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An unsteady vortex lattice method model of a horizontal axis wind turbine operating in an upstream rotor wake

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Abstract.

An unsteady formulation of the vortex lattice method, *VLM*, is presented that uses a force-free representation of the wake behind a horizontal axis wind turbine, *HAWT*, to calculate the aerodynamic loading on a turbine operating in the wake of an upstream rotor. A Cartesian velocity grid is superimposed over the computational domain to facilitate the representation of the atmospheric turbulence surrounding the turbine and wind shear. The wake of an upstream rotor is modelled using two methods: a mean velocity deficit with superimposed turbulence, based on experimental observations, and a purely numeric periodic boundary condition. Both methods are treated as frozen and propagated with the velocity grid.

Measurements of the mean thrust and blade root bending moment on a three bladed horizontal axis rotor modelling a 5 MW *HAWT* at 1:250 scale were carried out in a wind tunnel. Comparisons are made between operation in uniform flow and in the wake of a similarly loaded rotor approximately 6.5 diameters upstream. The measurements were used to validate the output from the *VLM* simulations, assuming a completely rigid rotor. The trends in the simulation thrust predictions are found to compare well with the uniform flow case, except at low tip speed ratios where there are losses due to stall which are yet to be included in the model. The simple wake model predicts the mean deficit, whilst the periodic boundary condition captures more of the frequency content of the loading in an upstream wake. However, all the thrust loads are over-predicted. The simulation results severely overestimate the bending moment, which needs addressing. However, the reduction in bending due to the simple wake model is found to reflect the experimental data reasonably well.

1. Introduction

The present trend towards increasingly large arrays of offshore horizontal axis wind turbines poses several new challenges in the modelling of aerodynamic loading on the rotor blades. In the lower intensity turbulence often present offshore, where it is desirable for connection reasons to keep wind park array packing relatively dense, wake impact has a significant influence on machine fatigue life. Low turbulent intensity leads to larger wake deficits and slower mixing of the wake [1]. This means that the wake is stronger at the downstream rotor position and leads to larger oscillation of the blades.

There has been much interest in the interaction between upstream rotor wakes and downstream turbines. Previous numerical studies have tended to focus on prediction of the power coefficient deficit and mean velocity profiles of the downstream turbine, e.g. [2] [3] and



[4]. Fletcher and Brown [5] used a vorticity transport model with lifting line representation of the blades to investigate the aerodynamic effects in more detail. They concluded that the power deficit decreases with increased spacing, but that the unsteadiness increased significantly, exacerbated by lateral offset of the upstream rotor. This will have a significant impact on the fatigue life of the blades. However, the details of the unsteady loading require further investigation.

The aim of this work is to model the unsteady loading on the rotor blades using a vortex lattice method code. The computational results are validated against wind tunnel measurements of the thrust loading at several lateral offsets of the upstream rotor and are implemented in the preliminary quantification of the effect of the impact of an upstream rotor wake.

2. Model Overview

An unsteady formulation of the vortex lattice method, *VLM*, has been developed in order to model the wake behind a horizontal axis wind turbine, *HAWT*, and to calculate the aerodynamic loading on the blades. The *VLM* is able to implicitly model some of the key unsteady and 3D effects, e.g. dynamic inflow and tip losses, on the turbine rotor improving the accuracy of the load calculations compared with simpler approaches, particularly in a spatially varying inflow across the rotor disc. Furthermore, in order to develop statistics of the unsteady loads, long run times will be required meaning the reduced computational cost over full Navier-Stokes solutions is preferred. The *VLM* utilises a mesh of vortex ring singularities to solve for potential flow over thin lifting surfaces with camber, pitch and twist, as detailed in e.g. [6].

