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Abstract. Current inverse optimization-based treatment planning for radiotherapy requires a set of complex DVH objectives to be simultaneously minimized. This process, known as multi-objective optimization, is challenging due to non-convexity in individual objectives and insufficient knowledge in the tradeoffs among the objective set. As such, clinical practice involves numerous iterations of human intervention that is costly and often inconsistent. In this work, we propose to address treatment planning with convex imputing, a new-data mining technique that explores the existence of a latent convex objective whose optimizer reflects the DVH and dose-shaping properties of previously optimized cases. Using ten clinical prostate cases as the basis for comparison, we imputed a simple least-squares problem from the optimized solutions of the prostate cases, and show that the imputed plans are more consistent than their clinical counterparts in achieving planning goals.

1. Introduction

Radiation therapy planning is a challenging process that relies heavily on dose planner expertise to achieve clinical planning objectives. Specifically, the dose planner must balance the conflicting objectives of target coverage and organ-sparing – both of which depend on patient geometry – by iteratively modifying dose-volume goals. Moreover, the dose planner must ensure that the optimal plan does not suffer from unwanted spatial dose phenomena, such as hot spots. This heavy dependence on operator judgment risks solution sub-optimality and plan inconsistency, in addition to the high cost of human operator time and effort. Therefore, it is important to design techniques to streamline the planning process to improve plan quality and consistency and to increase patient throughput.

In this study, we propose a novel method of convex imputing [1] for radiotherapy treatment planning to overcome these limitations. Specifically, we aim to obtain a latent imputing function whose M-estimator best approximates the behaviour of previously optimized clinical plans. Since previously validated and approved clinical plans reflect both the quantitative goals of target coverage and organ-sparing and the qualitative dose-shaping goals of the physician, the imputing operator would "inherit" such properties implicitly and impose such on new plans. In doing so, we establish a method that: 1) automates the treatment planning process; 2) enables learning of planning priorities; and 3) consistently produces high quality plans.

To establish the feasibility and efficacy of this novel method, we tested our imputing framework on a set of ten clinical IMRT prostate cases that were previously optimized. In Section II, the fundamental theory of convex imputing will be introduced and our specific formulation will be developed. In

Section III, important results will be reported and discussed. In Section IV, we will conclude with a summary of our work.

2. Methods

To begin, we consider an optimization problem where solution x is obtained by optimizing an objective function OF subject to constraints, where both OF and the constraints depend on a parameter set p. The goal of convex imputing is to learn the form of OF, i.e. weights w, given paired observations of previously optimized solutions x_k and their associated parameters p_k. It is assumed that the constraint functions are known and that OF is both convex and affine (Eq. 1).

$$minimize \ \underbrace{\sum_{n=1}^{N} w_n f_n(x,p)}_{OF} \quad s.t. \quad g_i(x,p) \leq 0 \ for \ i = 1, \dots, m; \ A(p)x = b(p) \qquad (1)$$

For a given p, solution x is optimal if the dual variables λ and v satisfy the KKT conditions. By contrast, for a given p, solution x is approximately optimal if the KKT conditions approximately hold. Thus, approximate optimality interprets the KKT conditions are small residuals, as shown below in Equations 2a-d.

$$r_{ineq} = \big(g_i(x,p)\big)_+, \qquad i = 1, \dots, m \qquad (2a)$$

$$r_{eq} = A(p)x - b(p) \qquad (2b)$$

$$r_{stat} = \nabla_x\big(\text{OF}(x,p)\big) + \sum_{i=1}^{m} \lambda_i \nabla_x \, g_i(x,p) + A(p)^T v \qquad (2c)$$

$$r_{comp} = \lambda_i g_i(x,p), \qquad i = 1, \dots, m \qquad (2d)$$

Since the primal residuals, r_{ineq} and r_{eq}, are fixed for each $\{x_k, p_k\}$ and do not depend on weights w, they cannot be used to impute weights w. Therefore, to impute weights w we solve the following problem:

$$w^{imputed} \quad = \quad \underset{w}{argmin} \sum_{k=1}^{K} \underbrace{\phi\big(r_{stat}^k, r_{comp}^k\big)}_{\substack{non-neg. \\ convex \ pen.}} \quad s.t. \quad \underbrace{\lambda^k \geq 0}_{low \ bnd.}, \ \underbrace{w \geq 0}_{pos.}, \ \underbrace{\sum_{n=1}^{N} w_n \geq 1}_{norm.} \qquad (3)$$

