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Passive vibration suppression using inerters for a multi-storey building structure

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Abstract. This paper investigates the use of inerters for vibration suppression of a multi-storey building structure. The inerter was proposed as a two-terminal replacement for the mass element, with the property that the applied force is proportional to the relative acceleration across its terminals. It completes the force-current mechanical-electrical network analogy, providing the mechanical equivalent to a capacitor. Thus allows all passive mechanical impedances to be synthesised. The inerter has been used in Formula 1 racing cars and applications to various systems such as vehicle suspension have been identified. Several devices that incoporate inerter(s), as well as spring(s) and damper(s), have also been identified for vibration suppression of building structures. These include the tuned inerter damper (TID) and the tuned viscous mass damper (TVMD). In this paper, a three-storey building model with an absorber located at the bottom subjected to base excitation is studied. Four simple absorber layouts, in terms of how spring, damper and inerter components should be arranged, have been studied. In order to minimise the maximum relative displacement of the building, the optimum parameter values for each of the layouts have been obtained with respect to the inerter's size.

1. Introduction

New possibilities for passive control of mechanical systems have been investigated recently making use of the inerter, introduced by Smith in [1]. The inerter is a mechanical device with two terminals and it has the property that the generated force is proportional to the relative acceleration between its two terminals. It is proposed in [2] that the traditional analogy between mechanical and electrical is to compare the physical masses, dampers and springs to the electrical capacitors, resistors and inductors, in the force-current analogy. However, a conventional mass is a device with one terminal connected to the ground, which limits the design and achievable performance of mechanical systems. This limitation can be avoided by substituting the mass with the inerter.

Applications of the inerter to vehicle suspension [3, 4, 5], control of motorcycle steering instabilities [6], vibration absorption [1], building suspension control [7, 8] and railway vehicle suspension [9, 10, 11] have been identified . The results showed that the performance of the systems can be significantly improved with the use of inerters. The inerter has been successfully deployed in Formula One racing in 2005, under the name of J-damper [5].

Mitigating seismic response of a structure is very important in civil engineering. Traditional passive control devices include base isolation systems [12, 13] and widely used seismic dampers (e.g. viscous damper [14]). Some of the possible applications of the inerter device to the civil engineering have been studied in [15, 16, 17, 8]. Ikago *et al.* [16] presented the tuned viscous damper (TVMD) as the control device and analysed the performance of a SDOF system with the use of the TVMD. The optimum response of a MDOF building structure with the use of the TVMDs in every storey has been obtained in [15]. Inerter-like devices known as inertial dampers have been introduced in [17] and the response reduction with the inertial dampers also has been investigated. In 2013, Lazar *et al.* [8] proposed a tuned inerter damper (TID) by substituting the mass of the widely used TMD with an inerter. It has been showed in [8] that the performance with a TID mounted between the structure and the ground can be better than that with a TMD at the top. For a conventional TMD [18], the mass ratio is typically taken to be a few percent of the entire structural mass, since adding large mass to the structure introduces additional structural loading. A big advantage of the inerter is that it can have a high inertance with low mass because of gearing. Considering the manufacture and installation problem of the control device, in our research, we restrict the inerter's size in a reasonable range for the potential absorber configurations.

In this paper, we present four candidate configurations of the control device for suppressing vibrations of a building. The relative displacement of the storeys to that of the base is chosen as the performance index and using this index, an optimisation objective function is proposed. By using the patternsearch and fminsearch functions in MATLAB, the optimum suppression device configuration is obtained with respect to the inerter's size.

This paper is structured as follows. In Section 2, an idealised building model is considered for performance analysis. The objective function and the optimisation approach are also proposed. In Section 3, four candidate configurations are shown and the optimum layout is obtained versus the inerter's size. The displacement response comparison with the specific inerter's size is given, as well. Finally, we draw some conclusions in Section 4.

