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Mathematical modeling and validation of the carburizing of low carbon steels

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Abstract. This paper shows the mathematical modeling of heat and mass transfer in transient state of cylindrical bars of low carbon steel subjected to carburizing process. The model solution for the two phenomena was performed using a one-dimensional analysis in the radius direction, using the numerical method of finite differences; also a sensitivity analysis by varying the coefficient of convective heat transfer (h) is performed. The modeling results show that this carburization steel is strongly dependent on h . These results suggest that if it can increase the value of h in this kind of process could reduce the time of process for this heat treatment. Additionally, an experimental procedure was established by carburization of a steel AISI SAE 1010, which develops cementing solid phase and the specimen steel and micrographic hardness profiles obtained from samples of the specimen analysis was performed, to determine the penetration depth of the carbon and validate this result over the values obtained by the computer model.

1. Introduction

This paper presents the mathematical model of the carburizing at $1173K$ over a cylindrical bar made of AISI SAE 1010. This model is made based on the coupling phenomena of heat and mass transfer, on the other hand an experimental validation of the results obtained in the model is made. The results of the model and the experiment show great similarities in concentrations for different carburizing times, also a sensitivity analysis is performed by varying the coefficient of convective heat transfer (h) getting as a result that this process is strongly dependent on this parameter.

2. Mathematical modeling

The mathematical model is taking into account the phenomena of heat and mass transfer. In order to solve this, the finite difference numerical method, specifically implicit method in the radial direction of the bar is used [1–4].

2.1. Thermal modeling

To solve the thermal model, the physical properties like thermal conductivity (K) and specific heat (C_p) were taken as variables and temperature dependent [5, 6]. It is proceeded to propose the general equation for heat transfer with variables properties:

$$\frac{1}{r} \left(\frac{\partial}{\partial r} kr \frac{\partial T}{\partial r} \right) = \rho C_p \frac{\partial T}{\partial t} \quad (1)$$



2.1.1. Boundary conditions As a one-dimensional analysis is performed, the model needs two boundary conditions, these are in the center and the outer surface of the bar. For the center, isolated border mirror image is used.

$$\frac{\partial}{\partial r} k \frac{\partial T}{\partial r} + k \frac{\partial^2 T}{\partial r^2} = \rho C_p \frac{\partial T}{\partial t} \quad (2)$$

Furthermore, the surface has a convection boundary condition equal to the conductive heat transfer in the bar [7].

$$-k \frac{\partial T(r, t)}{\partial r} = h(T_\infty - T(r, t)) \quad (3)$$

2.1.2. Initial condition The heat transfer begins with an initial uniform temperature across the bar setted as 293K and for this case h is fixed in 20W/m²K.

$$T_0 = 293K \quad \forall \quad (r, 0) \quad (4)$$

2.2. Mass transfer

Mass transfer is described by Fick's second law, with variable diffusion coefficient in function of the temperature [8]. The equation that describes this process is:

$$\frac{1}{r} \left(\frac{\partial}{\partial r} D r \frac{\partial \rho_c}{\partial r} \right) = \frac{\partial \rho_c}{\partial t} \quad (5)$$

In terms of the mass fraction of carbon.

$$\frac{1}{r} \left(\frac{\partial}{\partial r} D r \frac{\partial w_c}{\partial r} \right) = \frac{\partial w_c}{\partial t} \quad (6)$$

2.2.1. Boundary conditions There are two boundary conditions: these are in the center and the outer surface of the bar. For the center, isolated border mirror image is used.

$$\frac{\partial}{\partial r} D \frac{\partial w_c}{\partial r} + D \frac{\partial^2 w_c}{\partial r^2} = \frac{\partial w_c}{\partial t} \quad (7)$$

On the surface, the initial condition is similar to the heat transfer, mass diffusion in the bar is equals to the mass transfer convection boundary.

$$-D \frac{\partial w_c(r, t)}{\partial r} = h_m(w_\infty - w_c(r, t)) \quad (8)$$

Where h_m is convection mass transfer coefficient, which is obtained maching the Nusselt and Sherwood numbers.

2.2.2. Initial condition The initial condition for the mass transfer is an uniform carbon concentration in the bar at time zero:

$$w_c = 0.1\% \quad \forall \quad (r, 0) \quad (9)$$

3. Experimental validation

The experimental validation of the model was performed with samples of steel AISI SAE 1010, of 12.7mm diameter, which were subjected to a carburizing process carried out with mineral carbon at a temperature of 1173K for three different times (18000, 21600 and 25200s). With the carburizing made, it is proceeded to polish the samples, which are attacked with a reagent that highlights the case-hardened layer. The reagent used was Nital 0.2%. Subsequently it was measured in the microscope the depth of this layer using a graduated ocular. Metallographic study was conducted in order to supplement the data obtained through the microhardness test that was done on the cross-sectional area of the specimen. The procedure for making microhardness was according to the standard ASTM E92, ASTM E384 and ISO R81 for hardness vickers (HV) [9].