The strength of the singularities on each blade is determined using a surface boundary condition which specifies that the velocity should be tangential. The velocity induced on the blades by the blade singularities is given by the Biot-Savart Law, which, assuming a straight line vortex filament, may be expressed as

$$\mathbf{V}_n = \mathbf{C}_{mn} \cdot \Gamma_m \quad (1)$$

where $\mathbf{V}$ is the induced velocity, Γ is the circulation strength, $\mathbf{C}$ is the matrix of influence coefficients and subscripts m and n denote the vortex ring and calculation point, respectively. The influence coefficient is a geometric quantity, defined as

$$\mathbf{C}_{mn} = \frac{1}{4\pi} \frac{\mathbf{r}_1 \times \mathbf{r}_2}{|\mathbf{r}_1 \times \mathbf{r}_2|^2} \left(\frac{\mathbf{r}_0 \mathbf{r}_1}{|\mathbf{r}_1|} - \frac{\mathbf{r}_0 \mathbf{r}_2}{|\mathbf{r}_2|} \right) \quad (2)$$

where $\mathbf{r}_0$ is the length vector of the vortex filament, $\mathbf{r}_1$ is the vector from the start and $\mathbf{r}_2$ is the vector from the end of the filament to the calculation point.

Equating the blade induced velocity to the known free stream, rotational and wake induced velocity components at each time step gives the circulation strength of the blade singularities. In order to capture the unsteady effects, a time stepping procedure is employed, whereby the trailing edge vortex rings are shed to form the wake at the end of each time step.

2.1. Wake treatment

The wake is treated as force-free: each of the wake nodes is convected according to its local velocity, including the self induced velocity of the wake singularities. The number of vortex rings in the wake increases with each time step, hence the self induced velocity computation is of $O(t^2)$. To reduce the computational cost, the force-free representation is restricted to the near wake, extending one diameter downstream. Subsequently, the far wake is frozen, meaning that it is propagated with the free stream velocity only. This reduces the induced velocity computation to $O(t)$.

2.2. Loads

The aerodynamic loading on the turbine blades is calculated using the Joukowski theorem, as applied in [7]. The vortex ring is decomposed into straight line filaments and the contribution of each vortex filament is calculated and combined to give the load on the vortex ring:

$$\mathbf{F}_m = \sum_{i=1}^4 \rho \Gamma_m \mathbf{U}_i \times \mathbf{dl}_i + \rho S_m \frac{\partial \Gamma_m}{\partial t} \mathbf{n}_m \quad (3)$$

where $\mathbf{F}$ is the force vector, ρ is the fluid density, $\mathbf{U}$ is the total velocity, $\mathbf{dl}$ is the length of the vortex filament, S is the panel area, t is time and $\mathbf{n}$ is the unit normal vector and subscripts m and i denote the vortex ring and vortex filaments, respectively. This loading is summed to give the total load on the blades and is then resolved into the rotor plane and out-of-plane components, giving the power and thrust, respectively.

2.3. Velocity grid

To facilitate the input of empirical incident velocity data, a Cartesian grid is superimposed over the computational domain, extending 1 diameter upstream and 10 diameters downstream of the rotor. Velocities are defined at each of the grid nodes in order to facilitate the input of wake impacting, atmospheric turbulence and yaw. The grid nodes are propagated according to the mean free stream velocity at hub height, taking into account the yaw of the flow. The nodal velocities propagate with the grid, assuming a frozen velocity field, and are linearly interpolated to give the local velocity at any given calculation point.

2.4. Upstream wake models

Two different approaches to modelling the impact of an upstream wake have been investigated. As a first approximation, an axisymmetric mean velocity deficit is superimposed on the velocity grid. Whilst this approach is rather simplistic, it is proposed that the principle impact of the upstream rotor wake on the thrust is a result of the mean velocity deficit. Additionally, the $1P$ variation in loading as the downstream rotor passes into and out of the upstream rotor wake will be one of the stronger unsteady responses which should be sufficiently captured by this model. As proposed by Ainslie [8], the wake mean profile is assumed to be Gaussian:

$$\frac{\Delta U}{U_\infty} = 0.07 \Lambda e^{-(r/0.8R)^2} \quad (4)$$

where ΔU is the velocity deficit, Λ is the tip speed ratio, r is the radial distance from the axis of rotation, U_∞ is the mean free stream velocity and R is the rotor radius. The Ainslie model is found to give a more accurate representation of the mean velocity deficit and wake width compared with other models in [9]. For the purposes of comparison, the values of the coefficients were empirically defined using observations of the wake velocity deficit behind the rotor used in the experiments. These coefficients give similar centre line velocity deficits as [8]. Furthermore, a random turbulent velocity field with intensities and correlations matching the experimental observations is superimposed to simulate the turbulent component of the wake.