The first term in Equation 4 penalizes deviation of the stationarity and complimentary slackness residuals from 0, thereby making the proposed forward problem (Eq. 1) approximately optimal with respect to the paired observations $\{x_k, p_k\}$. For our formulation, we selected the L_2 norm to be used for this purpose, i.e. $\phi\big(r_{stat}^k, r_{comp}^k\big) = \big\|r_{stat}^k\big\|_2^2 + \big\|r_{comp}^k\big\|_2^2$. The inequalities in Equation 3 assure that: 1) the dual function of the imputed OF gives the lower bound of the forward problem for observations $\{x_k, p_k\}$; 2) the imputed OF is convex; and 3) the imputed solution is not trivial, e.g. $w = 0$. Since both r_{stat}^k and r_{comp}^k are linear, the imputing problem is a simple least-squares formulation and may be efficiently solved to its global minimum. The imputed objective function $\sum_{n=1}^{N} w_n^{imputed} f_n(x,A)$ can then be used to automatically optimize future instances of the forward problem that will be consistent with previously optimized solutions x_k Thus, by applying this method to radiotherapy, the imputed forward problem may be used to automatically optimize new cases such that the resulting

optimal plans reflect the quantitative dosimetric goals of the institution and the qualitative dose-shaping goals of the physician present in training cases.

In designing an appropriate forward problem to be imputed, we utilized three main premises. First, forward problems with complex/exotic penalties are less efficient to optimize than forward problems with simple penalties. Thus, it was desirable to design the forward problem to have simple penalties, so we restricted the *OF* to have quadratic penalties only. Second, while problem geometry largely limits achievable organ-sparing [2], planning consistency is more indicative of organ priority. For example, the rectum absorbs more dose than the femoral heads due to its geometric proximity with the PTV, but rectum-sparing is prioritized higher than femoral-head sparing. Thus, it was desirable to minimize this effect by normalizing structure penalties with an average measure of prescription violation, so we normalized penalties for each structure s with an inter-case average of dose variance $\overline{\sigma_s^2} = \frac{1}{C} \sum_c \sigma_s^2(case\ c)$. Third, penalizing the PTV and organs alone would not be sufficient to capture dose-shaping behaviours present in the clinical plans. Thus, several ring structures, which were created from successive volume expansions of the PTV in 0.5 cm radial increments, were used to help shape dose falloff from the PTV.

3. Results and Discussion

Consistently delivering high-quality treatment plans is crucial for the efficacy of radiotherapy. Excepting machine failures and inter-operator contouring differences, treatment planning is the largest obstacle to achieving this goal since dose planners work under time constraints and submit plans of variable quality. In this section, we will demonstrate how convex imputing may be used to improve plan quality and consistency.
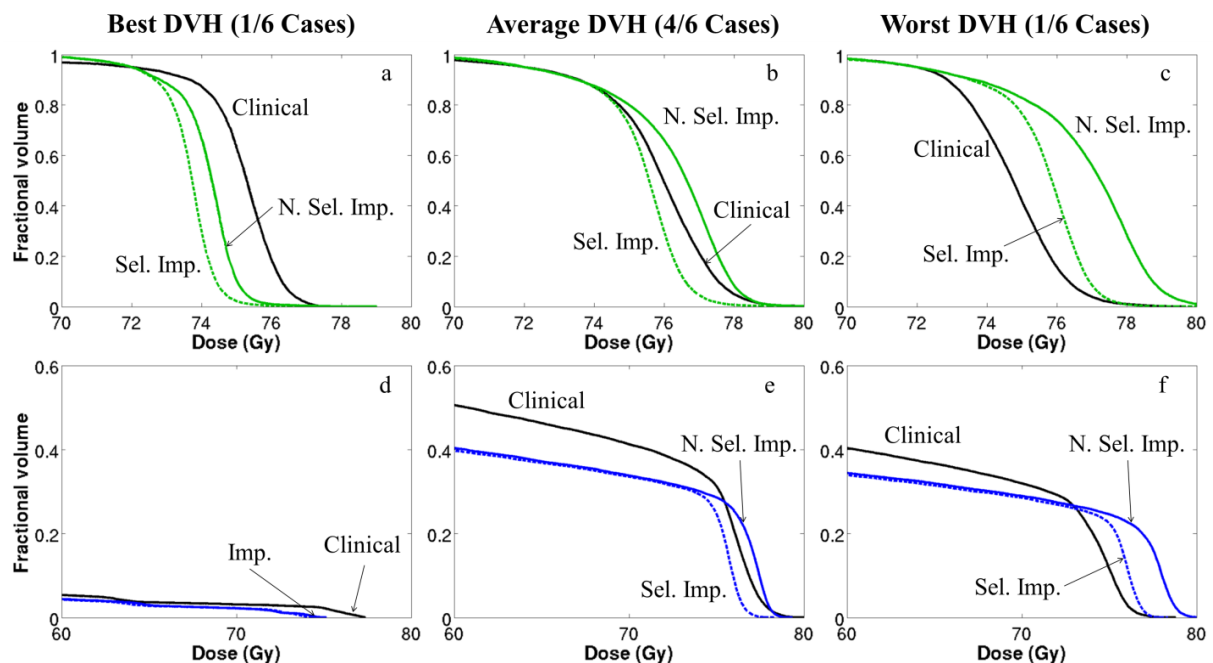

Figure 1. DVH results: PTV (a-c) and Bladder (d-f).

