with the optimal design. When $\zeta_2 > 0$, it is found that the maximum power of both optimal and suboptimal designs is achieved when $m_2 + A_{22} = m_o$ for the given wave condition. This means that $m_2 + A_{22} = m_o$ is the optimal mass of the submerged body in the presence of ζ_2 . Moreover, for the same $m_2 + A_{22}$, the increase of ζ_2 always results in the decrease of absorption power. Therefore, one should design ζ_2 as small as possible when $m_2 + A_{22}$ is decided. The output power of the single body system with the same floating buoy is also plotted as a comparison. It is found that the absorption power of a two-body system is significantly increased in the presence of a large viscous damping on the submerged body.

The maximum absorption power of the optimal design under different wave conditions is plotted in Figure 7. The results obtained by Fanles [24] (Eq. (27)) when $\zeta_2 = 0$ is also plotted as a comparison.

$$P_{max} = \frac{\rho g^3 A^2}{4\omega^3} \quad (27)$$

where, P_{max} is the maximum absorption power for an axisymmetric two-body system oscillating in heave without consideration of viscous damping; ρ is the density of water; g is the standard gravity; A is the amplitude of incident wave; ω is the frequency of incident wave.

In Figure 7, when $\zeta_2 = 0$, the results of Eq. (21) match well with Fanles' results. Based on the previous analysis, for the given 10m floating buoy, this theoretical maximum can be achieved as long as the power take-off is designed as Eq. (19) and $m_2 + A_{22} \leq m_c$. When $\zeta_2 > 0$, the design of the

submerged body is unique in order to achieve the maximum absorption power, which is $m_2 + A_{22} = m_o$ for the given 10m floating buoy under certain wave frequency. The maximum power decreases dramatically at low frequency with the increase of ζ_2 , but it remains almost the same at high frequency. The reason is that the increase of ζ_2 dramatically decreases $(c_{pto})_{subopt}$ in Eq. (17) at low frequency, as one can find from Figure 5(b). The displacement of the floating buoy and submerged body decrease while the relative motion remains almost the same in Figure 7(b) when ζ_2 increases from 0.05 to 0.1.

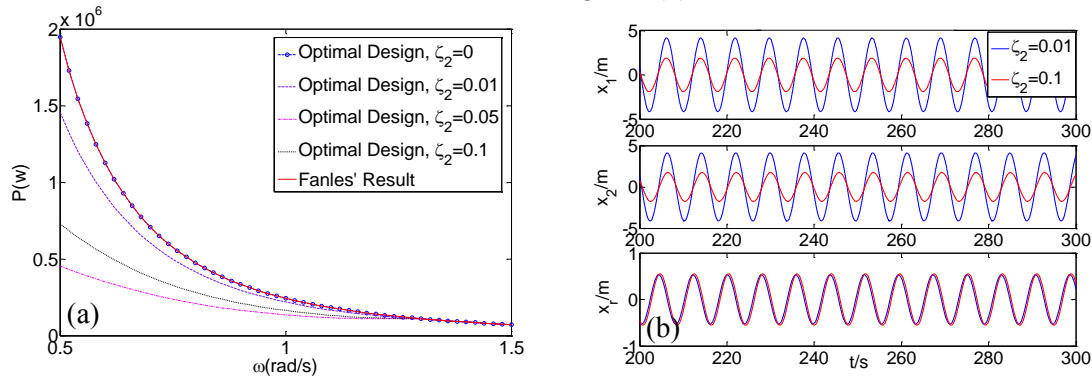


Figure 7. (a). Comparison of maximum absorption power of optimal design and Fanle's [24] results with different wave frequency. (b) Motion of the floating and submerged body subjected with optimal design for different wave frequency. For the results presented in (b), the wave amplitude and frequency are 1m and 0.8rad/s, respectively. The floating buoy is a cylinder with a diameter of 10m and draft of 3.5m. $m_2 + A_{22} = m_c$ is calculated with Eq. (26).

In conclusion, based on the results in Figures 4-7, the overall performance of the optimal design is better than the suboptimal design when $m_2 + A_{22} \leq m_c$. This is reasonable, since one of the natural frequencies of the system matches with the incident wave frequency by design (k_{pto}) with Eq. (19). The natural frequency of the suboptimal design matches with the excitation frequency only when $m_2 + A_{22} = m_o$. Therefore, when $m_2 + A_{22} > m_o$, the performance of the optimal design worsens with the increase of $m_2 + A_{22}$ since the natural frequency of the two-body system moves always from the excitation frequency. When $\zeta_2 > 0$, the absorption power of both the optimal and suboptimal designs decreases dramatically for the same design of $m_2 + A_{22}$. In this situation, the optimal design of the submerged body is unique, which is $m_2 + A_{22} = m_o$. In addition, the overall performance of the two-body system is better than the single body system even if one accounts for a large viscous damping on the submerged body.