A purely numerical approach to the modelling of the upstream wake is proposed as a periodic boundary condition (BC). In this model, the wake induced velocity is calculated in a plane at the relevant downstream position corresponding to the turbine spacing. This is then input into the inlet of the velocity grid as a velocity field and propagated across the rotor with the free stream. The velocities are mean filtered over the grid generation period removing any higher frequency noise resulting from the numerical methods. The result can be interpreted as a semi-infinite row of turbines, which might correspond to the flow field in the third or later rows of a wind farm.

3. Wind Tunnel Validation

Measurements of the mean loads on a three bladed, horizontal axis, 0.5 m diameter (D) rotor modelling a 5 MW turbine at 1:250 scale were obtained for a range of tip speed ratios, TSR . The experiments were performed in the Honda wind tunnel at Imperial College London. The working section has a $3 \times 1.5\text{ m}$ cross section and is 9 m long. The rotor was instrumented with strain gauges at the root of one of the blades and on the support strut to measure the blade root bending moment and thrust. The blade has a span of 0.2 m and mean aerodynamic chord of 0.026 m , giving a planform area of $5.2 \times 10^{-3}\text{ m}^2$. Readings were taken in a uniform flow of 12 m s^{-1} with the turbine positioned on the centre line of the tunnel. Figure 1 shows a schematic of the experimental setup, including the positions of the strain gauges.

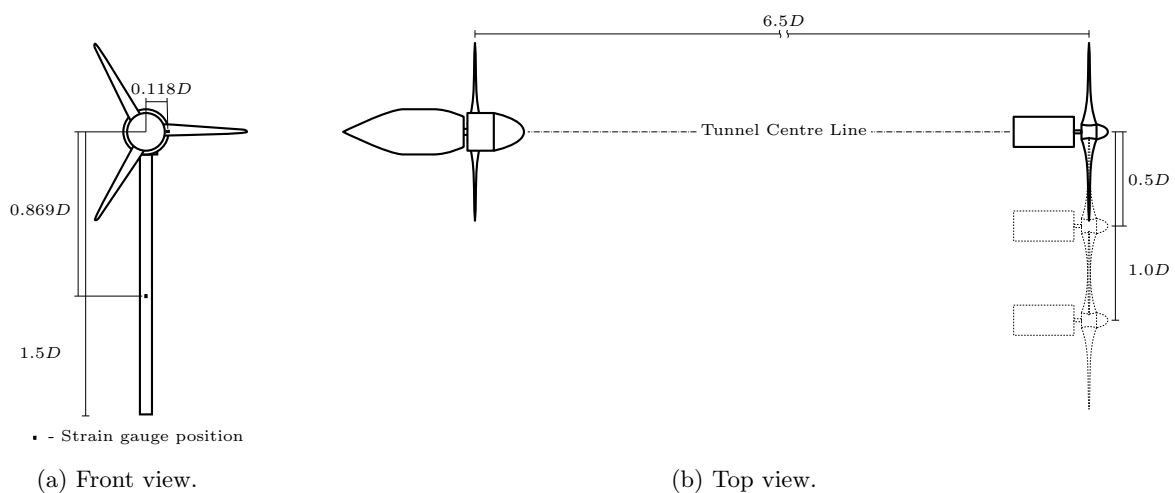


Figure 1: Schematic of experimental setup, $D = 0.5\text{ m}$.

To investigate the effect of operating in the wake of an upstream turbine, a similarly loaded rotor is then placed approximately 6.5 diameters upstream of the instrumented rotor in the working section. Three upstream rotor alignments are examined: with the rotors aligned; with the upstream rotor offset by $0.5D$; with the upstream rotor offset by $1D$. In each case the mean blade root bending moment and thrust are recorded for a range of TSR with both turbines rotating at the same angular velocity.

The tunnel blockage, defined as the swept area of the turbine divided by the cross sectional area of the tunnel, for the single instrumented rotor is 4.4% . With the upstream rotor $1D$ offset the blockage doubles, however, it is still deemed to be sufficiently low to neglect wall interference on the measurements.