2. Building model and objective function

In this section, an idealised building model with three storeys is introduced and the dynamic equations are derived in the Laplace domain. Then we introduce an objective function with the relative displacement chosen as the performance index. The comparison between the objective function and the original Den-Hartog tuning method [19] is also given to show the potential advantage of the proposed objective function.

2.1. The building model

We consider a three-storey building model shown in Figure 1, with equivalent floor mass m and equivalent inter-storey elasticity k. Since the structural damping is relatively small compared with that of the absorber, it is taken to be zero in our research. The control system is modelled between the first storey and the ground because it generates a force relevant to the relative velocity and this makes the installation of the absorber much easier. Note that only one control device is used at a time. f_d in Figure 1 represents the force generated by the suppression device. In this paper, we fix the parameters of the three storey building model as $m = 1\,\mathrm{kNs^2/m}$ and $k = 1500\,\mathrm{kN/m}$. The parameters for the building model are the same as these used in [8].

The equations of motion for the system in Figure 1 are written in absolute coordinates as

$$
\begin{aligned}
m\ddot{x}_1 + 2kx_1 &= kx_2 + kr + f_d,\\
m\ddot{x}_2 + 2kx_2 &= kx_1 + kx_3,\\
m\ddot{x}_3 + kx_3 &= kx_2.
\end{aligned}
$$

where in the Laplace domain $f_d(s) = sY(s)(R(s) - X_1(s))$ with $Y(s)$ is the transfer function of the control system from force to the relative velocity and $f_d(s)$, $X_i(s)$, $R(s)$ are the Laplace transforms of $f_d(t)$, $x_i(t)$, $r(t)$, respectively. By defining the relative displacement

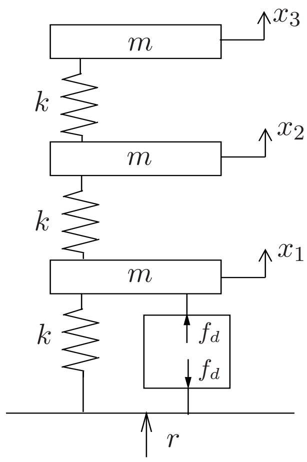

Figure 1: Schematic representation of an idealised building and lower floor suppression device.

$z_i = x_i - r$, $(i = 1, 2, 3)$, the steady-static equation of motion with respect to the relative displacement, in matrix form, in the Laplace domain is

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} s^2 Z + \begin{bmatrix} 2k + sY(s) & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix} Z = -\begin{bmatrix} m \\ m \\ m \end{bmatrix} s^2 R$$

where $Z = X - R$ represents the vector of relative storey displacements in the Laplace domain.

2.2. Objective function and optimisation approach

Many design criterions for a vibration absorber are proposed in [20], such as the absolute displacement, the absolute acceleration. We consider the displacement of each storey relative to that of the base as the performance index in this paper. The objective function is defined as

$$J_\infty = \max \left(\|T_{R \to Z_i}(j\omega)\|_\infty \right), \ i = 1, \cdots, n \tag{1}$$

where $T_{R \to Z_i}$ denotes the transfer function from R to Z_i, $\|T_{R \to Z_i}(j\omega)\|_\infty$ is the standard H_∞-norm, which represents the maximum magnitude of $T_{R \to Z_i}$ across all frequencies. Although researchers always consider the frequency range around the first fundamental frequency in cost functions, particularly, for these based on comfort, they often apply a weight distributions in the frequency domain. Here, it is the method that is important, so we select a simple unity weighting. In addition, the objective function might be based on the factors other than the relative displacement, for example, the inter-storey drift, the absolute acceleration, however, these are not considered here. In our research, J_∞ represents the biggest H_∞-norm among all the storeys. The optimisation is carried out to minimise J_∞.

