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Mathematical-statistical model for predicting the tendency of the mechanical integrity of a buried pipeline

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Mathematical-statistical model for predicting the tendency of the mechanical integrity of a buried pipeline

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Abstract. This paper is intended to use a process model stochastic renewal to analyze the disintegration of the pipeline due to the time of use and service conditions, classifying the pipeline into segments according to the conditions to which it is exposed (temperatures, pressures, speed flow, environmental conditions, etc.). The model will determine the material wear over time, periods of renewal of the material and life chances at any time.

1. Introduction

The transport of dangerous pipeline fluids are exposed to dependent and no dependent threats of time. The deterioration of pipeline with time is inevitable despite different coatings and protections that help extend the life of the material used, but eventually require renovation or change [1]. The disintegration pipeline thickness depends on many variables (temperature, pressure, flow rate, environmental conditions, etc.) that are not easy to relate all at once [2]. This model is intended to apply to predict the wear of the material, refresh rates and probabilities of life at any time t.

2. Wear equation

There are several factors that determine the pipeline desintegration, then it is assumed that the rate of change of the thickness of the pipe wall is proportional to $E(t)$ the remaining material in time t [3]. Mathematically it can be expressed as:

$$\frac{dE}{dt} = -\lambda E \tag{1}$$

Where $E(t) = 1 - d(t)/h$ represents the remaining percentage of the thickness of the pipe wall in a time t, $\lambda > 0$, a proportionality constant having wear duct over time and conditions thereof, the minus sign represents the disintegration suffered reducing material thickness with time. The Figure 1 represents: Pit length (Lp), maximum pit depth at a time (d), nominal wall thickness of the pipe (h), and nominal outside diameter of the pipe (D).

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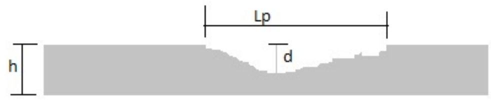

Figure 1. Corrosion parameters.

ASME B31 [4] sets a maximum length (L) of permissible sting, calculated as:

$$L = 1.12 * \sqrt{\left((\frac{\frac{d}{h}}{1.1\frac{d}{h} - 0.15})^2 - 1 \right) * D * h} \tag{2}$$

If $Lp > L$ is considered the critical pitting and suggests renew the material, then the solution of equation (Equation 1) as a function of driveability and maximum lengths to predict areas as required by the ASME B31 [4] is taken and possibly the probability of life with respect to time t in a process of renewal.

Solving the equation (1) is $f(t) = E(t) = C_0 \exp(-\lambda t), t \geq 0$, where $C0 = 1$, representing the 100% the thickness of the pipe wall in a time $t = 0$, when $d(t = 0) = 0$. Therefore

$$\frac{d(t)}{h} = 1 - E(t) = 1 - \exp(-\lambda t) \tag{3}$$

The corrosive maximum length allowed in a time t, according to the above equation and the standard ASME B31 [4] is given by $L_{max} = 1.12 * \sqrt{\left[(\frac{1-\exp(-\lambda t)}{1.1(1-\exp(-\lambda t))-0.15})^2 - 1 \right] * D * h}$, the maximum permitted corrosive area at a time t is $A_{max}(t) = 0.893\frac{L_{max}(t)}{\sqrt{Dh}}$ and the pressure maximum (P') is $P'(t) = 1.1P\exp(-\lambda t)$, where P is the value of the relationship established by maximum allowable pressure eleoducto.

3. Renewal processes

The pipeline has a lifespan that could be modeled by a random variable T. Once the pipeline fails or is in poor conditions of service is removed or restructured. This process generates a collection of random variables $T_1, T_2, T_3...$, positive and independent representing the succession of pipeline lifetimes put into operation. It is clear that whenever revive restructuring process probabilistically.

be $Q_n = T_1 + T_2 + T_3...T_n, Q_0 = 0$ representing the time of the n-th renewal and N_t as the number of renewals in time t [5, 6].

The density function of the form is considered $f(t) = \exp(-\lambda t)$ y $F(t) = \int_0^t f(s)ds$ [6, 7] representing a distribution function of the lifetimes of the pipeline.

The distribution function of N_t depends on $F(t)$, as indicated by the equation $P(N_t = n) = \exp(-\lambda t)\frac{(\lambda t)^n}{n!}$ [5].

The number of renewals that are made to the pipeline during a given time t, then defined $\Lambda(t) = E(N_t)$ (Expected value of the number of renewals at a time t) renewal and function that satisfies the following proposition.