3.1. Results

Figure 2 shows the mean thrust coefficients on the instrumented rotor observed during the experiments for each of the upstream rotor positions. The thrust coefficient on the rotor is defined as

$$C_T = \frac{F_N}{\frac{1}{2}\rho U_\infty^2 \pi R^2} \quad (5)$$

where F_N is the thrust. For both offset positions of the upstream rotor the percentage reduction in the thrust coefficient, relative to the uniform flow case, is reasonably constant: $21 - 22\%$ for the $0.5D$ offset rotor and $8 - 10\%$ for the $1D$ offset rotor. With the upstream rotor aligned, the

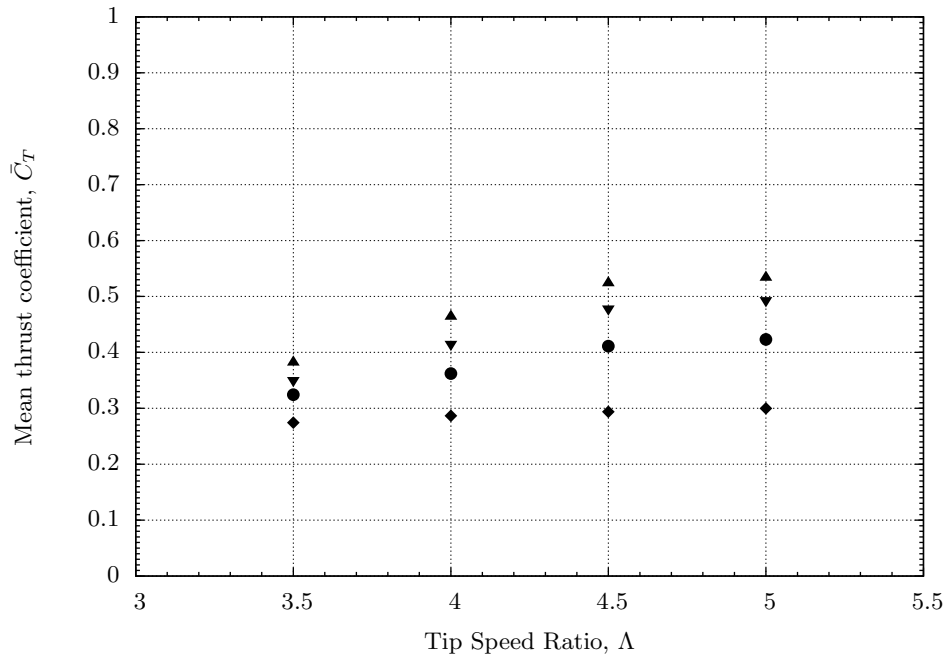


Figure 2: Mean thrust coefficient for different tip speed ratios and upstream conditions: (▲) uniform flow, (◆) aligned upstream rotor, (●) 0.5D offset upstream rotor, (▼) 1D offset upstream rotor.