For MIMO systems, the design of the absorber is normally carried out for the fundamental mode, with initial tuning based on the assumption that the natural frequencies are well separated, hence the contributions from higher modes will be ignored. In reality, the modal cross coupling has a deleterious effect on the tuning in some cases. Hence, we propose equation (1) as the objective function to avert this problem. To show the potential advantage of the objective function (1), we optimise the building model used in [8] with the same configuration TID. The inerter's size is fixed as 499 kg, which is as the same as that in [8]. Then we obtain $k_d = 142.7\,\mathrm{kN/m}$, $c_d = 3.263\,\mathrm{kNs/m}$ by optimising J_∞ with MATLAB. The authors of [8] chose the values of spring and damper as $k_d = 138.6\,\mathrm{kN/m}$, $c_d = 2.5\,\mathrm{kNs/m}$ based on Den Hartog

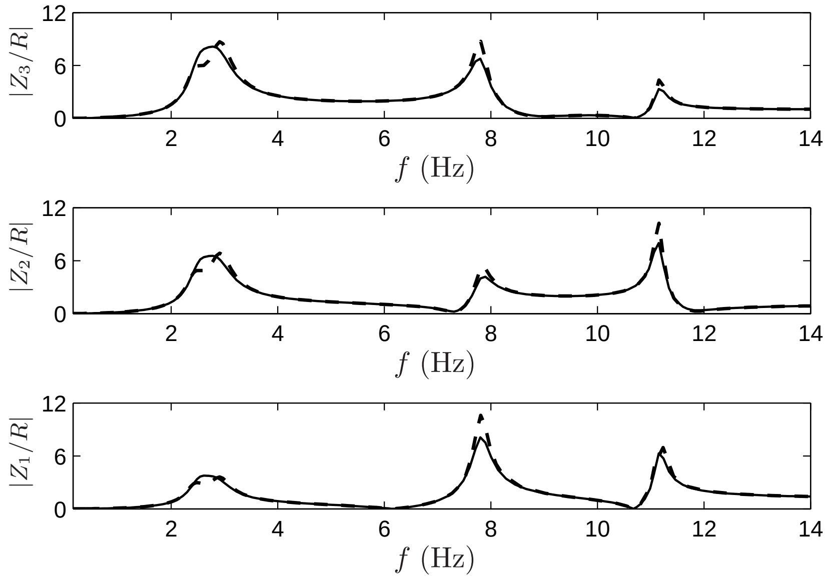

Figure 2: Displacement comparison: optimised TID using (1) with $k_d = 142.7\,\mathrm{kN/m}$, $c_d = 3.263\,\mathrm{kNs/m}$ (thin line) and TID proposed in [8] with $k_d = 138.6\,\mathrm{kN/m}$, $c_d = 2.5\,\mathrm{kNs/m}$ (thick dashed line).

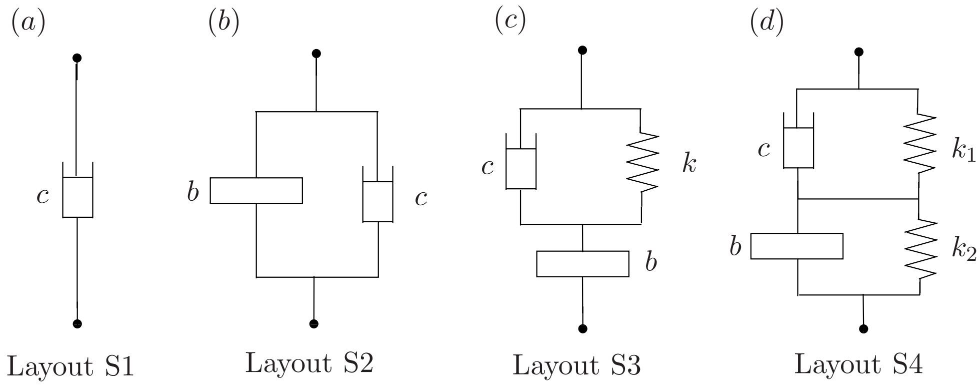

Figure 3: Candidate suppression device layouts.

tuning method [19]. The displacement responses of the three storeys in Figure 1 with a TID using these two set of values have been shown in Figure 2. It can be seen that with objective function (1), the TID device results in much smaller displacements of all the three floors in the vicinity of the second and third fundamental frequencies. Comparing with the displacement response in [8], although the displacement of the first storey in the first fundamental frequency obtained from the objective function J_∞ is slightly bigger, the max response displacement of the first fundamental frequency is smaller (for the third storey).

