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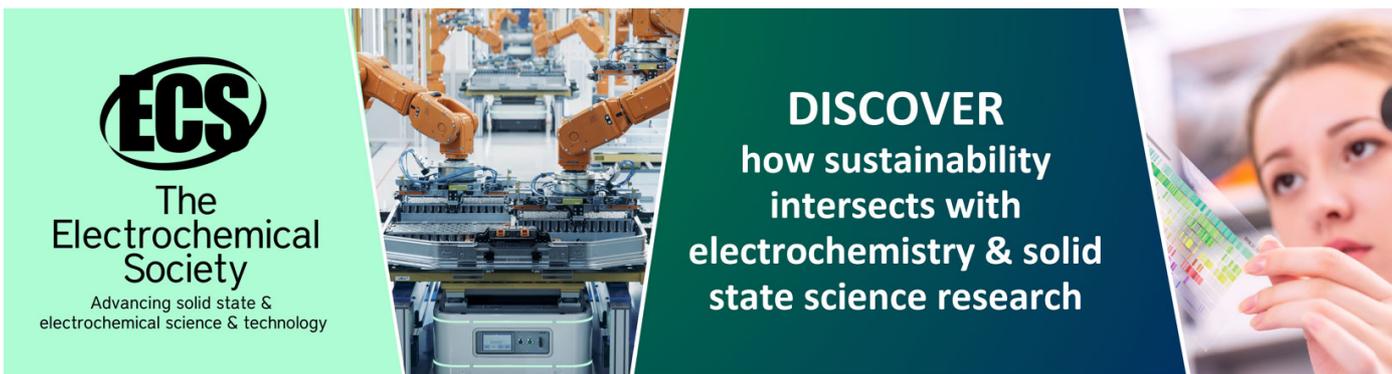
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# Generation of photon pairs through parametric processes in nonlinear waveguides with the account of losses

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**Abstract.** We present an analytical description of the process of spontaneous four-wave mixing in a cubic nonlinear fiber with linear losses. We consider the generation of photon pairs in the fiber when in the input of fiber is fed the pumping wave and single signal photon. The focus of attention is on three cases: when the signal photon propagates in the fiber without generating of biphotons; when the photon pair is generated; and when the photon is lost in the fiber. We also consider the cascade processes, but do not give them an analytical description because of their smallness. Description of the biphotons generation process we provide using the Schrödinger-type equation, and take into account the losses in the fiber through the introduction of the virtual beam splitters. We demonstrate the effectiveness of the generation of photon pairs through parametric processes.

## 1. Introduction

A lot of quantum communication systems, including quantum cryptography [1, 2] and quantum teleportation [3, 4] schemes, are based on the generation of biphoton pairs for the transmission of quantum information. Most modern quantum communication systems are using the effect of spontaneous parametric down-conversion (SPDC) as a source of biphotons [5, 6]. The generation of biphotons through effect of SPDC in  $\chi^{(2)}$ -nonlinear fibers has been described in [7]. However, this effect requires the fibers with the  $\chi^{(2)}$ -nonlinearity, whose crystal lattice has no inversion center [8]. Thus, the production of such fibers is entailed with some difficulties. Therefore, in this paper we consider the generation of photon pairs through the effect of spontaneous four-wave mixing (SFWM), which occurs in standard  $\chi^{(3)}$ -nonlinear fiber-optic communication lines.

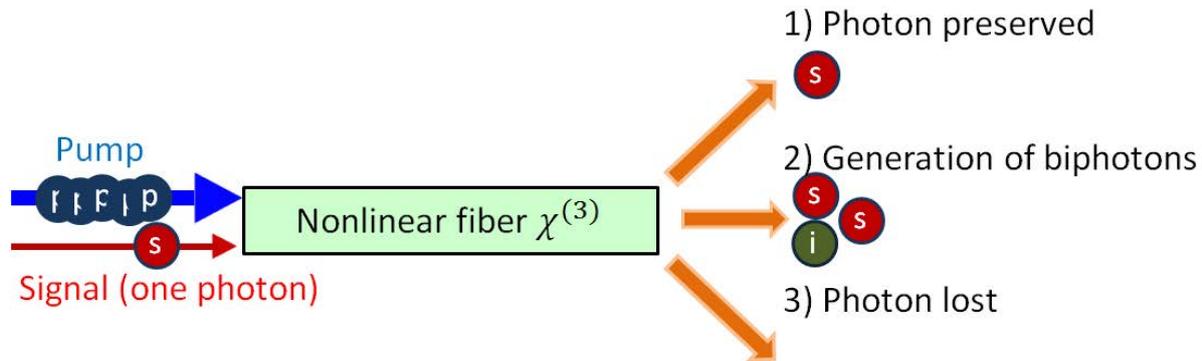
For efficient flow of the SFWM process it is necessary that, along with the pump signal at frequency  $\omega_p$ , in the fiber present single photon of frequency  $\omega_s$  at which we want to generate the signal photons of photon pairs. The frequencies of idler photons  $\omega_i$  of biphotons pairs will be determined from the condition  $2\omega_p = \omega_s + \omega_i$ .

