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# The importance of risk-aversion as a measurable psychological parameter governing risk-taking behaviour

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**Abstract.** A utility function with risk-aversion as its sole parameter is developed and used to examine the well-known psychological phenomenon, whereby risk averse people adopt behavioural strategies that are extreme and apparently highly risky. The pioneering work of the psychologist, John W. Atkinson, is revisited, and utility theory is used to extend his mathematical model. His explanation of the psychology involved is improved by regarding risk-aversion not as a discrete variable with three possible states: risk averse, risk neutral and risk confident, but as continuous and covering a large range. A probability distribution is derived, the "motivational density", to describe the process of selecting tasks of different degrees of difficulty. An assessment is then made of practicable methods for measuring risk-aversion.

## 1. Introduction

"Fear of failure explains all those rogue trader scandals, most of which started with a cover up", according to former banker turned best-selling author, Robert Kelsey [1], [2]. Kelsey says "It wasn't that I was unable to take the risks that are part and parcel of an investment banker's role ... I could take ludicrous risks in some of the most volatile trading environments on the planet ... It was my judgement of risk that was undermined by my fears [as was] my ability to sort good fear from bad fear [which] can lead to either paralysis ... or nonsensical leaps."

Such extreme behaviour by fearful people has been known to psychologists for many decade. As John W. Atkinson noted: "The tendency for anxious persons to set either extremely high or very low aspirations has been noted over and over again in the literature on level of aspiration" [3]. Atkinson was setting out to explain the behaviour observed with five-year-olds playing the game of hoop-the-peg, where greater rewards were on offer the farther back the child stood. Here children who had been found by independent tests to be highly achievement-motivated took up a throwing position that was a challenging but realistic distance from the peg. But the children found previously to have low achievement motivation were found to act differently. Either they stood almost on top of the peg or else, perplexingly, so far back that failure was almost certain.

Atkinson devised a simple, but powerful mathematical model in order to explain this apparently contradictory behaviour. But it is possible to improve on his explanation by developing a utility function with risk-aversion as its governing parameter. Risk-aversion is a well established parameter from the field of economics and decision science, and the fact that it can be defined precisely in mathematical terms means that Atkinson's results may be generalised, thus offering the potential for the accurate measurements of risk-aversion.

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## 2. John W. Atkinson's mathematical insights

Atkinson needed to devise a mathematical model in order to explain the apparently contradictory behaviour described above. For any task, he saw the "motivation to succeed", $D_s$ as the product of three factors: the incentive to succeed, $I_s$, the "expectancy" or probability of success, $p_s$, and the "motive to succeed", $M_s$. This would be counterbalanced by the "motivation to avoid failure", $D_f$, found by multiplying the incentive to avoid failure, $I_f$, by both the "expectancy" ( = probability) of failure, $p_f$, and the "motive to avoid failure", $M_f$. The resultant motivation to achieve, $D_r$, would then be the difference between the motivations to succeed and to avoid failure:

$$D_r = D_s - D_f = M_s p_s I_s - M_f p_f I_f \qquad (1)$$

Moreover, he modelled the incentive to succeed as the complement of the probability of success:

$$I_s = 1 - p_s = p_f \qquad (2)$$

reasoning that the more difficult a task was, the greater would be the gain in both personal satisfaction and kudos when it was achieved. Atkinson foresaw this model carrying over to more general tasks faced in life, commenting that "the ranking of occupations according to their prestige ... clearly suggests that occupations accorded greater prestige are also more difficult to obtain".

Atkinson accorded failure a psychological weighting that was different from that accorded to lack of success, which might attract only a null weighting. He took losing as being painful in its own right, with the dissatisfaction and embarrassment at failing with an easy task being greater than those associated with failure in a difficult task. He considered the incentive to avoid failure proportional to its probability of success, with a unity constant of proportionality:

$$I_f = p_s \qquad (3)$$

The greater the task's chance of success, the greater would be the dissatisfaction and shame if the subject failed at that task. (The link between degree of shame and the extent to which individuals fear failure is explored further in [4]).