4. Design of Two-body Wave Energy Converters in Irregular Waves

In this section, the influences of the submerged body on the power take-off design and absorption power are discussed under irregular wave excitation. In the simulation, the floating buoy is the same as described in Section 3 with a diameter of 10m and draft of 3.5m. $A_{12}=A_{21}=b_{12}=b_{21}=b_{22}=0$. A Pierson-Moskowitz spectra shown in Eq. (28), which is an empirical equation that defines the distribution of energy with wave frequency in the fully developed sea, is used here to describe the irregular waves.

$$S(\omega) = 526H_s^2T_e^{-4}\omega^{-5}\exp(-1054T_e^{-4}\omega^{-4}) \quad (28)$$

where H_s is the significant wave height and T_e is energy wave period. The average absorption power of a wave energy converter in the irregular waves can be calculated through the spectrum integration with the following equation.

$$\overline{P_{irr}} = \int_0^\infty P_{ave}(\omega)S(\omega)d\omega \quad (29)$$

where, $\overline{P_{irr}}$ is the absorption power of a two-body system in irregular waves; $P_{ave}(\omega)$ is the absorption power in regular waves described in Eq. (15). When the floating buoy, submerged body and power take-off system are determined, P_{ave} in Eq. (15) is a function of the excitation wave frequency. $S(\omega)$ is the wave spectrum described in Eq. (28).

In Section 3, it was found that the maximum absorption power of the optimal design can be achieved if the spring stiffness is designed in such a way that one of the damped natural frequencies matches with the incident wave frequency. In irregular waves, however, it is hard to apply this design principle since the excitation is a spectrum instead of single frequency. Therefore, to implement suboptimal and optimal design defined in Section 2 in irregular waves, the excitation frequency appeared in Eqs. (17), (18), (19) and (21) is chosen as the frequency corresponding to the energy period in Eq. (28), i.e. $\omega = 1/T_e$.

A constrained complex method described in [35] is used to find the maximum absorption power for the two-body system described in this section. The problem is formulated as following: for a given wave condition (T_e and H_s) and submerged body ($m_2 + A_{22}, \zeta_2$), the absorption power of such a two-body wave energy converter can be written as a function of k_{pto} and c_{pto} as the following equations,

$$\begin{cases} P_{irr} = \int_0^\infty P_{ave}(\omega, k_{pto}, c_{pto}) S(\omega) d\omega \\ 0 \leq k_{pto} \leq 10^8 \\ 0 \leq c_{pto} \leq 10^8 \end{cases} \quad (30)$$

The initial points were chosen randomly from the parameter domain. The details about simplex method can be found in [35]. The maximum absorption power of a single body system with the same floating buoy subjected with passive damping control is also calculated numerically as a comparison. The absorption power is calculated with the following equation.

$$(P_{irr})_{single} = \int_0^\infty P_{ave}(c_{pto}, \omega) S(\omega) d\omega \quad (31)$$

where, $(P_{irr})_{single}$ is the absorption power of a single system with the same floating buoy in irregular waves; $P_{ave}(c_{pto}, \omega)$ is the absorption power of this single body system in regular waves. $S(\omega)$ is the wave spectrum described in Eq. (28).

Figure 8 compares the absorption power of optimal and suboptimal design in irregular waves. The maximum absorption power obtained from constrained complex method was also plotted here as a comparison. The straight line represents the maximum absorption power of single body system with the same floating buoy subjected with passive damping control under the same wave condition. Several very useful conclusions can be obtained for a two-body wave energy converter.

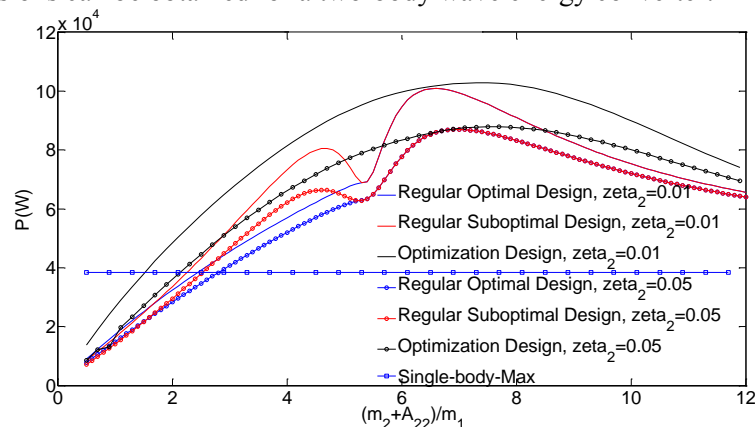


Figure 8. Absorption power of suboptimal and optimal design with different submerged body design. The optimization design represents the maximum absorption power obtained from constrained complex method. The straight line is the maximum absorption power for single body system with the same floating buoy. For the results plotted here, the irregular wave has a significant wave height of 2m and energy period of 10s. The floating buoy is a cylinder with a diameter of 10m and draft of 3.5m.