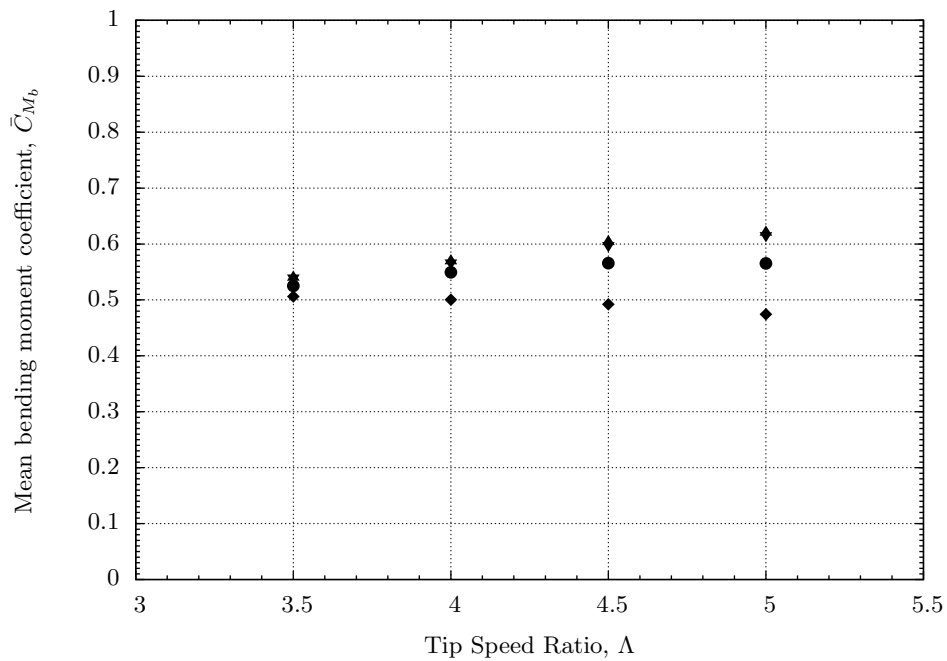


Figure 3: Mean bending moment coefficient for different tip speed ratios and upstream conditions: (▲) uniform flow, (◆) aligned upstream rotor, (●) 0.5D offset upstream rotor, (▼) 1D offset upstream rotor.

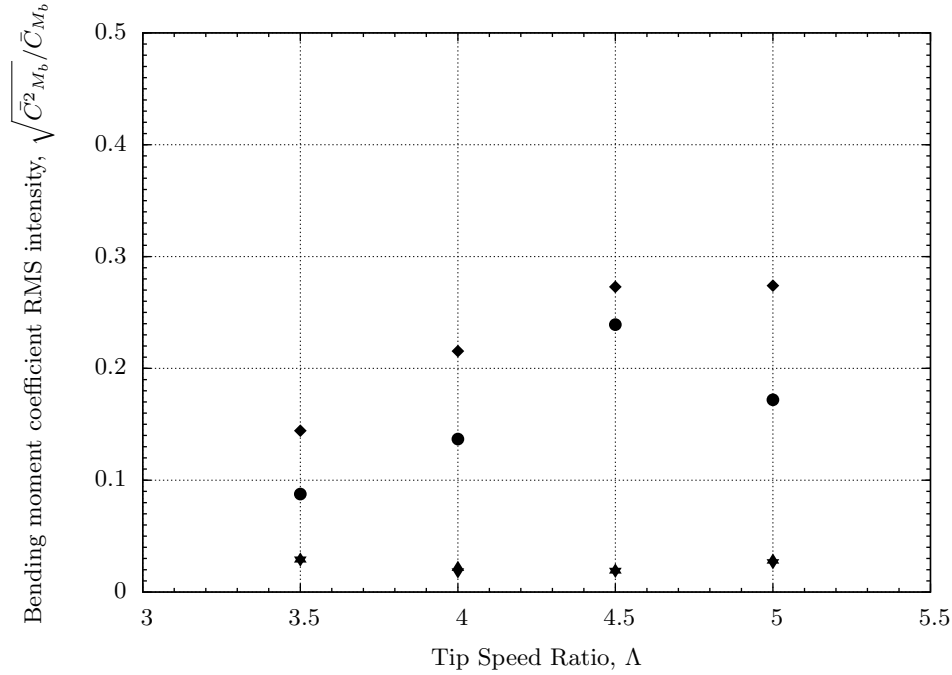


Figure 4: Normalised *RMS* of the bending moment coefficient for different tip speed ratios and upstream conditions: (▲) uniform flow, (◆) aligned upstream rotor, (●) 0.5*D* offset upstream rotor, (▼) 1*D* offset upstream rotor.

percentage reduction increases from around 28% to 44% at $\Lambda = 4.5$. This decrease in loading is most likely a result of the increased velocity deficit in the wake.

The bending moment coefficient on the blade is defined as [10]

$$C_{M_b} = \frac{M_b}{\frac{1}{2}\rho U_\infty S \Omega R^2} \quad (6)$$

where M_b is the bending moment, S is the planform area and Ω is the angular velocity. Figure 3 shows the mean bending moment coefficients observed during the experiment. The percentage reduction in bending moment for the aligned upstream rotor, compared with uniform flow, linearly increases from around 6% at $\Lambda = 3.5$ to 24% at $\Lambda = 5$. This agrees with field observations of [11] who found 30% reduction in the bending moment for a turbine operating at $\Lambda = 5$, 5*D* downstream of another turbine.

