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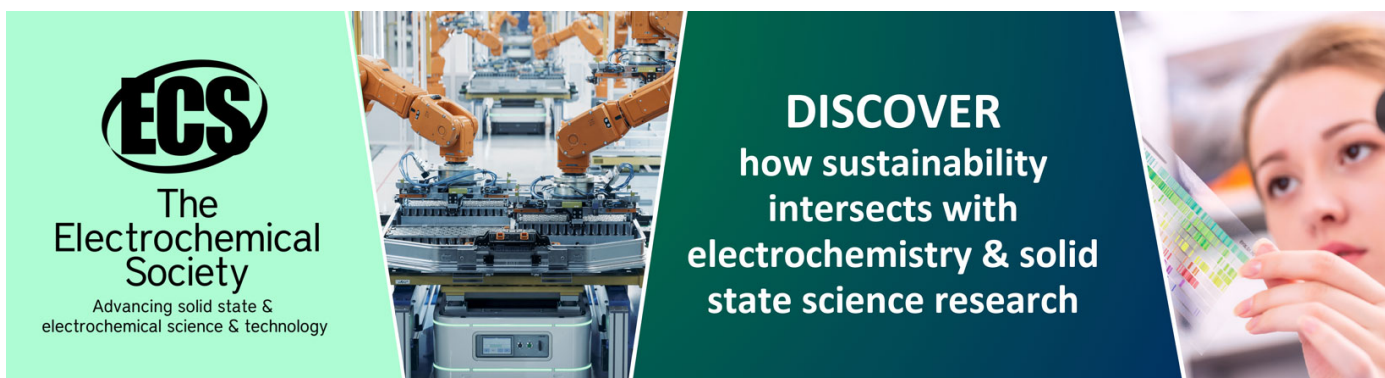
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Lumped Capacitance Model in Thermal Analysis of Solid Materials

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Abstract. The paper is devoted to the presentation of a method for measurement of thermal conductivity k , specific heat capacity c_p and thermal diffusivity α applying the lumped capacitance model (LCM) as a special case of Newton's model of cooling. At the specific experimental conditions resulting from the theoretical analysis of the used model, we present relatively very precise method for experimental determination of all three above mentioned thermal parameters for materials with different thermal transport properties. The input experimental data provide a cooling curve of the tested material obtained in special experimental arrangement. The evaluation of experimental data is realized by software the fundamental features of which are presented here. The statistical analysis of experimental data was performed (99% confidence interval P_{99}).

1. Introduction

The cooling of objects is often described by a law, attributed to Newton, which states that the temperature difference of a cooling body with respect to the surroundings decreases exponentially with time. Such behaviour has been observed during many laboratory experiments, which led to a wide acceptance of this approach. However, the heat transfer from any object to its surrounding is not only due to conduction and convection but also due to radiation. The latter does not vary linearly with temperature difference, which leads to deviations from Newton's law. A discussion of this topic is provided in work [1, 2, 3].

Specific properties of materials demand a large spectrum of measuring methods suitable for thermal parameters measurements. Some of them are based on a photoacoustic effect and they are described in [4].

Another group of experiments is based on the application of thermal impulse on the sample surface and its propagation through a sample. These impulse applications either have contact of the heat source with the sample or they are in a contactless (flash) mode. The fundamental theoretical work in this sense is [5]. The so-called flash method has a large spectrum of realizations [6].

Different contact impulse methods with different position of the heat source were described in [7]. The heat sources in this case usually have a meander shape. Heat generation in the measured sample can also be realized by mechanical stress. The sample is pulled in a tensile test machine and the generated heat is caused by deformation of its structure. The temperature decay of the pre-pulled sample is measured by a thermovision camera. This measuring system allows measuring specific heat capacity,



thermal diffusivity and thermal conductivity as well as the mechanical parameters usually obtained from a tensile test [8].

In this paper, we present a method for measurement of thermal conductivity k ($\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$), specific heat capacity c_p ($\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$) and thermal diffusivity α ($\text{m}^2\cdot\text{s}^{-1}$) applying the lumped capacitance model as a special form of Newton's model of cooling. Used method offer relatively reliable results and has a very good repeatability. Obtained results are statistically treated and they are also compared with known table values or results obtained on independent measurements.

2. Theory

For a heat flow from a sample to an environment with temperature T_∞ Newton's cooling law can be written in the form [9,10]

$$\frac{dQ}{dt} = -h_t \cdot S \cdot (T - T_\infty) \quad (1)$$

where Q is the heat given to a sample, h_t is the total heat transfer coefficient of a sample, S is the total heat flow area, T_∞ is the environment temperature.