3. Absorber layouts and optimisation results

In this section, four simple networks are introduced as candidates for the suppression device configurations of the building model in Figure 1. As shown in Figure 3(a), layout S1 is the commonly used damper with $Y(s) = c$. Figure 3(b) shows the configurations with an inerter in parallel with a damper, which is known as the viscous mass damper (VMD) and has the transfer function $Y(s) = bs + c$. The absorber configuration S3 shown in Figure 3(c) is a tuned inerter

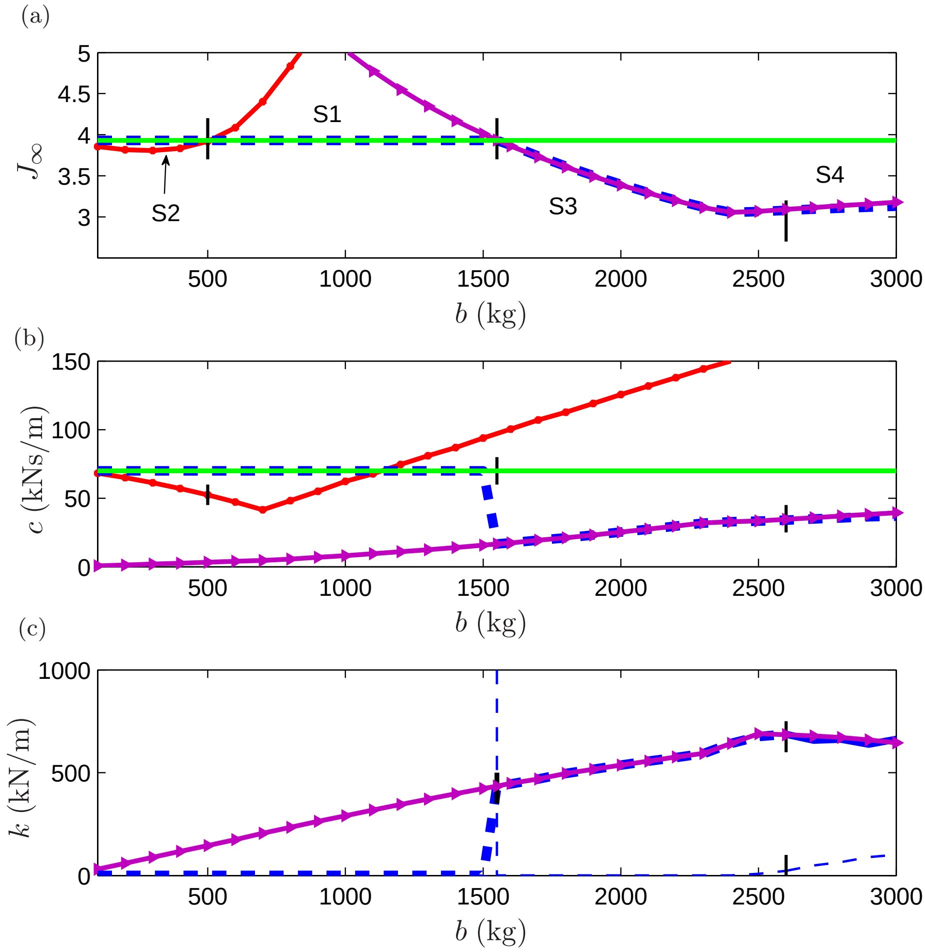

Figure 4: Results of optimisation study: (a) the optimum results, (b) the corresponding damping values, (c) the corresponding stiffness with layouts S1 (green line), S2(red line with dot marker), S3 (purple line with triangle marker) and S4 (dashed blue line, thick dashed blue for k_1, thin dashed blue for k_2).