In contrast, for the 1*D* offset upstream rotor there is negligible difference across the full range of *TSR* of less than 1%. In the case of the 0.5*D* offset upstream rotor, there is difference of 3 – 4% between $\Lambda = 3.5 - 4$, followed by a linear increase to 9% at $\Lambda = 5$. These trends concur with the results from the thrust coefficient measurements, and together indicate that the wake expansion is minimal leading to similar results in uniform flow and with a 1*D* offset upstream rotor. This agrees with the observations of [12] who found that wake expansion was less than 7% of the rotor radius.

The observed *RMS*, normalised by the mean, of the blade root bending moment is shown in Figure 4. For both the uniform flow and the 1*D* offset upstream rotor cases, the *RMS* is below 5%. This suggests minimal oscillation in the blade bending moment. The aligned upstream rotor case has linearly increasing *RMS* from around 15% at $\Lambda = 3.5$ to 27% at $\Lambda = 4.5$. Similarly, the 0.5*D* offset case has increasing *RMS* from 9% at $\Lambda = 3.5$ to a peak of around 24% at $\Lambda = 4.5$, but drops again to 17% at $\Lambda = 5$. The increased *RMS* in both the aligned and 0.5*D* offset cases is an

indication that the upstream wake includes significant turbulent components. This will increase the fatigue of the blade and result in a decreased life. The peak at $\Lambda = 4.5$, corresponding to 34 Hz rotational frequency, may be due to excitation of the strut resonant frequency which is around 10 Hz. The *RMS* of the thrust coefficient is dominated by the resonant response of the strut and hence was not examined further.

3.2. Comparison with simulations

Figure 5 shows a comparison of the *VLM* simulations with the experimental results for the mean thrust coefficient in uniform flow and in the wake of an aligned upstream rotor. In uniform flow, the simulations over-predict the thrust coefficient by around 30% of the experimental result, performing worse at the lower *TSR*. This is most likely due to a lack of a stall model which would cause greater losses at low *TSR*. The discrepancy in the thrust magnitudes may be a combination of factors, including the numerical discretisation, wake core model and losses associated with a flexible rotor blade.

Using the simple wake model, the simulation results over predict the aligned rotor thrust by 75–85% across the entire range, although the discrepancy decreases as *TSR* increases. The load deficit between the numerical results linearly increases from around 17% to 25%, resulting in a qualitatively similar trend to the experimental data, with around 10% less deficit. The periodic *BC* shows a slight increase in thrust of around 1%. This may be due to the fact that the velocity deficit was not allowed to fully develop due to the significant simulation time required.

Figure 6 shows a comparison of the mean bending moment between the simulations and experimental results in uniform flow and in the wake of an aligned upstream rotor. The simulation shows better agreement at higher *TSR* where both the thrust prediction and the rigid rotor assumption are more accurate: the rotation of the blade leads to a stiffening effect, which increases with the angular velocity. However, in the uniform flow case there is a discrepancy of over 70% at the lower end, with a roughly linear decrease to 20% at $\Lambda = 5$ and similar discrepancies in the aligned upstream rotor case for the simple wake model. Some of this may be attributed to the lack of a stall model, similar to the discrepancy in the thrust loading: the thrust is the principal force in determining the bending moment. However, there is a clear discrepancy between the simulations and experiments which needs to be addressed.

The reduction in bending moment between the uniform flow and aligned upstream rotor for the simple wake model in the simulations increases linearly from 15% at $\Lambda = 3.5$ to 20% at $\Lambda = 5$. This is similar to the trend in the experimental data, although the gradient is shallower and the magnitude greater. This suggests that the simple wake model is having a reasonable impact on the simulation predictions for the bending moment. Similar to the thrust loading, the periodic boundary condition doesn't capture the deficit in the bending moment. However, this model does appear to capture the shape of the bending moment curve slightly better, bearing in mind that the lack of a stall model means the low *TSR* results will be over-predicted.