Proposition 1 *Renewal function $\Lambda(t)$ satisfies the Equation 1.*

$$\Lambda(t) = F(t) + \int_0^t \Lambda(t - s)dF(s) \tag{4}$$

Where $\Lambda(t) = \lambda t$ and $F(t) = 1 - \exp(-\lambda t)$ satisfies the Equation 4.

3.1. Lifetimes
For this process three non-negative random variables defined: γ_t Remaining lifetime of the tube, δ_t Time elapsed tube life and β_t The total lifetime, as it is shown in Figure 2. Time remaining life is: $\gamma_t = Q_{N_{t+1}} - t$ (see Figure 2).

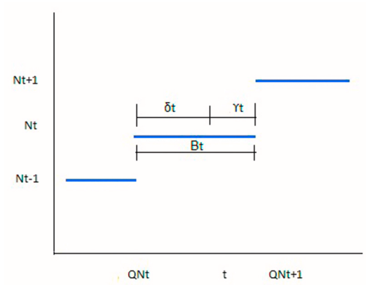

Figure 2. Graphic of useful life [5].

Proposition 2 *For $x > 0$, the function $h(t) = P(\gamma_t > x) = 1 - P(\gamma_t < x)$ satisfies the renewal Equation 1.*

$$h(t) = 1 - F(t + x) + \int_0^t h(t - s)ds \quad for \quad x > 0 \tag{5}$$

Where $h(t) = P(\gamma_t > x) = \exp(-\lambda x)$, satisfies the Equation 4 of the previous proposition. Lifetime elapsed is: $\delta_t = t - Q_{N_t}$ [5].

Proposition 3 *For $t \in (x, \infty)$, the function $g(t) = P(\delta_t > x) = 1 - P(\delta_t < x)$ satisfies the Equation 1.*

$$g(t) = 1 - F(t) + \int_0^{t-x} g(t - s)dF(s) \tag{6}$$

Where $P(\delta_t < x) = 1 - \exp(-\lambda x)$ for $0 < x < t$; if $x > t$ then $P(\delta_t < x) = 1$ and the total lifetime is (see Figure 2): $\beta_t = \gamma_t + \delta_t$ [5].

Proposition 4 *The function $g(t) = P(\beta_t > x) = 1 - P(\beta_t < x)$ renewal satisfies the Equation 1.*

$$g(t) = 1 - F(max\{t, x\}) + \int_0^t g(t - s)dF(s) \quad for \quad x > 0 \tag{7}$$

Where the total lifetime has the following distribution function; $P(\beta_t < x) = 1 - \lambda(1 + min\{t, x\}) \exp(-\lambda x)$ [5].

4. Result
The data of Table 1 is used to apply the equations discussed above (see Equations 1-7).

Table 1. To apply the model data.

Specifications	Measures	Specifications	Measures
Renewal Time	4 años	Pit depth	0.0405 in
Diameter	16 in	Pipe wall thickness	0.312 in
Length sting	0.3386 in	Design pressure	1684 in

Table 2 shows the predicted depth, length and pressure versus time t allowed by ASME.

Table 2. Prediction depth, length and maximum pressure allowed by ASME at time t.

Time (years)	Depth %	Length sting (in)	Pressure (psi)
4	0.130000011	46.40565441	1611.58798
5	0.159768307	15.32631443	1556.445188
7	0.216284056	5.624921409	1451.755416
9	0.268998436	3.876184499	1354.107297
12	0.341497025	2.84263713	1219.810911
15	0.406805417	2.334069374	1098.833646
20	0.501579111	1.87063979	923.274855
25	0.581210972	1.603842059	775.7647947

Table 3 shows the reliability of service life remaining, elapsed time and total for a t.

Table 3. Prediction of reliability (probability) of life.

Time (years)	$\Lambda(t)$	$P(\gamma_t > x)$	$P(\delta_t < x)$	$P(\beta_t < x)$
4	0.139262068	0.869999999	0.130000001	0.977369912
5	0.174077585	0.840231706	0.159768294	0.966625503
7	0.243708619	0.783715961	0.216284039	0.93975967
9	0.313339653	0.731001584	0.268998416	0.906346803
12	0.417786204	0.658502999	0.341497001	0.845437856
15	0.522232755	0.59319461	0.40680539	0.773389741
20	0.69631034	0.498420919	0.501579081	0.633282533

5. Conclusions

This model predicts the percentage of pit depth, lengths, pressures allowed by ASME and likely lifespan of the pipeline at a time t.

These equations can generate a permanent monitoring with respect to the likelihood of failure of transmission lines mitigating risk in the pipeline.

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