damper (TID) [8] with $Y(s) = bs(cs+k)/(bs^2+cs+k)$. Figure 3($d$) shows the configuration with an additional spring mounted with TID and it has been proposed in [4] as the suspension strut configuration for the vehicle system. The transfer function for the configuration of Figure 3(d) can be expressed as $Y(s) = (bs^2 + k_2)(cs + k_1)/s(bs^2 + cs + k_1 + k_2)$.

The mass ratio of a conventional TMD is typically taken to be a few percent of the entire structural mass, since adding large mass to the original system will add significant structural loading. Although the inerter can have a high inertance with a much lower mass because of its gearing, its size will be larger with a higher inertance. Hence, we want to limit the size of inerter in our absorber. All three configurations shown in Figure 3(b), (c), (d) have one inerter, and we choose the range of the inerter's size from 100 kg to 3000 kg, which is reasonable and achievable for designing the inerter. By optimising the objective function J_∞ shown in the equation (1) with the proposed four simple configurations shown in Figure 3, the optimum results, the damping values and the optimal stiffness for the four configurations in the whole range of the inerter's size have been shown in Figures 4(a), (b) and (c), respectively. Short vertical lines show the transition point in value of J_∞ between the four candidate configurations.

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Table 1: J_∞ optimisation with the four layouts shown in Figure 3 when $b = 300$ kg.

layout	J_∞	element values (c (kNs/m), k (kN/m))
S1	3.93	$c = 69.9$
S2	3.81	$c = 61.3$
S3	13.3	$c = 1.91,\ k = 88.6$
S4	3.93	$c = 69.9,\ k_1 = 3.12 \times 10^{-2},\ k_2 = 3.95 \times 10^{10}$

Table 2: J_∞ optimisation with the four layouts shown in Figure 3 when $b = 2000$ kg.

layout	J_∞	element values (c (kNs/m), k (kN/m))
S1	3.93	$c = 69.9$
S2	10.7	$c = 126$
S3	3.39	$c = 25.3,\ k = 536$
S4	3.40	$c = 25.3,\ k_1 = 536,\ k_2 = 1.51 \times 10^{-3}$

For Figure 4(c), it should be noted that the values of k_2 are not shown in the figure when $b \in [100\,\text{kg}, 1550\,\text{kg}]$, since those of k_2 are much larger than 1000 (kNs/m) and they are not shown for the clarity. Figure 4(a) shows that when $b \in [100\,\text{kg}, 500\,\text{kg}]$, the configuration S2 provides the best performance among the four configurations; When $b \in (500\,\text{kg}, 1550\,\text{kg}]$, the optimum configuration is S1; When $b \in (1550\,\text{kg}, 2600\,\text{kg}]$, S3 provides the smallest results of the objective function J_∞ and S4 is the optimum configuration when $b \in (2600\,\text{kg}, 3000\,\text{kg}]$. It also can be seen from Figure 4(a) that when $b \in [100\,\text{kg}, 1550\,\text{kg}]$, the optimum results of the configuration S4 are similar to those of S1. And when $b \in (1550\,\text{kg}, 2600\,\text{kg}]$, the configuration S3 and S4 provide similar performance. This phenomenon is because the configuration S4 can be simplified to the configuration S1 or S3 with some specific element values, for example, when the value of k_2 of the configuration S4 is very large and that of k_1 is relatively small, S4 can be reduced to a damper (S1). For the range $b \in (2600\,\text{kg}, 3000\,\text{kg}]$, the optimum results of J_∞ with the configuration S4 are smaller than those with the configuration S1 and S3. From Figure 4(a), it can be noted that when $b = 2400\,\text{kg}$, the configuration S3 provides the best performance with $J_\infty = 3.05$, $c = 32.9$ kNs/m, $k = 642.3$ kN/m and from Figures 4(a) and (b), when $b > 2400\,\text{kg}$, increasing the inerter's size results in an increase in the value of the objective function J_∞ as well as an increase in the damping value c.