(1) If one compares the maximum absorption power of a two-body system obtained from constrained complex method (black line) with that of a single body system (blue line with square), it is found that the maximum absorption power of a two-body system is more than twice of single-body system in the irregular waves considered here. Even if one considers $\zeta_2 = 0.05$, the maximum absorption power of the two-body system is still almost twice as that of single body system. This indicates that the absorption power of a single body system can be increased significantly by designing an additional body underneath in the irregular waves.

(2) The total mass of the submerged body plays a very important role in the two-body system. One can find that if the mass ratio is small ($(m_2 + A_{22})/m_1 < 1.5$), the absorption power of the two-body system obtained from the complex method (black line) is smaller than that of the single body system (blue line with square). Therefore, the total mass $m_2 + A_{22}$ of the submerged body should be large enough in order to get a better performance in irregular waves. For the system considered here, $(m_2 + A_{22})/m_1 > 4$ can guarantee an increase of absorption power compared with a single body system. In addition, if $m_2 + A_{22} \rightarrow \infty$, the two-body system is equivalent to a single body system, and the absorption power of the two-body system approaches that of the single body system as shown in Figure 8. The added mass of the submerged body A_{22} is usually a shape dependent constant if it is deployed in the deep water. This allows people to relate $m_2 + A_{22}$ with the dimension of the submerged body in the design stage. Take the floating buoy described in this paper as an example, if one wants to design a submerged sphere with $m_2 + A_{22} = 8m_1$. Based on the linear wave theory [24], the added mass of a submerged sphere is half of its mass, i.e. $A_{22} = 0.5m_2$. The diameter of the desired sphere is 14m. For other complicated shapes, one can resort to commercial software for the design of $m_2 + A_{22}$.

(3) The linearized damping due to viscous effects ζ_2 generally has a bad effect on the power absorption for a two-body system design. For all the absorption power plotted in Figure 8, increase in ζ_2 always results in a decrease of absorption power with the same $m_2 + A_{22}$. Based on [27], this damping term is mainly from drag force. Therefore, one should try to minimize the drag coefficient while designing the submerged body with desired mass. In this view, the submerged body is recommended to be designed with a streamline to decrease the drag damping when $m_2 + A_{22}$ is determined.

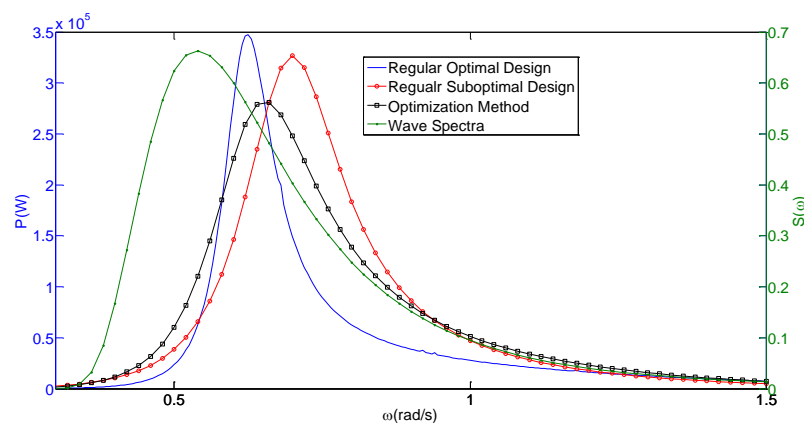


Figure 9. The wave spectra and absorption power of optimal and suboptimal design with different wave frequencies. For the results plotted here, the irregular wave has a significant wave height of 2m and energy period of 10s. The floating buoy is a cylinder with a diameter of 10m and draft of 3.5m. $m_2 + A_{22} = 4$ and $\zeta_2 = 0.05$.