For the lumped capacitance method solution of surface temperature T can be found from differential equation

$$m \cdot c_p \frac{dT}{dt} = -h_t \cdot S \cdot (T - T_\infty) \quad (2)$$

further

$$m = \rho \cdot V = \rho \cdot S \cdot L \quad (3)$$

where ρ is density of the sample, L is sample thickness and S is the effectively cooled surface.

In our case the sample is cooled on both side and so we can write

$$S^* = S_{\text{top}} + S_{\text{lower}} = 2S \quad (4)$$

After integration of equation 2 we subsequently obtain

$$\left[\rho S L c_p \ln(T - T_\infty) \right]_0^t = [-2hSt]_0^t \quad (5)$$

If we set

$$\tau = \frac{\rho \cdot c_p \cdot L}{2 \cdot h_t} \quad (6)$$

then

$$\left[\ln(T - T_\infty) \right]_0^t = -t/\tau \quad (7)$$

and finally for master function one can write

$$T = T_\infty + (T_0 - T_\infty) \exp(-t/\tau) \quad (8)$$

Relation among k , c_p and α has the form [10]

$$k = \alpha \cdot \rho \cdot c_p \quad (9)$$

where k is sample thermal conductivity, α is diffusivity, c_p is specific heat capacity and ρ is the sample density.

Validity of the model is verified by the Biot number Bi in the form

$$Bi = \frac{h_t L}{k} \ll 1 \quad (10)$$

where

$$h_t = h_c + h_r \quad (11)$$

represents so called combined heat transfer coefficient through convection with the coefficient of h_c and radiation with the coefficient of h_r .

The analysis of simulation results has shown that the radiation heat transfer coefficient observed at the temperature range of 25-26 °C is practically a linear function of temperature with increasing trend. However, the value of h_r coefficient within the monitored temperature interval does not change by more than 0.5%, which is why, taking into account the approximate 5% model accuracy, it can be approximated by its arithmetic average or median, which is less sensitive to deviation and extreme values.

The comparison shows that even at room temperatures and relatively very little temperature differences between the cooling body and the surroundings, the value of radiation heat flow density is considerably high. Radiation heat flow in the given case has an average of more than 23.9% share on the total heat discharge from the surface of the body. That is why the description of the cooling process must usually take into account both convective and radiation heat transfer mechanism, and it must take into account the combined nature of the thermal interactions of the body with the surroundings, which more accurately corresponds to the macroscopic description of the heat exchange processes than during the application of a simple exponential model of the first order which neglects radiation.

With increasing temperature differences between the cooling body and its surroundings, the possibility of approximation of the radiation heat transfer coefficient using a constant value is definitively lost. At higher temperatures and temperature differences, the temperature dependence of the convective heat transfer coefficient begins to show [9].

Higher temperatures can also show thermal dependence of other physical parameters concentrated in the relaxation time, thus losing the character of a constant value.

3. Experimental procedure and results

The measured samples must have an area of about $(10 \times 10 \times 2) \text{ mm}^3$ and must be finely ground. Matt black spray-paint is applied on all sides of the samples in order to ensure they have the same emissivity. Every sample has been measured ten times; standard deviation as well as the values of 99 percent confidence intervals (P99) were calculated [11].

The equipment consists of a thermally insulated measuring chamber [12]. The cover of the chamber has an opening for a pyro-electric sensor Raytek THERMALERT MID 02. The sample is heated above room temperature in a thermostat ($\sim 40^\circ\text{C}$). After removal from the thermostat, the sample is quickly placed into the measuring chamber which will be closed.

It is clear that at the beginning of the sample cooling the transient process take place. In the process of the sample cooling the relaxation time is not constant in whole range. To judge this process we have to choose proper interval where τ is approximately constant. From equation 9 we obtain relation

$$\tau = -t / \ln \varphi \quad (12)$$

where

$$\ln \varphi = \ln[(T - T_\infty) - (T_0 - T_\infty)] \quad (13)$$

Relation 13 allows to set proper, nearly constant interval of τ from its time dependence. This interval is used for next calculation of desired thermal parameters. Typical transient process for Cu sample is in Figure 2