Nevertheless, the model Atkinson produced had some deficiencies. Literally interpreted, it pointed to the anxious individual preferring a task where success was either completely certain or completely impossible, but still having no net motivation to undertake either. To cover this, Atkinson invoked a further influence, "e.g. social pressures", as necessary to persuade the individual to take on either the easiest available or the hardest available task, but did not include this in the model. Moreover, Atkinson's model was unable to distinguish between degrees of anxiousness beyond just three states, which might be characterised as risk averse, risk neutral and risk confident. Yet common experience suggests that people vary in their levels of anxiety in a more gradual way than this ternary system would suggest, underlining the need to have a fuller model. This paper combines Atkinson's insights with utility theory to construct the more general mathematical model outlined in the next section.

## 3. Modelling the hoop-the-peg game using utility theory

The reward associated with success in a task may be described by a utility function of which the argument is the incentive for success. The utility of success is then modelled using a Power utility function [5], with risk-aversion, $\varepsilon$, as parameter:

$$u_s = a_s (1 - p_s)^{1-\varepsilon} \qquad (4)$$

where $a_s$ is a constant that reflects the fact that the utility function is correct to a positive multiplier. Its explicit inclusion will facilitate the later comparison with the utility of failure which will be considered next. The utility function of equation (4) becomes identical to Atkinson's incentive for success, $u_s = I_s$, when $\varepsilon = 0$ and $a_s = 1$, in much the same way that a utility function applied to money becomes either the money or a multiple of it when $\varepsilon = 0$.

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The penalty associated with failure in a task may be described by a disutility function, the argument of which will be the incentive to avoid failure. In choosing the form of the disutility function, we note that, while utility functions are usually assumed concave, reflecting satiation with good things, disutility functions may be taken generally to be convex, reflecting an accelerating growth in dissatisfaction as bad things accumulate [6]. While successive increments of utility can be expected to fall in size under the addition of equal increments of wealth, the increments of disutility for successive increments of debt are likely to increase in absolute value, implying that the disutility of being £20,000 in debt is more than 10 times worse than being indebted to the tune of £2,000. Accordingly, a suitable form for disutility is the Power disutility function, which has the same form as the Power utility function, except that the risk-aversion parameter, $\varepsilon$, is preceded by a plus rather than a minus sign. It is assumed that the individual's utility and disutility functions will use the same value of risk-aversion. Thus the disutility of failure, $v_f$, is given by:

$$v_f = a_f p_s^{1+\varepsilon} \qquad (5)$$

where the constant, $a_f$, reflects the fact that the utility function is correct to a positive multiplier.

The disutility function of equation (6) becomes identical to Atkinson's incentive to avoid failure, $v_f = I_f$, when $\varepsilon = 0$ and $a_f = 1$, in a similar way to that in which a disutility function applied to debt becomes simply the debt when $\varepsilon = 0$. The utility of failure is then the negative of the disutility of failure: $u_f = -v_f$.

Let us apply the utility model developed above to a task with success probability, $p_s$. The expected utility, $y(\varepsilon, p_s)$, for a person with risk-aversion, $\varepsilon$, is then

$$y(\varepsilon, p_s) = E[U(\varepsilon)] = p_s u_s + (1-p_s)u_f = a_s p_s (1-p_s)^{1-\varepsilon} - a_f (1-p_s) p_s^{1+\varepsilon} \qquad (6)$$

While Atkinson's model of the hoop-the-peg game assigns a discrete uniform probability distribution to tasks with success probabilities ranging from 0.1 and 0.9, the model may be generalised by assuming there to be a continuum of tasks with success probability, $p_s$, conforming to a uniform distribution between 0 to 1. The probability density for success probability, $f(p_s)$, will then obey simply: $f(p_s) = 1$ for $0 \le p_s \le 1$. Hence the expected utility, $y_T(\varepsilon)$, across all tasks for a person with risk-aversion, $\varepsilon$, will be:

$$y_T(\varepsilon) = \int_{p_s=0}^{1} y(\varepsilon, p_s) f(p_s) dp_s = a_s \int_{p_s=0}^{1} p_s (1-p_s)^{1-\varepsilon} dp_s - a_f \int_{p_s=0}^{1} (1-p_s) p_s^{1+\varepsilon} dp_s = \frac{a_s}{(2-\varepsilon)(3-\varepsilon)} - \frac{a_f}{(2+\varepsilon)(3+\varepsilon)} \qquad (7)$$