(4) Different from the results in regular waves, the performance of the suboptimal design is better than the optimal design when $m_2 + A_{22} \leq m_c$, as shown in Figure 8, especially when $2 < m_2 + A_{22} < m_c$. Despite having the peak power of in Figure 9, the bandwidth of optimal design is narrower than that of the suboptimal design and complex optimization method. In addition, one should notice that the absorption power of the suboptimal design gets closer to results obtained by the constrained complex

method when we increase ζ_2 from 0.01 to 0.05, especially when $m_2 + A_{22} > 10$. Therefore, if one designed the submerged body with a large added mass and drag damping, like a heave plate used in the Powerbuoy, the suboptimal design can be used and the absorption power is very close to the maximum power.

5. Conclusion

A frequency domain model of a two-body wave energy converter is considered in this paper by accounting for the viscous damping. The closed-form solution of the suboptimal and optimal absorption power and corresponding power take-off parameters are obtained. The suboptimal and optimal designs of the two-body wave energy converter are defined based on the obtained closed-form solutions. Despite the linearized drag damping and hydrodynamic simplification considered in the simulation, the results are believed to be useful to understand the dynamics and design of a two-body wave energy converter.

A case study is conducted to investigate the influence of the submerged body on the performance of a two-body wave energy converter subjected to suboptimal and optimal designs in regular waves. If $\zeta_2 = 0$, the absorption power of the optimal design can always reach the theoretical maximum power when the total mass of the submerged body is smaller than the optimal mass. This is because the optimal design ensures one of the natural frequencies of the two-body system matches with the incident wave frequency. When the total mass of the submerged body is larger than the optimal mass (i.e. $m_2 + A_{22} > m_o$), the absorption power decreases with the increase of the total mass of the submerged body. This is because the frequency match condition does not hold any more. The optimal mass is actually the optimal design for the submerged body when the damping due to viscous effects on the submerged body is large than zero (i.e. $\zeta_2 > 0$). By comparing the absorption power of a two-body system with a single body system with the same floating buoy, it is found that the absorption power can be increased significantly even with a large damping due to viscous effects on the submerged body.

The suboptimal and optimal designs of the two-body wave energy converter are also studied under irregular wave excitation by accommodating the energy frequency of the irregular wave in the closed-form solution. It is found that the maximum absorption power of a two-body wave energy converter is more than twice that of a single body system with the same floating buoy for the system considered in this paper. Moreover, the total mass of the submerged buoy needs to be designed large enough in order to achieve a higher absorption power compared with single-body system. The shape dependent drag coefficient should be minimized while designing the submerged body with a desired total mass.

6. Acknowledgement

The authors would gratefully acknowledge financial support from Nation Science Foundation (NSF 1530122) and EPA P3 program (SU835532).

7. Reference

- [1] Hirohisa T. Sea trial of a heaving buoy wave power absorber. In: Proceedings of 2nd International Symposium on Wave Energy Utilization, Trondheim, Norway 1982.
- [2] Budal K, Falnes J. Wave power conversion by point absorbers: a Norwegian project. *International Journal of Ambient Energy* 1982;3:59-67.
- [3] Ferdinande V, Vantorre M. The concept of a bipartite point absorber. *Hydrodynamics of Ocean Wave-Energy Utilization*. Berlin: Springer; 1986; 217-226.
- [4] Nielsen K, Smed PF. Point absorber—optimization and survival testing. In: Proceedings of the Third European Wave Energy Conference, Patras, Greece 1998.
- [5] Weinstein A, Fredrikson G, Parks MJ, Nielsen K. AquaBuOY-the offshore wave energy converter numerical modeling and optimization. In: Proceedings of MTTS/IEEE Techno-Ocean'04 Conference, Kobe, Japan 2004.
- [6] Eriksson M, Waters R, Svensson O, Isberg J, Leijon M. Wave power absorption: Experiments in open sea and simulation. *Journal of Applied Physics* 2007;102:084910.