4. Results

The results of microhardness and depth of case-hardened layer for different test times are presented in Table 1. The HV decreases with depth of the case-hardened layer.

Table 1. HV and hardness rockwell C (HRC) of the specimens at different depth.

Time test [s]	0.2 [mm]	0.4 [mm]	0.6 [mm]	0.8 [mm]	depth [mm]	w_c [%]
18000	827-62	802-61	778-60	766-60	1.1	0.92
21600	850-63	812-62	795-61	778-60	1.35	1.05
25200	884-64	820-62	800-62	780-60	1.5	1.1

In Figure 1, the variation of the carbon concentration is observed at different carburizing times with respect to the radius of the specimen obtained with the mathematical model. In this Figure a match with the experimental measurements shown in Table 1 was observed [10,11].

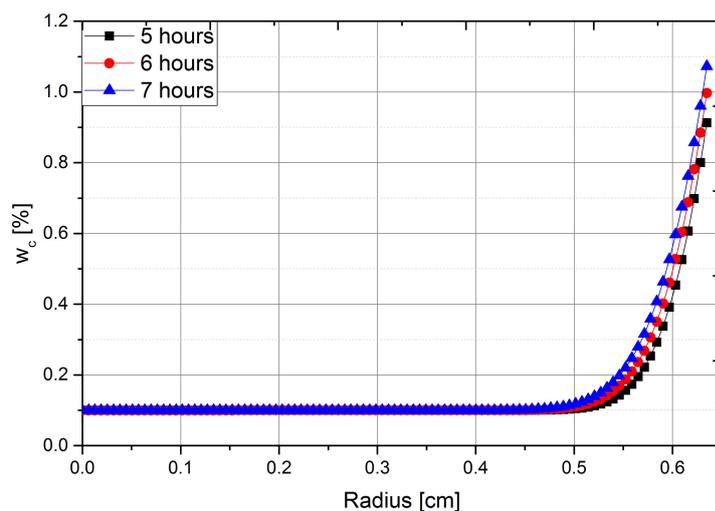


Figure 1. variation of the carbon concentration at different times carburizing.

In Figure 2 are shown results of the mathematical model varying h . These results show that carburizing process improves with increasing this parameter.

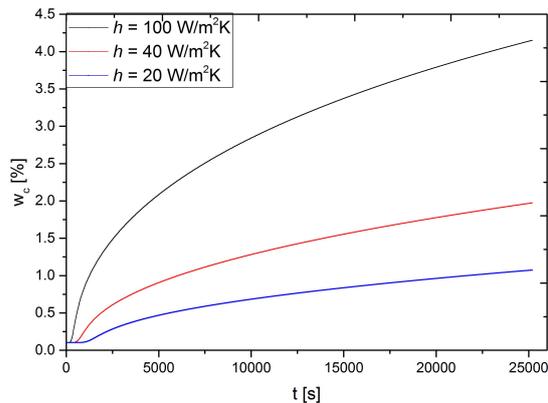


Figure 2. Behavior of the carbon concentration on the surface for different heat transfer coefficients.

In Figure 3, it is observed the difference in the carbon concentration at the surface between the mathematical model and experimental validation.

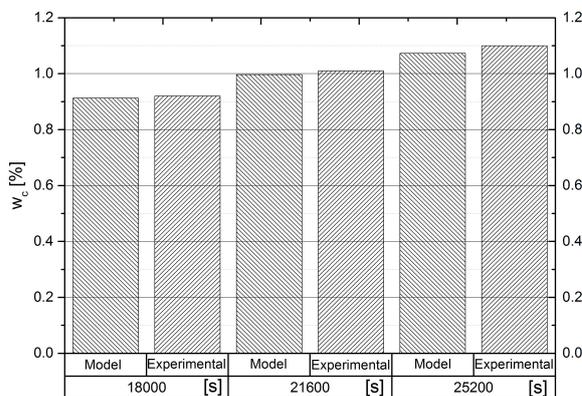


Figure 3. Variation of the carbon concentration in the surface between the mathematical model and experimental validation.

5. Conclusions

The results from the mathematical model show that the carburizing process is strongly dependent on the coefficient of convective heat transfer h . In Figure 2, it can be observed that an increase of 100% of this parameter could reduce the process time by 6 and it could be seen in cost reduction. The mathematical model applied for each carburizing time has a percentage relative error (E_t) less than 2% for concentrations on the surface, as shown in Figure 3. For this reason, the proposed mathematical model describes accurately the behavior of mass transfer and heat transfer occurring in the carburizing process.

6. References

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