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Uncertainty in airflow field parameters in a study of shock waves on flat plate in transonic wind tunnel

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Abstract. When flat plates are submitted to wind tunnel tests and the upstream Mach number is greater than 1.0 this gives rise to shock and expansion waves. This study experimentally evaluates the airflow field parameters ahead of and behind the shock region and compares the data obtained in the tests with the expected theoretical results. For the case of expansion waves only theoretical values are supplied in this preliminary part of the study. The drag and lift forces acting on the model being tested are also estimated from the pressure values measured on the plate surface. Data reduction includes evaluation of the uncertainties associated with the measured quantities, as well as their propagation to the output quantities, employing the Monte Carlo method. Numerical simulation of the phenomena using Computational Fluid Dynamics is also done. Data obtained analytically, experimentally and numerically are compared.

1. Introduction

Shock waves occur in nature when an object moves faster than the speed of sound and when the presence of the object causes an abrupt decrease in the airflow area. If the airflow area increases, a different phenomenon is observed: an expansion fan [1,2]. Shock and expansion waves cause changes in the gas properties and this is the aim of the present study: to experimentally evaluate the conditions associated with the shock and expansion caused by the presence of the flat plate in the airflow. Aerodynamic forces acting on the plate are also calculated. The uncertainties in the estimated flow field parameters are obtained using the Monte Carlo method. The mean value and standard deviation of measured parameters are computed and this information is used to determine the probability density function for output quantities [3]. The study of flat plates is important because they have typical characteristics of all supersonic airfoils. The inclined flat plate is the simplest case of lift airfoil. The tests were carried out in the TTP pilot transonic wind tunnel facility. In the first part of the test campaign, the flat plate was positioned at an angle of attack equal to -4° . The angle between the direction of the incident airflow and the direction of the plate is called angle of attack α .

2. The TTP pilot transonic wind tunnel

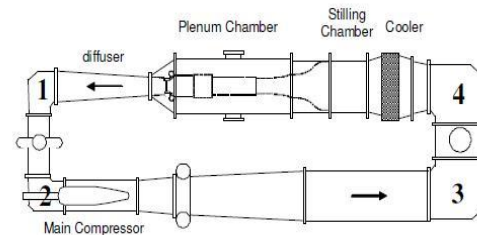
The wind tunnel TTP is located at the Aerodynamics Division of the Institute of Aeronautics and Space, IAE/ALA/TTP (figure 1a). The tunnel is continuously driven by an 830 kW main axial



compressor and can also be operated intermittently by means of an injection system, which supplies airflow for around 30 seconds. It is a variable-pressure wind tunnel with control capability to independently vary Mach number, stagnation pressure, stagnation temperature and humidity. The test section is 0.25 m high and 0.30 m wide and has longitudinally slotted walls to favor the uniformity of the airflow. At TTP, the Mach number range can vary from 0.20 to 1.30. Some configurations can be changed in order to allow better flow control such as reentry flaps position and the rate of forced mass extraction, determined by the Plenum Evacuation system, PES. A schematic picture of the aerodynamic circuit is presented in figure 1b, highlighting its main parts: the axial compressor, cooler, stilling chamber, plenum chamber, high velocity diffuser and corners (numbered from 1 to 4). Arrows indicate the direction of the airflow. The test section, which receives the test article, is located inside the plenum chamber.



a) The circuit.

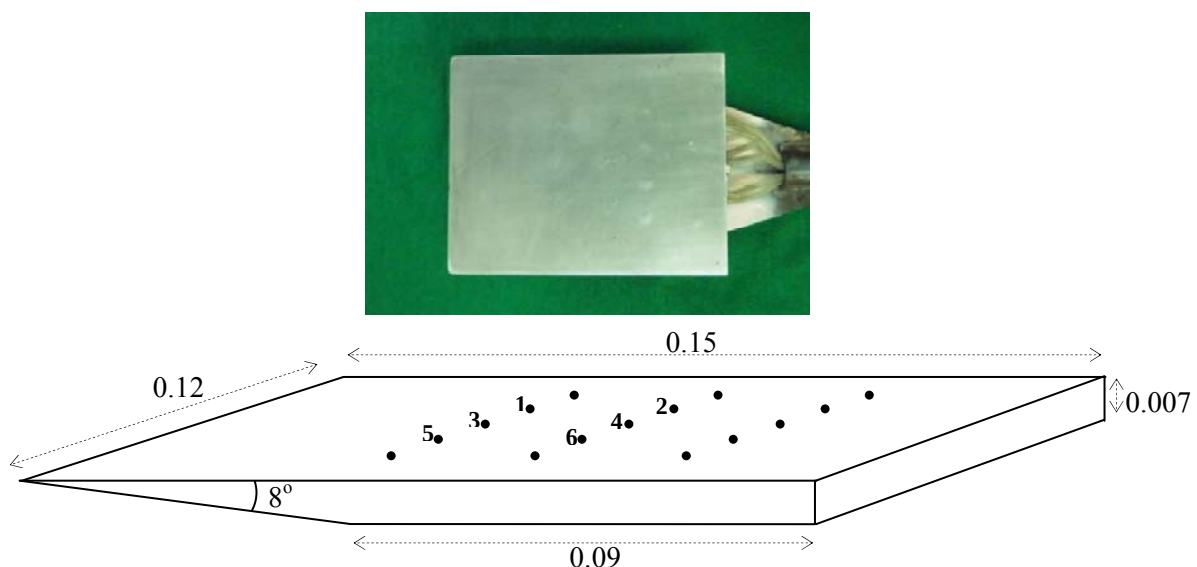


b) The schematic diagram.

Figure 1. The TTP wind tunnel.