Figure 7 shows the frequency spectra of the experimental aligned rotor case and the two *VLM* wake models for a *TSR* of 5. This indicates that the periodic *BC* is able to capture more of the features of the spectra than the simple wake model, despite using no empirical data. More significantly it identifies the peak at $s = 0.795$, corresponding to the rotational frequency, which is excited in the aligned rotor results and, therefore, assumed to be a wake impacting effect caused by a slight offset in the upstream wake leading to a load imbalance.

4. Conclusion

The simulations were found to predict the trends in the thrust loading well, although a stall model is required to improve the prediction at low *TSR*. The simple wake model predicted similar deficits in the loading, but was unable to identify the frequency content of the loads. Whereas the periodic boundary condition predicted more of the frequency content, it was unable

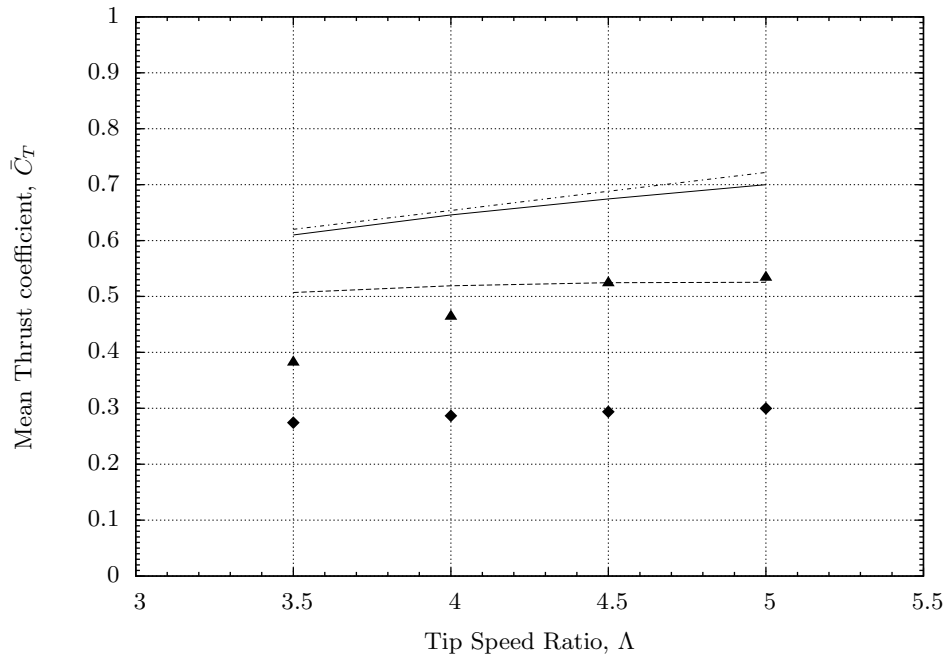


Figure 5: Comparison of mean thrust coefficient with *VLM* model: Experimental results: ($\blacktriangle$) uniform flow, ($\blacklozenge$) aligned upstream rotor; Simulations: (—) uniform flow, (- - -) simple wake model, (- · -) periodic boundary condition.