We have compared the optimum configuration with the other three configurations for two set of b values. For $b = 300$ kg, the configuration S2 is the optimum layout obtained from Figure 4(a) and the optimal parameter settings for the four candidate configurations shown in Figure 3 are illustrated in Table 1. It can be seen that the optimum results of the configurations S1 and S4 are the same, which can also be seen from the Figure 4(a). The value of J_∞ with the configuration S2 has an improvement of 3% comparing with that of S1 and S4. In comparison with the configuration S3, 71.4% improvement of J_∞ can be obtained with S2. The displacement response of these four configurations with $b = 300\,\text{kg}$ has been shown in Figure 5. It can also be seen from Figure 5 that around the first fundamental frequency, the max displacement response occurs at third floor and the displacement with the configuration S2 is slightly smaller than that with the configuration S1 or S4, and significantly smaller than that with the configuration S3.

When $b = 2000$ kg, it can be seen from Figure 4(a) that the configuration S3 is the optimum

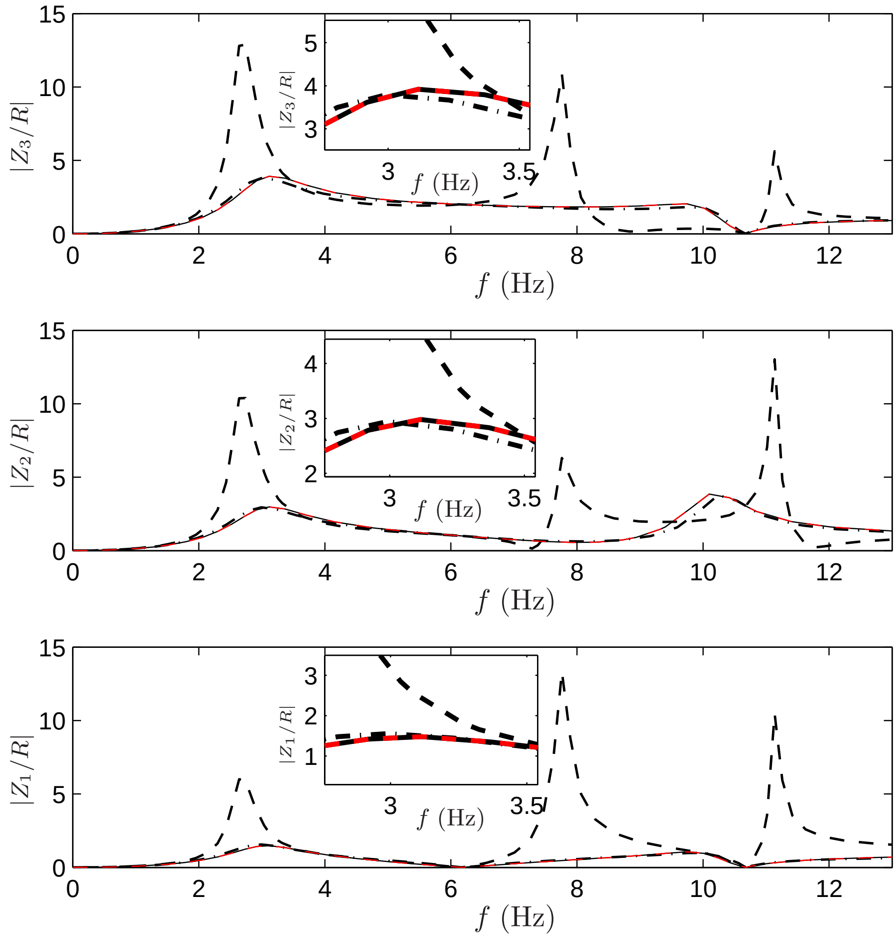

Figure 5: Displacement response comparison: configuration S1 of Figure 3(a) (black solid), S2 of Figure 3(b) (dash dotted), S3 of Figure 3(c) (dashed) and S4 of Figure 3(d) (red dashed) when $b = 300$ kg.