In betting, a fair game is one where the expected monetary gains are offset exactly by the expected monetary losses. By analogy, we may define a fair game when the rewards may be other than monetary as one where the expected net reward is zero for a risk-neutral person. Since $\varepsilon = 0$ for a risk-neutral person, a fair game implies, from equation (7):

$$y_T(0) = \frac{1}{6}(a_s - a_f) = 0 \qquad (8)$$

This will be satisfied when $a_s = a_f$. The risk-neutral person will regard taking part in such a game with equanimity, since, if he undertook each of the tasks within the game, he would not expect overall to lose, even if he would not expect to win overall either. The idea of a fair game may be expanded to encompass the notion of a "fair game *according to his lights*" if we replace the condition $y_T(0) = 0$ by $y_T(\varepsilon) = 0$, which will imply that a person with risk aversion, $\varepsilon$, would regard the game as fair in that if he undertook each of the tasks in it, he could expect to break-even in the sense that his psychological pain would be matched by his psychological gain. This condition may be satisfied by setting the weighting of utility of success (arbitrarily) at $a_s = 1$ and varying $a_f$. Solving equation (7) under these conditions gives

$$a_f = \frac{2+\varepsilon}{2-\varepsilon}\frac{3+\varepsilon}{3-\varepsilon} \tag{9}$$

Substituting back into equation (3) gives the expected utility for each task for someone with risk-aversion, $\varepsilon$, under the condition that he will regard the game as fair according to his lights, so that he will be prepared to play it:

$$y(\varepsilon, p_s) = p_s(1-p_s)^{1-\varepsilon} - \left(\frac{2+\varepsilon}{2-\varepsilon}\frac{3+\varepsilon}{3-\varepsilon}\right)(1-p_s)p_s^{1+\varepsilon} \tag{10}$$

It may be noted that this equation depends solely on the task's success probability and the individual's risk-aversion.

## 4. The choice of task for an individual of given risk-aversion

Plotting equation (10) shows that the choice of task is unimodal for a risk-confident individual, who will have a negative risk-aversion (Fig.1), while it is bimodal for a risk-averse individual (Fig. 2).

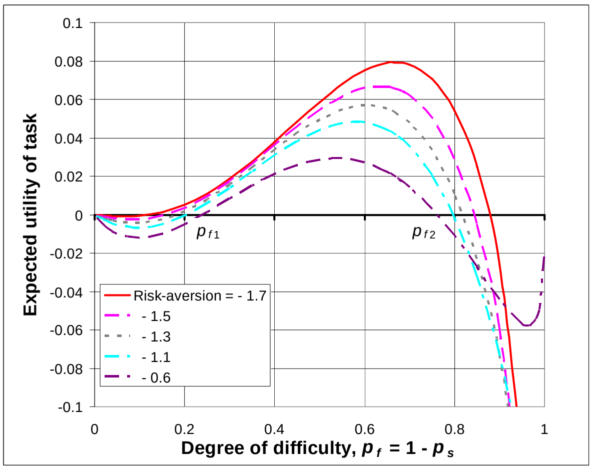


**Figure 1.** Risk confident individual, $\varepsilon < 0$

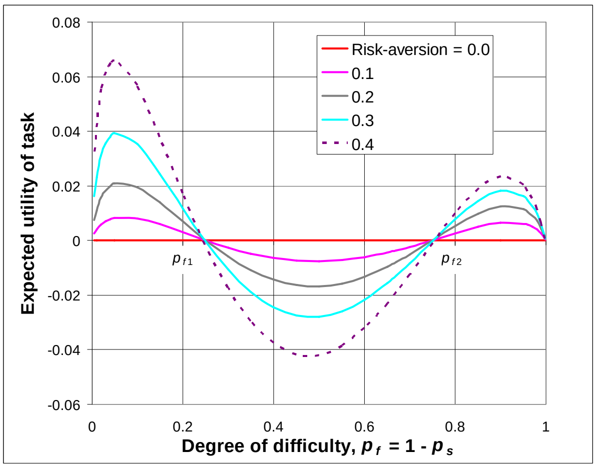


**Figure 2.** Risk averse individual, $\varepsilon > 0$

The curves are plotted against the degree of difficulty of the task, the failure probability, $p_f = 1 - p_s$. The curve at the risk-neutral value, $\varepsilon = 0$, coincides with the $p_f$-axis. For all other values of risk-aversion in the range defined by the model, $-2 \le \varepsilon \le 1$, the curve starts and finishes on the $p_f$-axis and also cuts it twice, at the values, $p_{f1}$ and $p_{f2}$, given by:

$$p_{f1} = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4\left(\frac{2+\varepsilon}{2-\varepsilon}\frac{3+\varepsilon}{3-\varepsilon}\right)^{-\frac{1}{\varepsilon}}}\;;\quad p_{f2} = 1 - p_{f1} \tag{11}$$

It is striking that, while risk-confident individuals are predicted to choose tasks that are challenging, but not impossible, the model suggests that people who are risk-averse may choose either a very easy task, or else one with a high degree of difficulty. This is exactly the behaviour reported by Atkinson.

The situation may be generalised by assigning a probability density for the chosen degree of difficulty that is proportional to the expected utility associated with that degree of difficulty provided the expected utility is positive, and zero otherwise. This yields a "motivational density", $h(\varepsilon, p_f)$, applicable to all tasks where the reward is the esteem and kudos associated with the completion of the task rather than a specific, direct benefit; such a benefit may well accrue later, but cannot be included in the calculation of utility because it is not known. Such a situation applies widely to tasks in life, including many associated with people's professional life. As graphed in Figures 3 and 4 against degree of difficulty, $p_f$, the motivational density is given by:

$$h(\varepsilon, p_f) = \begin{cases} \max\left\{0, \dfrac{(2+\varepsilon)(3+\varepsilon)p_f(1-p_f)^{1+\varepsilon} - (2-\varepsilon)(3-\varepsilon)(1-p_f)p_f^{1-\varepsilon}}{D}\right\} & \text{for } \varepsilon < 0 \\ \max\left\{0, \dfrac{(2-\varepsilon)(3-\varepsilon)(1-p_f)p_f^{1-\varepsilon} - (2+\varepsilon)(3+\varepsilon)p_f(1-p_f)^{1+\varepsilon}}{D}\right\} & \text{for } \varepsilon > 0 \end{cases} \quad (12)$$

$$\begin{aligned} D = &(1-p_{f2})^{2-\varepsilon}((2-\varepsilon)p_{f2}+1) - (1-p_{f1})^{2-\varepsilon}((2-\varepsilon)p_{f1}+1) \\ &+ (3+\varepsilon)p_{f2}^{2+\varepsilon} - (3+\varepsilon)p_{f1}^{2+\varepsilon} + (2+\varepsilon)p_{f1}^{3+\varepsilon} - (2+\varepsilon)p_{f2}^{3+\varepsilon} \end{aligned} \quad (13)$$