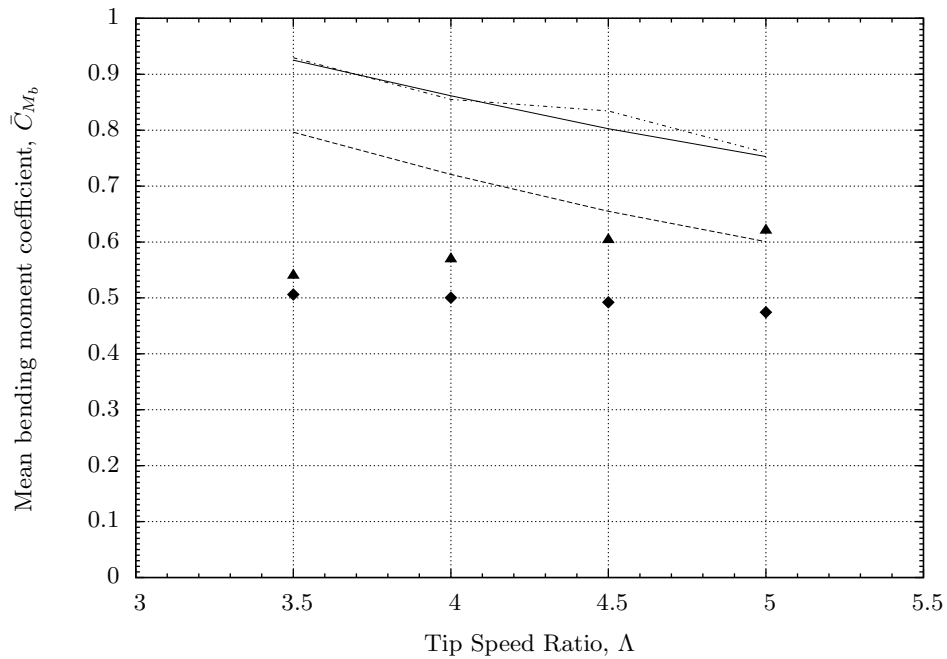


Figure 6: Comparison of mean bending moment coefficient with *VLM* model: Experimental results: ($\blacktriangle$) uniform flow, ($\blacklozenge$) aligned upstream rotor; Simulations: (—) uniform flow, (- - -) simple wake model, (- · -) periodic boundary condition.

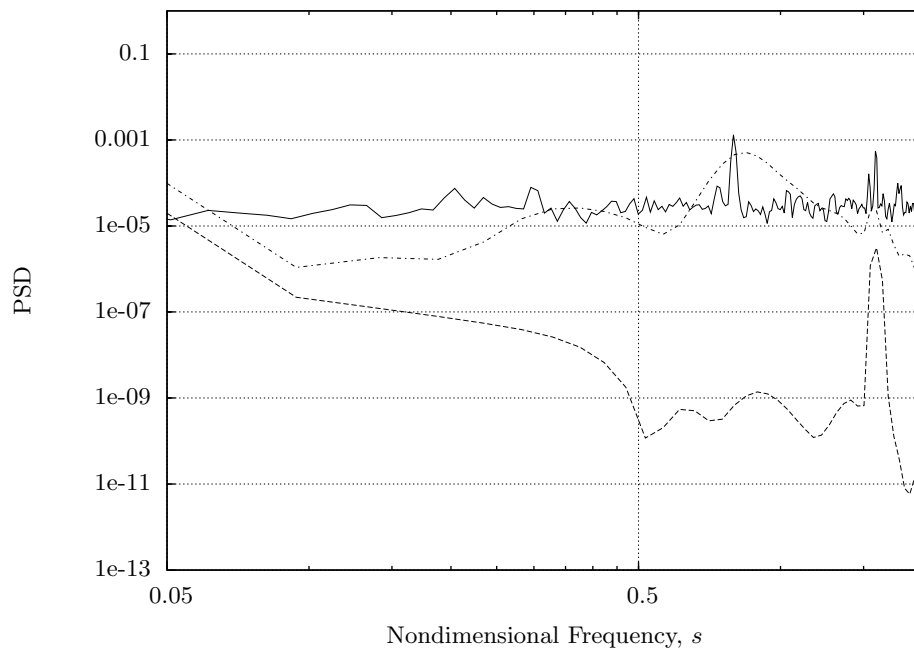


Figure 7: Comparison of frequency spectrum for *TSR 5* with *VLM* model: (—) experimental, (- - -) simple wake model, (- · -) periodic boundary condition.

to model the load deficit, and in fact increased the thrust loading by around 1%. However, the bending moment predictions are poorly modelled, suggesting that the loading distribution may be incorrect.

The experimental results for the offset upstream rotor suggest that an offset of $1D$ is sufficient to greatly improve the downstream rotor performance. However, it should be noted that the effect of atmospheric turbulence, which would increase wake expansion, has not been studied, although this is planned for future experiments.

To improve the predictions of the current wake model a stall model is required. Furthermore, the upstream wake models will need to be developed to capture both the fluctuating and mean deficit components of the wake impacting process. This will allow better prediction of the fatigue loads on a rotor operating in an offshore wind farm.

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