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To cite this article: Danila N Puzyrev and Arkadiy A Drozdov 2014 *J. Phys.: Conf. Ser.* **541** 012065

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Collimation and focusing of initially single-cycle paraxial optical beams

Danila N. Puzyrev and Arkadiy A. Drozdov

Department of Photonics and Optical Information Technology, ITMO University,
Saint-Petersburg 197101, Russia

E-mail: puzurdan@gmail.com, arkadiy.drozdov@gmail.com

Abstract. The paper reports theoretical results of collimation and focusing of spatio-temporal field structure that is forming due to far-field (Fraunhofer) diffraction of paraxial wave packet emitted by single-cycle radiation source. A spatial collimation of such waves results in a peculiar shape of spatio-temporal structure with the central part moving faster than its outlying areas. During the focusing of collimated beam, a transformation from 1.5-cycle to single-cycle and again to 1.5-cycle wave occurs along a path from collimating to focusing mirrors with equal focal lengths.

1. Introduction

With the development of conversion technology of near-IR femtosecond radiation in to terahertz (THz) frequency range, it become possible to obtain single-cycle electromagnetic waves with a spectrum in the range of 0.1-10 THz [1–3]. Such radiation usually has widespread application in detection systems of drugs and explosive materials, for medical diagnostics and other applications [4–7]. Optimization of optical systems that emit, collimate and focus such waves is one of the important problem in THz science nowadays. It is also well-known that initially single-cycle pulse is transformed to 1.5-cycle pulse over the entire wave packet in far field region [8]. The properties of single-cycle THz pulses propagating through a focus of parabolic mirrors or lens have been previously studied both theoretically and experimentally [9–11]. The first noninterferometric observation of axial π phase shift for single-cycle THz pulses passing through a focus was demonstrated in the experiment [12]. The first direct observation of the 2D focused few-cycle THz field distribution by CCD based THz imaging system was shown in the work [13]. To our knowledge, detailed investigation of spatio-temporal field dynamics after collimation and subsequent focusing of initially single-cycle pulses has not been performed.

In this paper, for the case of linearly polarized radiation with Gaussian transverse spatial distribution we theoretically investigate peculiarities of collimation and focusing of spatio-temporal field structure that is forming due to Fraunhofer diffraction of paraxial wave packet emitted by single-cycle radiation source.



2. Diffraction dynamics of spatio-temporal spectrum of single-cycle optical beams

Spatio-temporal dynamics of spectrum

$$g_{x,y}(\omega, k_x, k_y, z) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E_{x,y}(t, x, y, z) \exp(-i(\omega t + k_x x + k_y y)) dt dx dy \quad (1)$$

of paraxial radiation with electric field strength $E_{x,y}(t, x, y, z)$ in an isotropic homogeneous optical medium can be described by expression [8]

$$g_{x,y}(\omega, k_x, k_y, z) = C_{x,y}(\omega, k_x, k_y) \exp\left(-ikz \left(1 - \frac{(k_x^2 + k_y^2)}{2k^2}\right)\right), \quad (2)$$

where x, y, z are coordinates of Cartesian axes (z axis coincides with wave propagation direction and x, y coincide with transverse direction), Cartesian components of wave $E_{x,y}$ and its spatio-temporal spectrum $g_{x,y}$ are denoted by x, y indexes, C_x and C_y are assumed known spectral components of radiation at $z = 0$, ω is the temporal frequency of radiation, k_x and k_y are the spatial frequencies, $k = \frac{\omega n(\omega)}{c}$ is the wave number, $n(\omega)$ is the refractive index of a medium, c is the speed of light in a vacuum.

In this paper we restrict the analysis of collimation and subsequent focusing of paraxial Gaussian beams, which we assume linearly polarized along axis x for simplicity. So we will assume their spectrum of the form [8]

$$C(\omega, k_x, k_y) = \pi \rho^2 \exp\left(-\frac{\rho^2 (k_x^2 + k_y^2)}{4}\right) G_0(\omega), \quad (3)$$

that corresponds to initial electric field distribution

$$E(t, x, y) = \exp\left(-\frac{x^2 + y^2}{\rho^2}\right) F_0(t), \quad (4)$$

where ρ is the transverse beam width, $F_0(t)$ is temporal profile of wave, $G_0(\omega)$ is its temporal spectrum at $z = 0$.