layout and the Table 2 shows the optimal results and element values of the four configurations. It can be seen that the configuration S3 and S4 provide the similar performance. The value of J_∞ with the configuration S3 has an improvement of 13.7% comparing with that of the damper (S1) and an improvement of 68.3% comparing with that of the configuration S2. These conclusions can also be seen from the Figure 6. It can be noted from the figure that the max displacement response of the configuration S2 occurs in the second floor at the frequency $f = 10.1$ Hz and this is because of the definition of the objective function J_∞ shown in equation (1). It can also be seen that although the displacement of the first storey around the first fundamental frequency with the configuration S3 is bigger than that of the configuration S1 and S2, the max displacement response around the first fundamental frequency with the configuration S3 is significantly smaller than that of the configuration S1 and is slightly smaller than that of the configuration S2. Also note that the damping value c of the configuration S3 is smallest, almost reduced to 1/3 value for the pure damper.

4. Conclusions

This paper has studied the performance benefits of vibration suppression device with inerters. An idealised three-storey building model was considered for analyses and the performance index

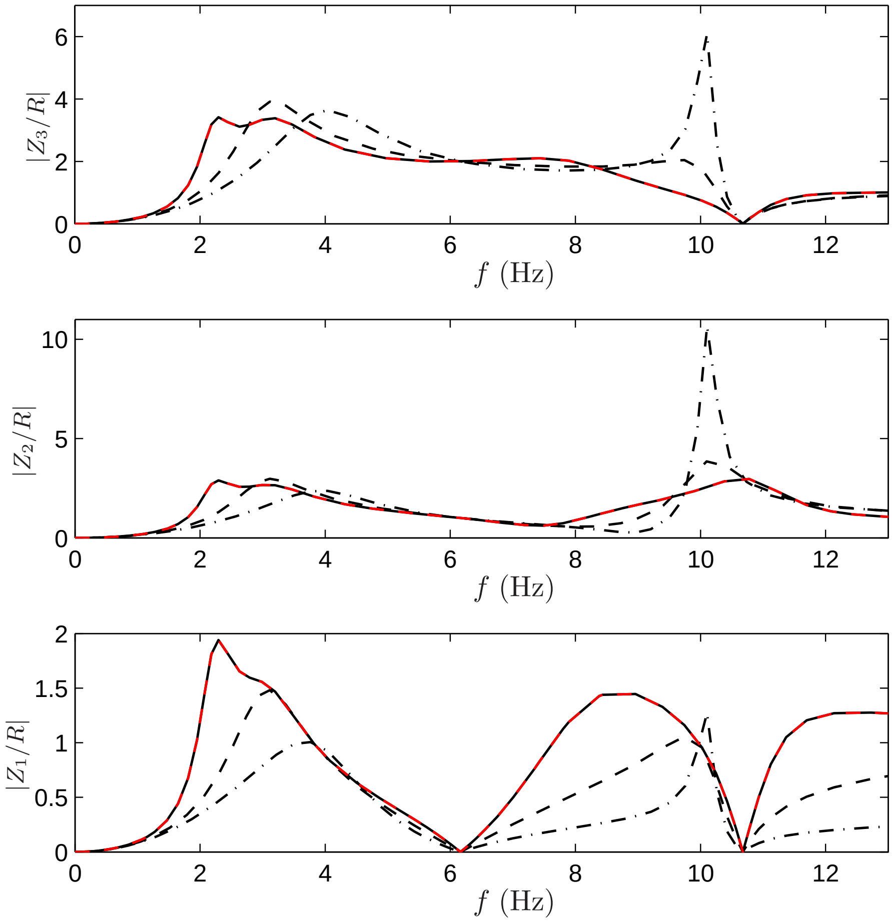

Figure 6: Displacement response comparison: configuration S1 of Figure 3(a) (black dashed), S2 of Figure 3(b) (dash dotted), S3 of Figure 3(c) (black solid) and S4 of Figure 3(d) (red dashed) when $b = 2000$ kg.