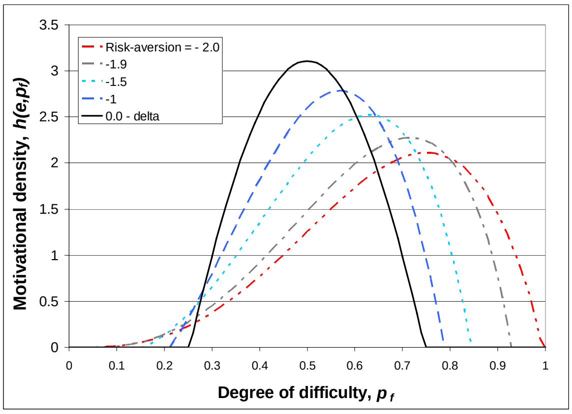


**Figure 3.** Risk confident individual, $\varepsilon < 0$

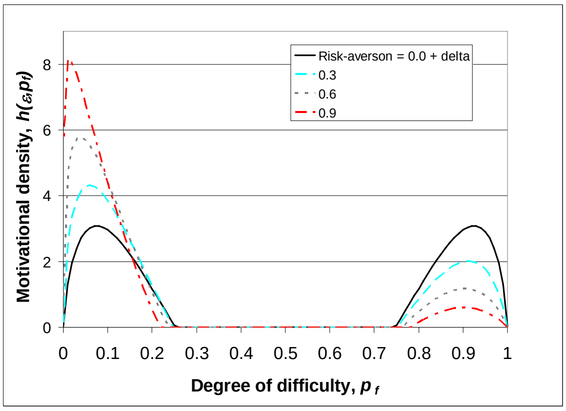


**Figure 4.** Risk averse individual, $\varepsilon > 0$

It is clear from Figure 3 that for risk confident people, their chosen degree of difficulty increases as risk-confidence grows (risk-aversion becomes more negative). However, the situation is more complex for risk averse individuals, where the distribution is bimodal. From Figure 4, people with a positive but very low risk-aversion will choose either a very easy task, with a probability of failure of 25% or less, or else a very hard task, with a probability of failure of 75% or more. As they become more risk averse, they will tend to prefer easier tasks, but their tendency to choose very difficult tasks persists, as shown by the lobe in the motivational density curve centred around the high degree of difficulty, $p_f \approx 0.9$.

Accurate measurement of risk-aversion becomes possible as soon as the motivational density has been defined mathematically. Setting a series of generally similar tasks where the degree of difficulty needs to be chosen will enable an average chosen degree of difficulty to be calculated, from which the individual's risk-aversion may be deduced. Figures 5 and 6 show the relationship between risk-aversion and the mean for both risk confident and risk averse subjects, with Figure 5 showing also the mode for risk confident persons. These figures show that the average degree of difficulty tends to 0.5 as risk-aversion, $\varepsilon$, approaches zero both from above and below. Even though Figures 3 and 4 make clear that the mode divides into two as $\varepsilon$ increases past 0.0, Figures 5 and 6 show that, on average, more risk averse individuals prefer easier tasks, confirming the intuitive interpretation of that parameter. A rational individual will experience different levels of risk-aversion depending on the decision facing him [7], [8], but nevertheless psychological tests interpreted via equations (12) and (13) may provide a ranking of the risk-aversions of different individuals facing similar decisions.

## 5. Conclusions on the importance of the risk-aversion parameter

It has been observed previously that risk-aversion, acting as the governing parameter of a utility function, can provide a mathematical explanation for panic: at a very high level of risk-aversion, the decision maker may act rashly because he is unable to distinguish between acting to enhance safety or else behaving in such a way as to diminish it [7], [8]. This study extends the field in which a utility

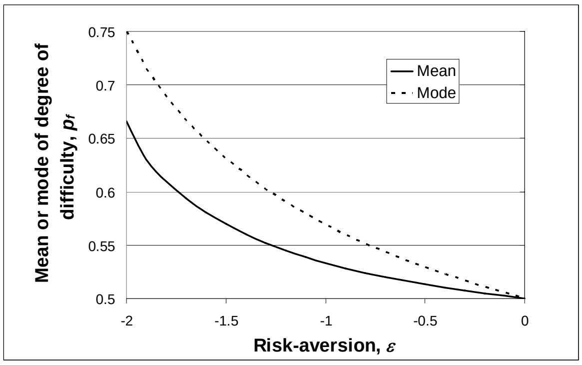


**Figure 5.** Risk-confident individual, $\varepsilon < 0$

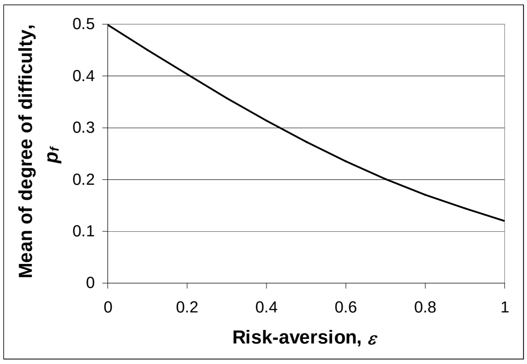


**Figure 6.** Risk-averse individual, $\varepsilon > 0$

function with risk-aversion as parameter can provide an insight into human behaviour, in this case into actions where the potential rewards are not monetary (at least in the first instance), but satisfaction, esteem and kudos. It has been shown once again how individuals who are highly risk averse may act in the rash way observed in practice. Methods of applying the newly derived motivational density as the basis for measuring risk-aversion have been discussed, while further methods will be discussed in a companion paper [9].

## 6. Acknowledgements

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