For boundary condition (3) expression (2) takes form

$$g(\omega, k_x, k_y, z) = \pi \rho^2 \exp\left(-\frac{\rho^2 (k_x^2 + k_y^2)}{4}\right) \left(1 - i \frac{2cz}{\rho^2 n(\omega) \omega}\right) \exp\left(-i \frac{n(\omega) \omega z}{c}\right) G_0(\omega). \quad (5)$$

Using the transform formula for spatial dependence of temporal spectrum of radiation,

$$G(\omega, x, y, z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} g(\omega, k_x, k_y, z) \exp(i(k_x x + k_y y)) dk_x dk_y \quad (6)$$

and relation (5), we can present the expression (6) for diffraction dynamics of radiation in form

$$G(\omega, x, y, z) = \frac{T(\omega, z) \omega}{T(\omega, z) \omega - i} \exp\left(-\frac{(x^2 + y^2)}{\rho^2} \frac{(T(\omega, z) \omega)^2}{1 + (T(\omega, z) \omega)^2} \left(1 + \frac{i}{T(\omega, z) \omega}\right) - ikz\right) G_0(\omega), \quad (7)$$

where $T(\omega, z) = \frac{\rho^2 n(\omega)}{2cz}$.

For far-field region ($z \gg \frac{\pi \rho^2 n(\omega)}{\{\lambda\}_{\min}}$, where $\lambda_{\min}$ is minimum wavelength from wavelength range where the majority of initial radiation energy is trapped [8]) relation (7) is simplified to form

$$G_F(\omega, x, y, z) = T(\omega, z) \omega (T(\omega, z) \omega + i) \times \exp \left(-\frac{(x^2 + y^2)}{\rho^2} (T(\omega, z) \omega)^2 - ik \left(z + \frac{(x^2 + y^2)}{2z} \right) \right) G_0(\omega). \quad (8)$$

For further analysis of paraxial Gaussian beams we restrict ourselves by the case of dispersionless media when we can present the refractive index in form $n(\omega) = n_0 = \text{const}$. Atmospheric air for THz radiation is a characteristic example of such medium [14, 15] at considered distances in this paper. We also assume the single-cycle wave of source with spectrum $G_0(\omega) = -\frac{\sqrt{\pi}}{2} E_0 \tau_p^2 i \omega \exp \left(-\left(\frac{\tau_p \omega}{2} \right)^2 \right)$ [16], where τ_p is input pulse duration at $z = 0$, E_0 is correspond initial amplitude of electric field strength of wave.

Using the Fourier transform formula

$$E(t, x, y, z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} G(\omega, x, y, z) \exp(i\omega t) d\omega \quad (9)$$

and relation (8) we can find the spatio-temporal distribution of electric field in far-field region in form [see figure 1(a) below]

$$E_F(t', x, y, z) = E_0 T(z) \frac{\tau_p^2}{\tau_F^3} \left(2 \frac{T(z)}{\tau_F^2} \left(3 - 2 \frac{t'^2}{\tau_F^2} \right) t' + \left(1 - 2 \frac{t'^2}{\tau_F^2} \right) \right) \exp \left(-\frac{t'^2}{\tau_F^2} \right), \quad (10)$$

where we introduce $\tau_F^2 = \tau_p^2 + \left(\frac{\rho N_0}{c} \right)^2 \frac{(x^2 + y^2)}{z^2}$ and "delay time" $t' = t - \frac{N_0}{c} \left(z + \frac{(x^2 + y^2)}{2z} \right)$.

Red and blue areas correspond to maximum positive and maximum negative field values of two-dimensional images. Here we take $\rho/\lambda_{\max} = 10$, where $\lambda_{\max}$ is the wavelength corresponding to the maximum of spectrum of input wave packet [16, 17]. For example, for terahertz single-cycle waves this value is usually $\lambda_{\max} = 300 \mu\text{m}$. We also consider electric field emitted by the source (e.g., at $z = 0$) in the form of a single-cycle pulse with Gaussian transverse distribution [see figure 1(b) below]

$$E(t, x, y) = E_0 \exp \left(-\frac{x^2 + y^2}{\rho^2} \right) \frac{t}{\tau_p} \exp \left(-\frac{t^2}{\tau_p^2} \right). \quad (11)$$

As follows from this figure and relation (10), in far field region a single-cycle beam becomes 1.5-cycle for the whole wave package. We note that this feature was also observed in [8].