Two sets of imputed objectives were learned for six test cases from separate training sets: 1) the non-selectively imputed objective for each test case was learned from the other nine cases; and 2) the selectively imputed objective for each test case was learned from four cases possessing excellent PTV homogeneity and high-dose bladder-sparing. In Figure 1, the imputed plans were clustered into three categories according to how well they performed w.r.t. the clinical plans in terms of PTV homogeneity and high-dose bladder-sparing. The selectively imputed plans possessed greater PTV homogeneity and

high-dose bladder-sparing than: 1) the clinical plans in the majority of cases and 2) the non-selectively imputed plans in all cases. In addition, the selectively imputed plans improved upon the non-selectively imputed plans and clinical plans in terms of rectum-sparing and bulb-sparing, as shown by the characteristic cases in Figure 2.
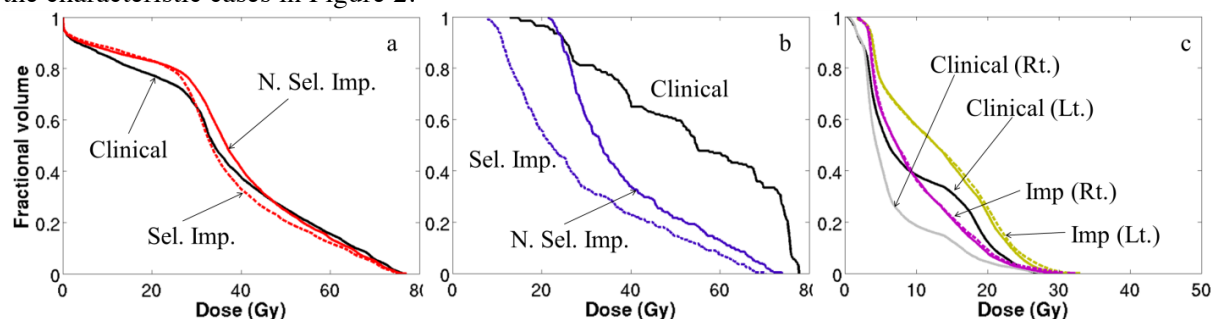

Figure 2. DVH results: Rectum (a), Penile Bulb (b), Femoral Heads (c).

In Figure 3, the non-selectively imputed plan is shown to possess relatively large hot spots whereas the selectively imputed plan does not, further demonstrating that imputing may be improved via proper case selection. In addition, both imputed plans possessed dose falloff from the PTV that was more conformal than that of the clinical plan, and the selectively imputed plan possessed dose falloff that was more conformal than the non-selectively imputed plan, as evidenced by the 50% isodose line.
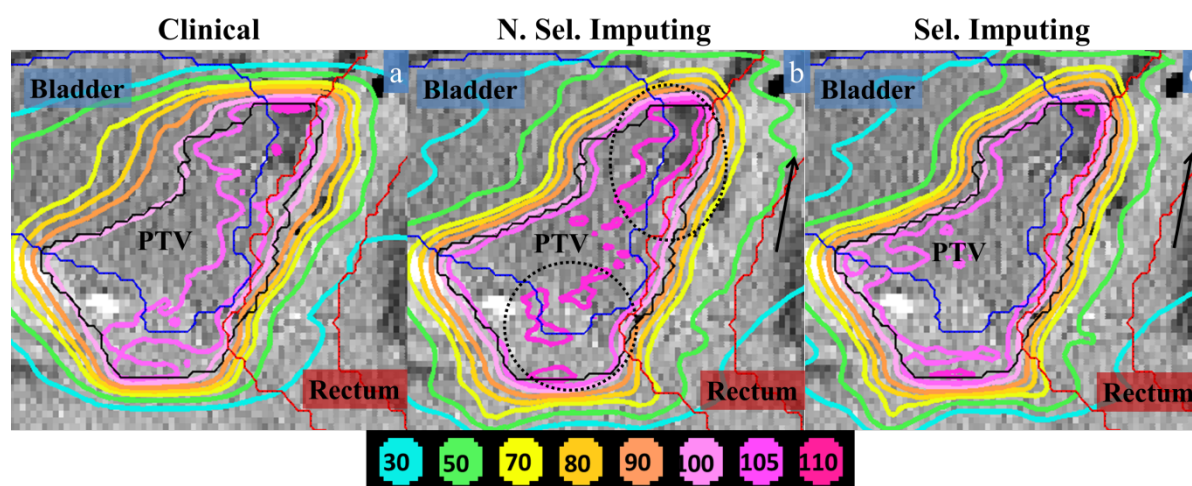

Figure 3. Dose maps. Isodose lines shown. PTV (black), Bladder (dark blue), Rectum (dark red).

4. Conclusions

We have developed a novel method that uses convex imputing to automate the treatment planning process by learning the form of a convex objective function from a training set of previously optimized cases. We demonstrated that only a small number of training cases are needed to learn the DVH and dose-shaping preferences of the physician, and showed that the imputed plans are more consistent than their clinical counterparts in achieving planning goals.

References

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