was chosen as the maximum displacement of the storeys relative to the base. Four basic absorber configurations were introduced as suppression device. Considering the importance of mass ratio and the difficulty of manufacture, the numerical optimisation of these four configurations were carried out in a specific range of inerter's size. From the optimisation results, the optimum configuration was proposed with respect to the inerter's size. A comparison of the displacement response of the building model with the optimum layout and the other configurations has also been investigated to show the effectiveness of the proposed device. From the building considered here, we find that the TID configuration is beneficial in reducing the cost function beyond that achieved by a pure damper if an inertance greater than 1500 kg is considered. In addition, when using the TID with the inertance above 1500 kg, the size of the damper is significantly smaller than that needed for an optimal pure damper. It also should be noted that using the TID in its corresponding optimal range can not guarantee the smallest displacement of all the storeys because of the objective function used in this paper.

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Council.

References

[1] Smith M C 2002 *IEEE Trans. Autom. Control* **47** 1648–1662
[2] Firestone F 1933 *J. Acoust. Soc. Amer.* **4** 249–267
[3] Papageorgiou C and Smith M C 2006 *IEEE Trans. Control Syst. Technol.* **14** 423–435
[4] Smith M C and Wang F C 2004 *Vehicle Syst. Dyn.* **42** 235–237
[5] Smith M C [19 August,2008] http://www.eng.cam.ac.uk/news/stories/2008/mclaren
[6] Evangelou S, Limebeer D J N, Sharp R S and Smith M C 2006 *IEEE Contr. Syst. Mag.* **26** 78–88
[7] Wang F, Su W and Chen C 2010 *P. I. Mech. Eng. C-J. MEC.* **224** 1650–1616
[8] Lazar I F, Neild S A and Wagg D J 2014 *Earthquake Eng. Struc.* **43** 1129–1147
[9] Wang F C and Liao M K 2010 *Vehicle Syst. Dyn.* **48** 619–643
[10] Jiang J Z, Matamoros-Sanchez A Z, Goodall R M and Smith M C 2011 *Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility* **50** 263–276
[11] Jiang J Z, Matamoros-Sanchez A Z, Zolotas A, Goodall R and Smith M 2013 *P. I. Mech. Eng. F-J. Rai*
[12] Ordonez D, Foti D and Bozzo L 2003 *Earthquake Eng. Struc.* **321** 151–164
[13] Malhotra P K 1997 *Earthquake Eng. Struc.* **265** 515–528
[14] Priestley M J N and Grant D N 2005 *J. Earthquake Eng.* **9** 229–255
[15] Ikago K, Sugimura Y, Saito K and Inoue N 2011 Seimic displacement control of multiple-degree-of-freedom structures using tuned viscious mass dampers *Proc. 8th Int. Conf. Structural Dynamics* (Leuven, Belgium)
[16] Ikago K, Saito K and Inoue N 2012 *Earthquake Eng. Struc.* **41** 453–474
[17] Takewaki I, Murakami S, Yoshitomi S and Tsuji M 2012 *Structural Control and Health Monitoring* **19** 590–608
[18] Soong T T and Dargush G F 1997 *Passive energy dissipation systems in structural engineering* (Chichester, West Sussex, England: John Wiley & Sons)
[19] DenHartog J P 1940 *Mechanical Vibration* (York, PA, USA: McGraw Hill)
[20] Warburton G B 1982 *Earthquake Eng. Struc.* **10** 381–401