3. Spatio-temporal collimation and focusing dynamics of paraxial wave packet emitted by single-cycle source of radiation

First, we set collimating mirror at distance z_0 from radiation source with focal length $f_{\text{col}} = z_0$ and reflection function $R_{\text{col}}(\omega, x, y) = \exp(ik(x^2 + y^2)/2f_{\text{col}})$ in far field region. We neglect the effects of optical-system aberrations. Temporal spectrum of wave packet at distance z_0 from the source of radiation will be determined by following expression

$$G_{\text{col}}(\omega, x, y, z_0) = G_F(\omega, x, y, z_0) R_{\text{col}}(\omega, x, y), \quad (12)$$

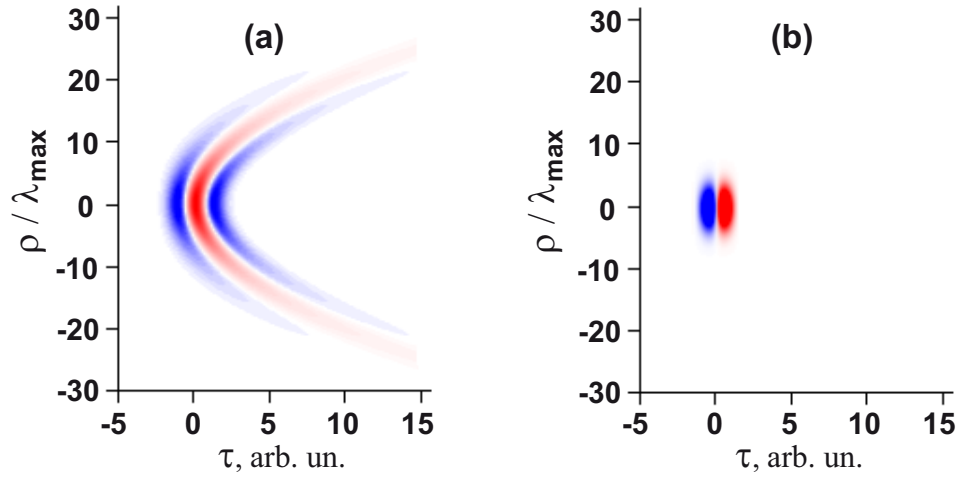


Figure 1. Spatio-temporal structure of distribution of the electric field of radiation in far-field region (10) (a) and its structure at $z = 0$ (b).

where $G_F(\omega, x, y, z_0)$ is given by relation (8).

As follows from relation (12), the temporal spectrum of radiation immediately after collimating mirror takes form

$$G_{\text{col}}(\omega, x, y, z_0) = T(\omega, z_0)\omega (T(\omega, z_0)\omega + i) \exp\left(-\frac{(x^2 + y^2)}{\rho^2}(T(\omega, z_0)\omega)^2 - ikz_0\right) G_0(\omega). \quad (13)$$

It can be seen from (13), collimation simplifies Fourier spectrum of incident on collimating mirror radiation [see (8) and (13) for comparison]. So the spatio-temporal distribution of electric field immediately after collimating mirror will be described by relation

$$E_{\text{col}}(t'', x, y, z_0) = E_0 T(z_0) \frac{\tau_p^2}{\tau_F^3} \left(2 \frac{T(z_0)}{\tau_F^2} \left(3 - 2 \frac{t''^2}{\tau_F^2} \right) t'' + \left(1 - 2 \frac{t''^2}{\tau_F^2} \right) \right) \exp\left(-\frac{t''^2}{\tau_F^2}\right), \quad (14)$$

where we introduce the retarded time $t'' = t - \frac{N_0}{c}z$ which is not depending on transverse coordinates now. We now consider (14) as a new boundary condition for dynamics of collimated radiation and show in figure 2 (a). It can be seen that specially shaped 1.5-cycle spatio-temporal structure with increasing duration from beam axis to periphery is formed immediately after collimating mirror.

According to (13) spatio-temporal spectrum of collimated radiation become

$$g_{\text{col}}(\omega, k_x, k_y, z_0) = \pi \rho^2 \frac{T(\omega, z_0)\omega + i}{T(\omega, z_0)\omega} \exp\left(-\frac{\rho^2}{4} \frac{k_x^2 + k_y^2}{(T(\omega, z_0)\omega)^2} - ikz_0\right) G_0(\omega). \quad (15)$$

The general solution for spatio-temporal spectrum of collimation dynamics by taking into account (2) can be presented in form

$$g_{\text{col}}(\omega, k_x, k_y, z) = g_{\text{col}}(\omega, k_x, k_y, z_0) \exp\left(-ik(z - z_0) \left(1 - \frac{(k_x^2 + k_y^2)}{2k^2}\right)\right). \quad (16)$$