This paper is organized as follows. In Section 2 gives a description of the process of biphotons generation, possible variants of photon propagation in the fiber and scheme of losses. In Section 3 provides an analysis of the obtained equations. Also in this section we describe the mostly effective modes of biphotons generation for different coefficients of losses in the fiber. Summary and conclusions of the work we present in Section 4.



## 2. Process description

The focus of our attention is on three cases (Figure 1): 1. The signal photon propagates in the fiber without generating of biphotons; 2. The photon pair is generated; 3. The photon is lost in the fiber.



**Figure 1.** Possible cases of signal photon propagation through  $\chi^{(3)}$ -nonlinear fiber with losses in presence of pumping.

We consider the following state of a single photon propagating within the fiber:

$$|1\rangle = a_s^\dagger(\omega_s)|0\rangle. \quad (1)$$

We write the Hamiltonians for the case when it is 1 photon in the input, before their action on that photon:

$$\begin{aligned} \hat{H}_{nl}(z) = & \int d\omega_s \beta_s^{(0)}(\omega_s) a_s^\dagger(\omega_s) a_s(\omega_s) \\ & + \int d\omega_i \beta_i^{(0)}(\omega_i) a_i^\dagger(\omega_i) a_i(\omega_i) \\ & + \int d\omega_s \int d\omega_i [\chi^{(3)} E_p^2(z, \omega_s + \omega_i) a_s^\dagger(\omega_s) a_i^\dagger(\omega_i) \\ & + \chi^{(3)} E_p^{*2}(z, \omega_s + \omega_i) a_s(\omega_s) a_i(\omega_i)], \end{aligned} \quad (2)$$

To avoid difficult calculations, we will hold the renormalization:  $\chi(z) \ll 1$  to  $\chi(z) = 1$ , and  $A \ll 1$  to  $A = 1$ . This renormalization will not affect the final results and the dynamics of the process. In fact, we consider a weak-pump regime  $A \ll 1$ , because we are interested in the generation of a single photon pair rather than cascading processes that we neglect due to their smallness.

$$\begin{aligned} \hat{H}_{bs}(z) = & \int d\omega_s \sqrt{2\gamma_s(\omega_s)} [a_s(\omega_s) b_s^\dagger(z, \omega_s) \\ & + a_s^\dagger(\omega_s) b_s(z, \omega_s)] \\ & + \int d\omega_i \sqrt{2\gamma_i(\omega_i)} [a_i(\omega_i) b_i^\dagger(z, \omega_i) \\ & + a_i^\dagger(\omega_i) b_i(z, \omega_i)], \end{aligned} \quad (3)$$

$$\hat{H}(z) = \hat{H}_{nl}(z) + \hat{H}_{bs}(z). \quad (4)$$

Now we write the Hamiltonians after their action on the signal photon:

$$\begin{aligned} \hat{H}_{nl}(z)|1\rangle &= \int d\omega_s \beta_s^{(0)}(\omega_s) a_s^\dagger(\omega_s) a_s(\omega_s) |1\rangle \\ &+ \int d\omega_i \beta_i^{(0)}(\omega_i) a_i^\dagger(\omega_i) a_i(\omega_i) |1\rangle \\ &+ \int d\omega_s \int d\omega_i [\chi^{(3)} E_p^2(z, \omega_s + \omega_i) a_s^\dagger(\omega_s) a_i^\dagger(\omega_i) |1\rangle \\ &\quad + \chi^{(3)} E_p^{*2}(z, \omega_s + \omega_i) a_s(\omega_s) a_i(\omega_i) |1\rangle], \end{aligned} \quad (5)$$

$$\begin{aligned} \hat{H}_{bs}(z)|1\rangle &= \int d\omega_s \sqrt{2\gamma_s(\omega_s)} [a_s(\omega_s) b_s^\dagger(z, \omega_s) |1\rangle \\ &\quad + a_s^\dagger(\omega_s) b_s(z, \omega_s) |1\rangle] \\ &+ \int d\omega_i \sqrt{2\gamma_i(\omega_i)} [a_i(\omega_i) b_i^\dagger(z, \omega_i) |1\rangle \\ &\quad + a_i^\dagger(\omega_i) b_i(z, \omega_i) |1\rangle], \end{aligned} \quad (6)$$

$$\hat{H}(z)|1\rangle = \hat{H}_{nl}(z)|1\rangle + \hat{H}_{bs}(z)|1\rangle. \quad (7)$$