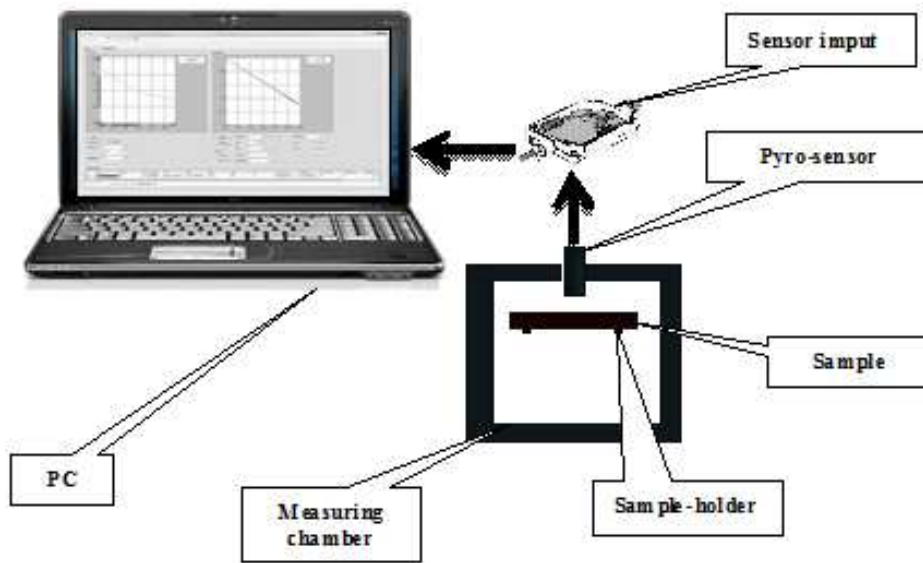


Figure 1. Block schema of the apparatus

The heart of the device is software that provides data processing. The time constant is the basis for determining other heat-transport parameters of the material. It is a reason, why the program uses in the first step search for τ . Equation 8 which represents the master equation of the problem is linearized by logarithmic function and we obtain relation $\ln(T-T_{\infty}) = \ln(T_0-T_{\infty}) - t/\tau$ which is fitted by using the method of the smallest squares and the term $1/\tau$ indicates the slope of line. The initial temperature T_0 is known from the measurement. The relaxation time τ has been determined and this value is used to determine of c_p .

In the second step we start to find further unknown physical value c_p . Because the Equation 6 expressing the relation of c_p versus h_t has infinite number of solutions, which means that only fitting procedures can be used to determine the required thermal and physical quantities. The basic prerequisite is the knowledge or at least an estimate of the total heat transfer coefficient h_t from the material to the surrounding environment. According to experimental analysis in this step, we can set $h_t \approx (15-25)$. For further unknown c_p we set qualified estimate according to the type of material – metals, plastics, laminates, etc. (table values).

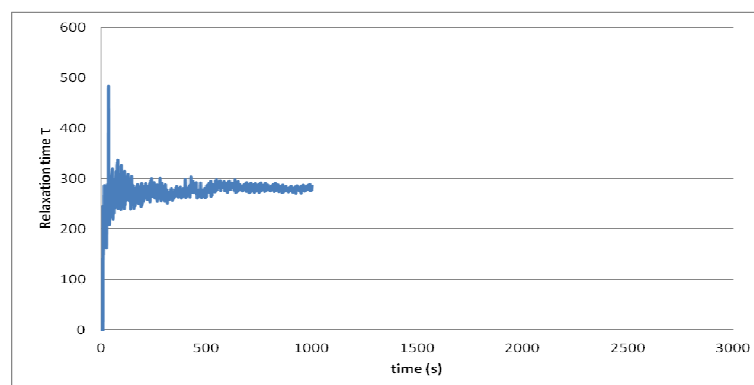


Figure 2. Transient process in *Cu* sample

The algorithm in the initial step is based on the intersection of the given and possible intervals of both quantities. Once the common intervals of meaningful values have been determined, the algorithm tries

to find a single value using the interval division method by gradually decreasing the interval of both required values. After this procedure c_p and experimental h_t are known and they are used for evaluation of thermal conductivity and thermal diffusivity. Similar procedure is used for the further unknown pair of thermal parameters α and k , where the equation 9 is applied and we can rewrite it as $\alpha \rho c_p = k$, $B c_p = k$, $B = \alpha \rho$ and where for both quantities (α, k) will use qualified estimates according to the type of material at the beginning of fitting (see Table 1).

The size of the sample and its weight is also used to determine the density of the sample material.

Finally, if we have determined all k , c_p and α it is necessary to verify validity of the condition given by relation 4, which is satisfying for model validity (Biot number have to be smaller than 0.1).