Expression (16) according to (15) takes form

$$g_{\text{col}}(\omega, k_x, k_y, z) = \pi \rho^2 \frac{T(\omega, z_0)\omega + i}{T(\omega, z_0)\omega} \times \exp\left(- (k_x^2 + k_y^2) \frac{\rho^2}{4T(\omega, z_0)\omega} \left(\frac{1 - iT(\omega, z_0)\omega(z - z_0)/z_0}{T(\omega, z_0)\omega} \right) - ikz\right) G_0(\omega). \quad (17)$$

Using the relation (17) we can describe the spatial dependence of temporal spectrum dynamics of collimated radiation at arbitrary distance after mirror by expression

$$G_{\text{col}}(\omega, x, y, z) = \frac{T(\omega, z_0)\omega (T(\omega, z_0)\omega + i)}{1 - iT(\omega, z_0)\omega(z - z_0)/z_0} \times \exp\left(- \frac{x^2 + y^2}{\rho^2} \frac{(T(\omega, z_0)\omega)^2}{1 - iT(\omega, z_0)\omega(z - z_0)/z_0} - ikz\right) G_0(\omega). \quad (18)$$

The results of numerical simulation of spatio-temporal field evolution at different distances from collimating mirror in linear optical medium from equation (18) are shown in figure 2 (b-e). It is shown that due to diffraction the central part of this complex structure is moving faster than outlying areas.

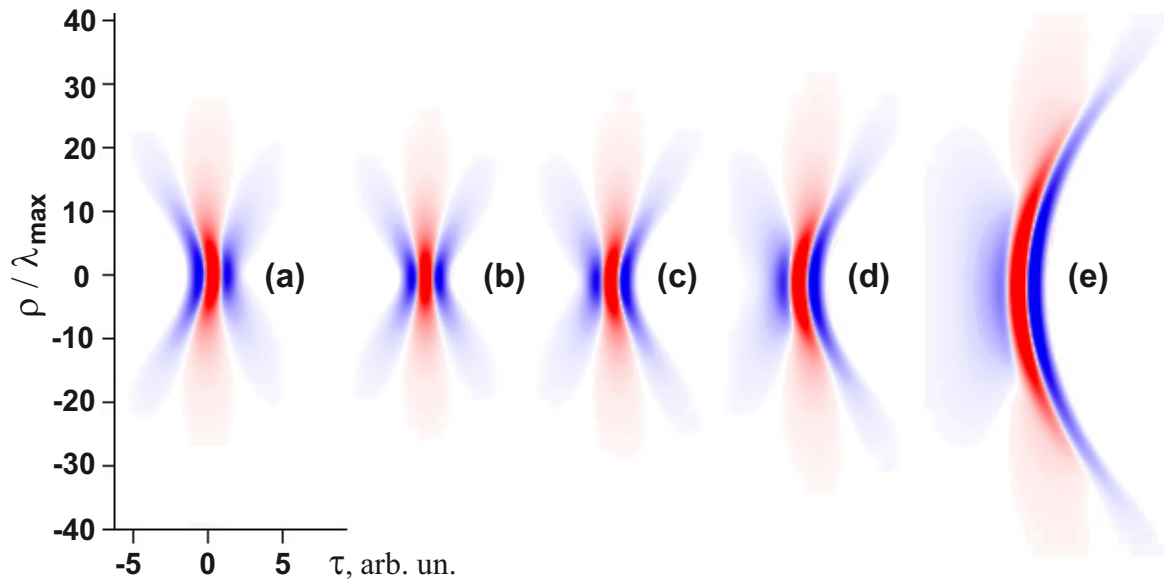


Figure 2. Spatio-temporal structure of the electric field of collimated wave packet immediately after collimating mirror (a) and during its further evolution in linear optical medium (b-e) at the distances f_{col} (b), $2f_{\text{col}}$ (c), $3f_{\text{col}}$ (d), $6f_{\text{col}}$ (e) from mirror. τ is the normalized time.

We now consider focusing features of presented in figure 2 collimated radiation by focusing mirror with focal length $f_{\text{foc}} = f_{\text{col}}$. Spatio-temporal distributions of the electric field of radiation at the focus of mirror are presented in figure 3 (a-e). Figure 3 shows that during the focusing of collimated radiation, optical wave at the focus is changing from 1.5-cycle to a single-cycle and again to 1.5-cycle pulse along the way from collimating to focusing mirrors. It can be seen from figure 3 (c), that focused electric field of radiation with an accuracy to π phase shift coincide with initial field of single-cycle wave packet when distance between collimating and focusing mirrors is twice of their focal length.