Now we write the Eqs.(5)-(7) in the simplified form, taking into account the standart properties of the operators ( $a^\dagger a |n\rangle = n|n\rangle$ ;  $a|0\rangle = 0$ ;  $a|n\rangle = n^{1/2}|n-1\rangle$ ;  $a^\dagger|n\rangle = (n+1)^{1/2}|n+1\rangle$ ) [9]:

$$\begin{aligned} \hat{H}_{nl}(z) |1\rangle &= \int d\omega_s \beta_s^{(0)}(\omega_s) |1\rangle \\ &+ \int d\omega_s \int d\omega_i [\chi^{(3)} E_p^2(z, \omega_s + \omega_i) a_s^\dagger(\omega_s) a_i^\dagger(\omega_i) |1\rangle], \end{aligned} \quad (8)$$

$$\begin{aligned} \hat{H}_{bs}(z)|1\rangle &= \int d\omega_s \sqrt{2\gamma_s(\omega_s)} [a_s(\omega_s) b_s^\dagger(z, \omega_s) |1\rangle \\ &\quad + a_s^\dagger(\omega_s) b_s(z, \omega_s) |1\rangle] \\ &+ \int d\omega_i \sqrt{2\gamma_i(\omega_i)} [a_i(\omega_i) b_i^\dagger(z, \omega_i) |1\rangle], \end{aligned} \quad (9)$$

We seek a solution for a given state at a distance of  $z$  as (we assume that the losses are weak):

$$\begin{aligned} |\Psi(z)\rangle &= \Phi_1(z) a_s^\dagger |0\rangle + \Phi_2(z) a_i^\dagger a_s^\dagger |0\rangle \\ &+ \int_0^z dz_l \tilde{\Phi}_3^{(i)}(z, z_l) b_s^\dagger(z_l) |0\rangle \\ &+ \int_0^z dz_l \tilde{\Phi}_4^{(i)}(z, z_l) a_i^\dagger b_s^\dagger(z_l) a_s^\dagger |0\rangle \\ &+ \int_0^z dz_l \tilde{\Phi}_5^{(s)}(z, z_l) b_i^\dagger(z_l) a_s^\dagger a_s^\dagger |0\rangle \\ &+ \dots \end{aligned} \quad (10)$$

Here we neglect terms of higher order, to avoid cumbersome writing formulas. However, we take into account that in the fiber both photons may be lost.

In our paper  $E_p(z, \omega_p)$  - is a function proportional to the pump amplitude  $A$  at the frequency  $\omega_p$  and inversely proportional to the cubic nonlinearity  $\chi^{(3)}$ . Also,  $E_p(z, \omega_p)$  is normalized to the attenuation with the loss coefficient  $\gamma_p$ :

$$\chi^{(3)} E_p^2(z, \omega_p) = A^2 e^{-2\gamma_p z}. \quad (11)$$

According to our notation all the propagation constants  $\beta$  is calculated with respect to the propagation constant of the pump  $\beta_p$ . Therefore, the physical meaning of  $\Delta\beta = \beta_s + \beta_i$  - is a mismatch of nonlinear SFWM and term  $-2\beta_p$  is contained in  $\Delta\beta$ .

In our case the evolution of the state vector in assumptions describes by approximate Schrödinger equation for the biphoton wavefunction:  $d\Psi(z)/dz \approx -i\hat{H}(z)(|0\rangle + |\Psi(z)\rangle)$ .