After this analysis, the experiment proceeds with the presentation of data obtained from selected materials with relatively high or low value of k , c_p and α . Firstly, we present the measurements of thermal parameters obtained for Cu (Table 1).

To test the validity of results obtained for Cu by used LCM model, we have measured the same sample of copper also by Netzsch laser equipment LFA 427. Obtained result for diffusivity is $1,20 \cdot 10^{-4} \text{ (m}^2 \cdot \text{s}^{-1})$ at ambient temperature which is very encouraging result supporting the validity of presented results together with statistical analysis.

In the next experiment we tested samples of lead with lower transport parameters α and k then in previous case. Measured and table values together with statistical parameters are in Table 2.

As we can see from the presentation of the measured data, the conformity of the measured, tabular or referential data is very good, both for low-conductive material, as well as for materials with relatively good thermal transport properties. The width of P99 confidence interval is smaller than 10 percent in all samples under investigation. Biot number is smaller than 0.1 in all cases and so used LCM is valid.

Table 1. Thermal parameters, tabular values, Bi number and statistical parameters for copper

Material Cu	Measured	P ₉₉	Table value [13]
c_p (J·kg ⁻¹ ·K ⁻¹)	387.4±4.69	(372.17 - 402.63)	383
α (m ² ·s ⁻¹)	(1.24±0.01)·10 ⁻⁴	(1.19 - 1.28)·10 ⁻⁴	1.15 10 ⁻⁴
k (W·m ⁻¹ ·K ⁻¹)	393.30±4.82	(377.65- 408.95)	400
Bi	0.021		

Table 2. Thermal parameters, tabular values, Bi number and statistical parameters for lead

Material Pb	Measured	P ₉₉	Table value [13]
c_p (J·kg ⁻¹ ·K ⁻¹)	126,80±0,95	(123,71-129,89)	129
α (m ² ·s ⁻¹)	(2,42±0,01)10 ⁻⁵	(2,38-2,46). 10 ⁻⁵	2,44.10 ⁻⁵
k (W·m ⁻¹ ·K ⁻¹)	34,70±0,30	(33,73-35,68)	35
Bi	0,016		

The third sample under investigation was PVC (polyvinyl chloride). This sample has the smallest values of k and α . The results are presented in Table 3.

Table 3. Thermal parameters, tabular values, *Bi* number and statistical parameters for PVC

Material PVC	Measured	P_{99}	Table value [13]
c_p (J·kg ⁻¹ ·K ⁻¹)	1399±5,04	(1382,61-1415,39)	1400
α (m ² ·s ⁻¹)	(6,88±0,05)·10 ⁻⁸	(6,71-7,05) ·10 ⁻⁸	6,90·10 ⁻⁸
k (W·m ⁻¹ ·K ⁻¹)	0,14±0,01	(0,14-0,15)	0,14
Bi	0,08		

Now we compare presented method with those from other authors which are based on the exponential body cooling.

In the work [14] authors tested polyurethane matrix by pulse transient method (Thermophysical Transient Tester 1.02 [15]). They tested the application of first order exponential model. Process of cooling of specimen after its thermal stimulation was monitored during the following approximately 6000 seconds. In these experiments relaxation time τ and c_p was determined only.

Finally we can compare our method with this described in [16]. Experimental method used in this case may be summarized as follows. A thin slice of the material to be tested is held between a hot copper block and a cold copper base, which remains essentially isothermal during the measurement. After an initial transient the temperature difference between the block and the base decays exponentially, and the thermal conductivity of the sample of the material may be calculated from the exponent. Heat losses from the block to the surroundings and the influence of thermal contact resistance at the sample-block interface and the sample-based interface are established by calibration.

In comparison with previous works our method offers:

- Complete set of thermal constants obtained from one cooling curve, not only α or k .
- Method is fully contactless and demands on the sample preparation are small.
- Our apparatus is also able to measure materials with large scale of thermal parameters (see presented results).
- As we can see from the presentation of the measured data, the conformity of the measured, tabular or referential data is relatively very good.
- The width of P_{99} confidence interval is smaller than 10 percent in all samples under investigation.
- Biot number is smaller than 0.1 in all cases and so used LCM is valid.

4. Conclusion

The presented method used to measure k , c_p and α is based on Newton's model of cooling body, respectively on its modification in the form of lumped capacitance model. Software processing, based on qualified estimates of k , c_p and α and h_c , provides approximate solution of the characteristic equations, which allow relatively highly accurate determination of k , c_p and α . The advantage of the method is that it is contactless; it requires only small amounts of tested sample, whose surface treatment is not very demanding.

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