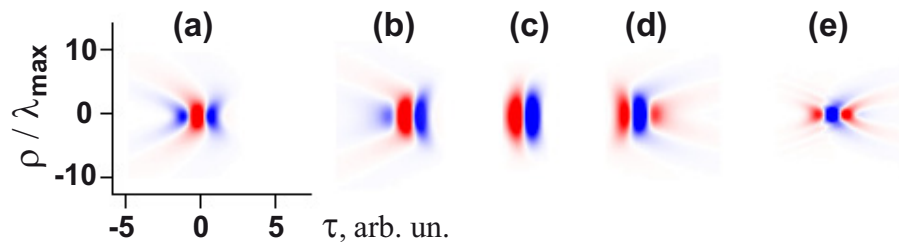


Figure 3. Spatio-temporal structure of distribution of the electric field of radiation at the focus of mirror during focusing of wave packet obtained immediately after the collimating mirror (a) and at the distances f_{col} (b), $2f_{\text{col}}$ (c), $3f_{\text{col}}$ (d), $6f_{\text{col}}$ (e) from it.

4. Conclusion

We have theoretically investigated the features of collimation and focusing of spatio-temporal field structure that is forming due to far-field diffraction of paraxial wave packet emitted by single-cycle radiation source. We have demonstrated that spatial collimation of such waves results in a peculiar shape of the spatio-temporal structure with the central part moving faster than its outlying areas during further evolution in free space. During the focusing of collimated wave packet a transformation from 1.5-cycle to single-cycle and again to 1.5-cycle pulse occurs along a path from collimating to focusing mirrors with equal focal lengths. We have also shown that focused electric field with an accuracy to π phase shift coincide with electric field of initial single-cycle radiation when the distance between collimating and focusing mirrors is twice of their focal lengths. These features shall be taken into account for the optimization of optical systems operating with single-cycle THz pulses.

Acknowledgments

Authors would like to thank Prof. Sergei Kozlov for helpful discussions. This work was financially supported by Government of Russian Federation (Grant 074-U01).

References

- [1] Bespalov V G, Gorodetski A A, Denisyuk I Y, Kozlov S A, Krylov V N, Lukomski G V, Petrov N V and Putilin S E 2008 *J. Opt. Technol.* **75** 636–642
- [2] Zhang X C and Xu J 2009 *Introduction to THz wave photonics* (N.Y.: Springer Science+Business Media)
- [3] Sajadi M, Wolf M and Kampfrath T 2014 *Appl. Phys. Lett.* **104** 091118
- [4] Lee Y 2009 *Principles of Terahertz Science and Technology* (Corvalis: Springer Science+Business Media)
- [5] Horiuchi N 2010 *Nat. Photonics* **4** 140
- [6] Grachev Y V, Kuklin I A, Gerasimov I V, Smolyanskaya O A, Kozlov S A and Bespalov V G 2010 *J. Opt. Technol.* **77** 731–733
- [7] Sakhnov S N, Leksutkina E V, Smolyanskaya O A, Usov A V, Parakhuda S E, Grachev Y V and Kozlov S A 2011 *Optics and Spectroscopy* **111** 257–261
- [8] Ezerskaya A A, Ivanov D V, Kozlov S A and Kivshar Y S 2012 *J. Infrared Millim. Terahertz Waves* **33** 926–942
- [9] Feng S, Winful H G and Hellwarth R W 1998 *Opt. Lett.* **23** 385–387
- [10] Hunsche S, Feng S, Winful H G, Leitenstorfer A, Nuss M C and Ippen E P 1999 *J. Opt. Soc. Am. A* **16** 2025–2028
- [11] Kužel P, Khazan M A and Kroupa J 1999 *J. Opt. Soc. Am. B* **16** 1795–1800
- [12] Ruffin A B, Rudd J V, Whitaker J F, Feng S and Winful H G 1999 *Phys. Rev. Lett.* **83**(17) 3410–3413
- [13] Jiang Z and Zhang X C 1999 *Opt. Express* **5** 243–248
- [14] Gurtler A, Winnewisser C, Helm H and Jepsen P 2000 *J. Opt. Soc. Am. A* **17** 74–83
- [15] Kulya M S, Grachev Y V, Bespalov V G and Kujanpaa V P 2013 *Nanosystems: Physics, Chemistry, Mathematics* **4** 206–213
- [16] Drozdov A A, Kozlov S A, Sukhorukov A A and Kivshar Y S 2012 *Phys. Rev. A* **86** 053822
- [17] Drozdov A A, Sukhorukov A A and Kozlov S A 2014 *Int. J. Mod. Phys. B* **28** 1442007