After the necessary calculations, we obtain the following equations for the evolution of biphoton wave functions with account of losses:

$$\begin{aligned} \partial\Phi_1(z)\partial z &= -(i\beta_s + \gamma_s)\Phi_1(z) \\ &- i2A^2 e^{-2\gamma_p z} \Phi_2(z), \Phi_1(0) = 1, \end{aligned} \quad (12)$$

$$\begin{aligned} \partial\Phi_2(z)\partial z &= -(i(2\beta_s + \beta_i) + 2\gamma_s + \gamma_i)\Phi_2(z) \\ &- iA^2 e^{-2\gamma_p z} \Phi_1(z), \Phi_2(0) = 0, \end{aligned} \quad (13)$$

$$\begin{aligned} \partial\tilde{\Phi}_3^{(i)}(z, z_l)\partial z &= 0, z \geq z_l, \\ \tilde{\Phi}_3^{(i)}(z_l, z_l) &= -i\sqrt{2\gamma_s}\Phi_1(z_l), \end{aligned} \quad (14)$$

$$\begin{aligned} \partial\tilde{\Phi}_4^{(i)}(z, z_l)\partial z &= -(i(\beta_s + \beta_i) + \gamma_s + \gamma_i)\tilde{\Phi}_4^{(i)}(z, z_l), \\ z \geq z_l, 8mu\tilde{\Phi}_4^{(i)}(z_l, z_l) &= -i\sqrt{2\gamma_s}\Phi_2(z_l), \end{aligned} \quad (15)$$

$$\begin{aligned} \partial\tilde{\Phi}_5^{(s)}(z, z_l)\partial z &= -(i2\beta_s + 2\gamma_s)\tilde{\Phi}_5^{(s)}(z, z_l), \\ z \geq z_l, 8mu\tilde{\Phi}_5^{(s)}(z_l, z_l) &= -i\sqrt{2\gamma_i}\Phi_2(z_l). \end{aligned} \quad (16)$$

Here we consider the pump in the form of Eq.(11).

### 3. An analysis of equations

Of particular interest to us are the Eqs.(12)-(14), because values  $|\Phi_{1,2}(z)|^2$  and  $\int_0^z |\Phi_3(z)|^2 dz$  represents a probability of density of three considered cases.

Equations (12)-(13) can be solved analytically when  $\gamma_p = 0$  and  $\Delta\beta^2 \neq -8A^4$ , where  $\Delta\beta = \beta_s + \beta_i$ :

$$\Phi_1(z) = e^{ik_0 z} \left[ \cos(\Delta k z) + \frac{\tilde{\beta}_s + \tilde{\beta}_i}{2\Delta k} i \sin(\Delta k z) \right], \quad (17)$$

$$\Phi_2(z) = -i \frac{A^2}{\Delta k} e^{ik_0 z} \sin(\Delta k z), \quad (18)$$

$$k_{1,2} = k_0 \pm \Delta k, \quad (19)$$

$$k_0 = \frac{-(3\tilde{\beta}_s + \tilde{\beta}_i)}{2}, \quad (20)$$

$$\tilde{\beta}_s = \beta_s - i\gamma_s, \tilde{\beta}_i = \beta_i - i\gamma_i, \quad (21)$$

$$\Delta k = \frac{1}{2} \sqrt{(\tilde{\beta}_s + \tilde{\beta}_i)^2 + 8A^4}. \quad (22)$$

In the absence of losses, when  $\gamma_s = \gamma_i = 0$ , we obtain  $k_{1,2} = \frac{1}{2}(-\beta_3 \pm \sqrt{8A^4 + \Delta\beta^2})$ , where  $\beta_3 = 3\beta_s + \beta_i$ ,  $\Delta\beta = \beta_s + \beta_i$ . Thus, if the condition  $\beta_3^2 = 8A^4 + \Delta\beta^2$  is satisfied, propagation constant  $k_1$  is zero.

When  $\gamma_p \neq 0$  the equations (12)-(13) can not be solved analytically in terms of elementary functions, so we will solve them by perturbation theory, assuming that  $A^2$  is small:

$$\Phi_1(z) = e^{-i\tilde{\beta}_s z}, \quad (23)$$

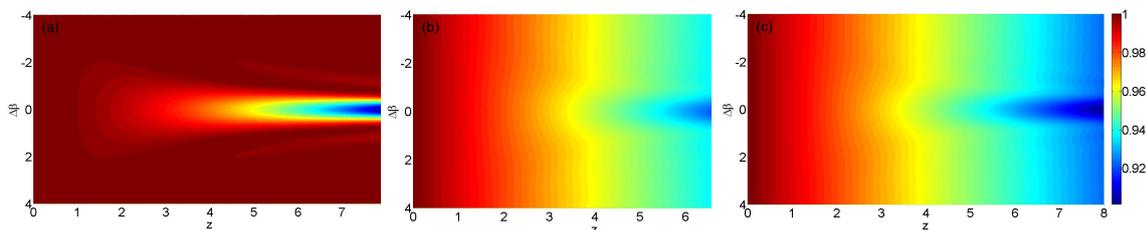
$$\Phi_2(z) = \frac{-2iA^2}{\tilde{\beta}_s + \tilde{\beta}_i + i2\gamma_p} \sin\left[\frac{z}{2}(\tilde{\beta}_s + \tilde{\beta}_i + i2\gamma_p)\right] \times e^{ik_0 z} e^{-\gamma_p z}, \quad (24)$$