- [7] Leijon M, Boström C, Danielsson O, Gustafsson S, Haikonen K, Langhamer O, Strömstedt E, Stålberg M, Sundberg J, Svensson O, Tyrberg S. Wave energy from the North Sea: experiences from the Lysekil research site. *Surveys in geophysics* 2008;29:221-40.
- [8] Payne GS, Taylor JR, Bruce T, Parkin P. Assessment of boundary-element method for modelling a free-floating sloped wave energy device. Part 1: Numerical modelling. *Ocean Engineering* 2008;35:333-341.
- [9] Payne GS, Taylor JR, Bruce T, Parkin P. Assessment of boundary-element method for modelling a free-floating sloped wave energy device. Part 2: Experimental validation. *Ocean Engineering* 2008;35:342-357.
- [10] Elwood D, Schacher A, Rhinefrank K, Prudell J, Yim S, Amon E, Brekken T, von Jouanne A. Numerical modeling and ocean testing of a direct-drive wave energy device utilizing a permanent magnet linear generator for power take-off. In: *Proceedings of 28th International Conference on Ocean, Offshore and Arctic Engineering*, ASME, Honolulu, Hawaii; 2009.
- [11] Elwood D, Yim SC, Prudell J, Stillinger C, von Jouanne A, Brekken T, Brown A, Paasch R. Design, construction, and ocean testing of a taut-moored dual-body wave energy converter with a linear generator power take-off. *Renewable Energy* 2010;35:348-54.
- [12] Waters R, Stålberg M, Danielsson O, Svensson O, etc. Experimental results from sea trials of an offshore wave energy system. *Applied Physics Letters* 2007; 90: 034105.
- [13] Carnegie Wave Energy. URL: <http://carnegiwave.com/>, last accessed data 2015.
- [14] Budar K and Falnes J. A resonant point absorber of ocean-wave power. *Nature* 1975;256:478-479.
- [15] Evans DV. A theory for wave-power absorption by oscillating bodies. *Journal of Fluid Mechanics* 1976;77: 1-25.
- [16] Mei CC. Power extraction from water waves. *Journal of Ship Research* 1976;20:63-66.
- [17] Falcao AF de O. Wave energy utilization: A review of the technologies. *Renewable and Sustainable Energy Reviews* 2010;14:899-918.
- [18] Engström J, Eriksson M, Isberg J and Leijon M. Wave energy converter with enhanced amplitude response at frequencies coinciding with Swedish west coast sea states by use of a supplementary submerged body. *Journal of Applied Physics* 2009;106: 064512.
- [19] Budal K and Falnes J. Interacting point absorbers with controlled motion. *Power from Sea Waves* 1980;381-399.
- [20] Babarit, A., Guglielmi, M. and Clément, A. H. Declutching control of a wave energy converter. *Ocean Engineering* 2009; 36:1015-1024.
- [21] Ocean Power Technologies. URL: <http://www.oceanpowertechnologies.com/>, last accessed data 2015.
- [22] Dick W. 2005 U.S. Patent No. 6,857,266. Washington, DC: U.S. Patent and Trademark Office.
- [23] Falnes J. Wave-energy conversion through relative motion between two single-mode oscillating bodies. *Journal of Offshore Mechanics and Arctic Engineering* 1999; 121: 32-38.
- [24] Falnes J. *Ocean waves and oscillating systems: linear interactions including wave-energy extraction*. Cambridge University Press, 2002.
- [25] Korde UA. Systems of reactively loaded coupled oscillating bodies in wave energy conversion. *Applied ocean research* 2003;25: 79-91.
- [26] Wu B, Wang X, Diao X, Peng W, Zhang Y. Response and conversion efficiency of two degrees of freedom wave energy device. *Ocean Engineering* 2014;76:10-20.
- [27] Beatty SJ, Hall M, Buckham BJ, Wild P, and Bocking B. Experimental and numerical comparisons of self-reacting point absorber wave energy converters in regular waves. *Ocean Engineering* 2015;104:370-386.
- [28] Ricci P. *Modelling, optimization and control of wave energy point-absorbers* [PhD thesis]. Instituto Superior Técnico; 2012
- [29] Falcao AF, Cândido JJ, Justino PA, and Henriques JC. Hydrodynamics of the IPS buoy wave energy converter including the effect of non-uniform acceleration tube cross section. *Renewable Energy* 2012; 41:105-114.

- [30] Henriques JCC, Lopes MFP, Gomes RPF, Gato LMC, and Falcao, AF. On the annual wave energy absorption by two-body heaving WECs with latching control. *Renewable Energy* 2012;45:31-40.
- [31] Yeung RW. Added mass and damping of a vertical cylinder in finite-depth waters. *Applied Ocean Research* 1981;3:119-133.
- [32] Zheng Y, Shen Y, You Y, Wu B, and Rong L. Hydrodynamic properties of two vertical truncated cylinders in waves. *Ocean Engineering* 2005;32:241-271.
- [33] Berggren L, Johansson M. Hydrodynamic coefficients of a wave energy device consisting of a buoy and a submerged plate. *Applied Ocean Research* 1992;14:51-58.
- [34] Sabuncu T and Calisal S. Hydrodynamic coefficients for vertical circular cylinders at finite depth. *Ocean Engineering* 1981;8:25-63.
- [35] Ravindran A, Reklaitis GV and Ragsdell KM. *Engineering optimization: methods and applications*. John Wiley & Sons, Chicago, 2006