$$\tilde{\beta}_s = \beta_s - i\gamma_s, \quad (25)$$

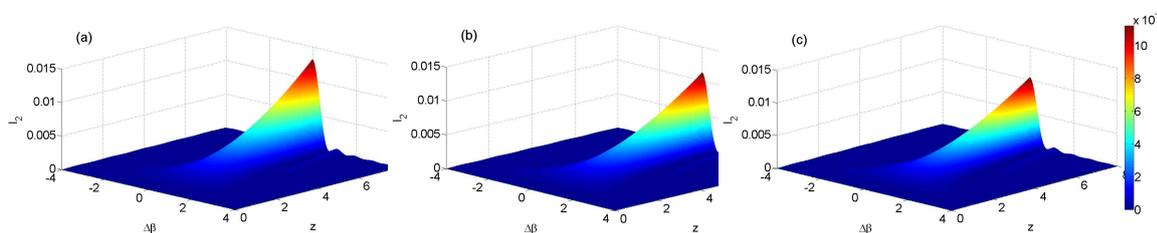
$$\tilde{\beta}_i = \beta_i - i\gamma_i, \quad (26)$$

$$k_0 = \frac{-(3\tilde{\beta}_s + \tilde{\beta}_i)}{2}. \quad (27)$$

In creating the figures, we calculate the normalized intensity of the photons generated by the SFWM effect. The intensity  $I_1(z) = |\Phi_1(z)|^2$  is equal to the average number of signal photons without generating photon pairs that can be detected at the coordinate  $z$ , per unit time. The intensity  $I_2(z) = |\Phi_2(z)|^2$  is equal to the average number of sets of three photons (signal photon + photon pair) at coordinate  $z$  per unit time.



**Figure 2.** Normalized number of signal photons  $I_1(z)$  without generating of biphotons in a single waveguide vs the phase mismatch  $\Delta\beta$  and propagation distance  $z$  for  $A = 0.12$ , and different losses: (a)  $\gamma_p = \gamma_s = \gamma_i = 0$ , (b)  $\gamma_p = 0, \gamma_s = \gamma_i = 0.005$ , (c)  $\gamma_p = \gamma_s = \gamma_i = 0.005$ .



**Figure 3.** Normalized number of signal photons with biphotons generated through SFWM  $I_2(z)$  in a single waveguide vs the phase mismatch  $\Delta\beta$  and propagation distance  $z$  for  $A = 0.12$ , and different losses: (a)  $\gamma_p = \gamma_s = \gamma_i = 0$ , (b)  $\gamma_p = 0, \gamma_s = \gamma_i = 0.005$ , (c)  $\gamma_p = \gamma_s = \gamma_i = 0.005$ .

First of all, we have to provide general analyze the dependence of the intensities  $I_1$  and  $I_2$  of coordinate  $z$  and phase mismatch  $\Delta\beta$ . This dependences are presented at [Figure 2 and Figure 3]: for different values of the loss coefficients. On both figures: a).  $\gamma_p = \gamma_s = \gamma_i = 0$

b)  $\gamma_p = 0, \gamma_s = \gamma_i = 0.005$ , a)  $\gamma_p = \gamma_s = \gamma_i = 0.005$ . Figure 2 is the above view to  $I_1(z, \Delta\beta)$ . Figure 3 is the side view to  $I_2(z, \Delta\beta)$ .

Note that the expressions for  $\tilde{\Phi}_3^{(i)}(z, z_l)$ ,  $\tilde{\Phi}_4^{(i)}(z, z_l)$  and  $\tilde{\Phi}_5^{(s)}(z, z_l)$  can be easily obtained from the Eqs.(14)-(16).

#### 4. Conclusion

In this paper we have performed the analysis of the effect of linear losses on SFWM in cubic nonlinear fiber, considering in detail states of single-photon, triple-photon (single-photon + biphoton) and without photon outputs under a different distances and phase mismatching. In particular, we provide a detail analysis of a case with small losses by perturbation theory. We also demonstrate the effectiveness of the generation of photon pairs through parametric processes.

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