3. The flat plate

Airfoils designed for supersonic flows are slender and have a sharp trailing edge. An extreme example of slender airflow is the flat plate. To represent the flat plate a wedge-like model is used (figure 2). The wedge is 0.15 m long, 0.12 m wide and 0.007 m thick. Its superior surface has 15 pressure taps connected to tubes which transmit the static pressure of the airflow to the instrumentation (figure 3). The diameter of the pressure tap holes is 0.5 mm. Only the six anterior central taps are included in the data reduction. The plate is fixed to the wind tunnel test section through a sting support (figure 4).

**Figure 2.** The wedge model picture and schematic. Unit: meter.

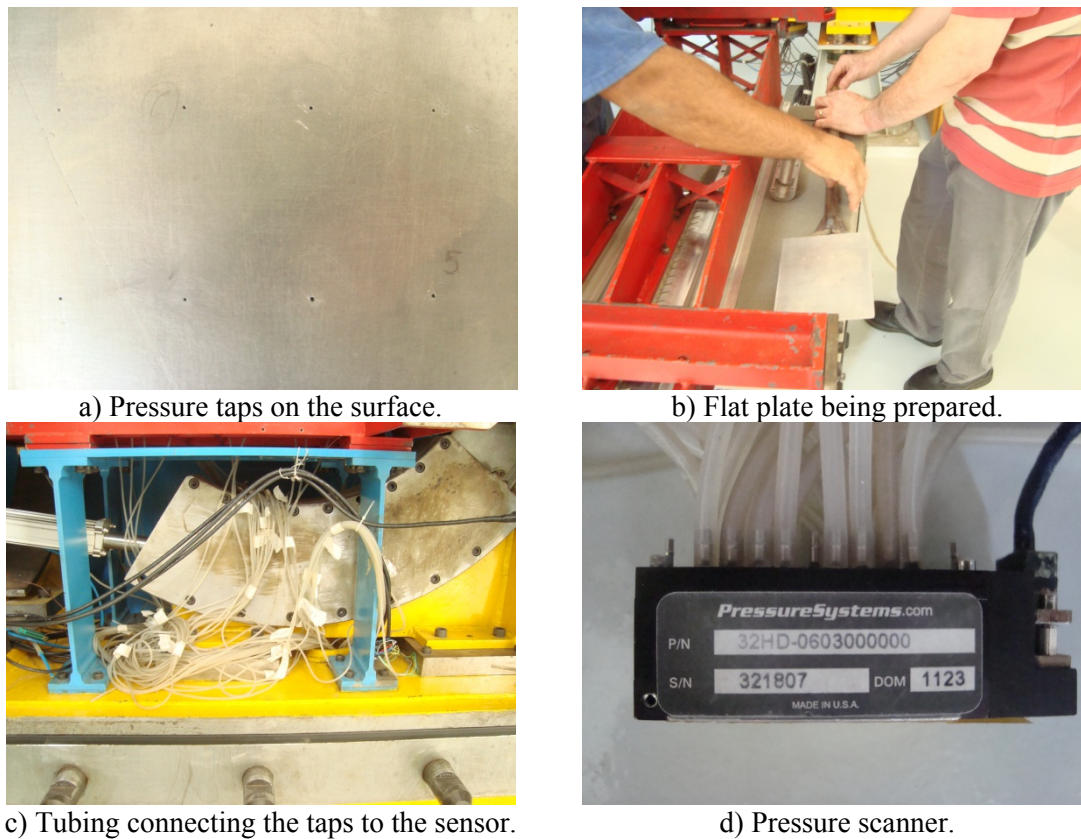


Figure 3. The flat plate and instrumentation.

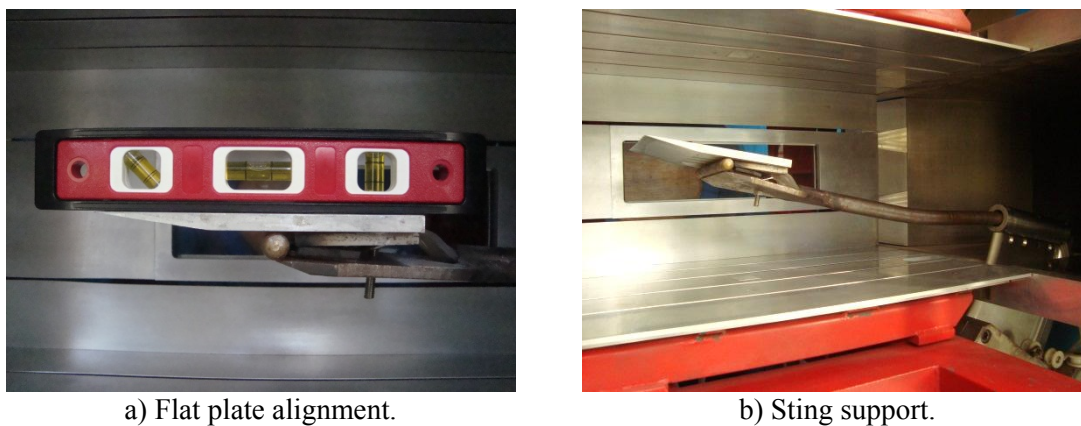


Figure 4. The flat plate installed in the TTP test section.

4. Methodology

In order to evaluate the airflow parameters ahead of the shock and expansion waves caused by the presence of the plate, the freestream airflow static pressure, p_1 , and total pressure, p_0 , were measured. The value of p_1 was obtained at the first pressure tap of the wind tunnel superior wall (figure 5a) and the measurement of p_0 was taken in the plenum chamber (figure 5b).

The mean value and standard deviation of the pressure signals, p_0 and p_1 , supplied by the instruments during the runs of the wind tunnel tests are computed. This information is used to determine the probability density function for Mach number using the Monte Carlo method [3].

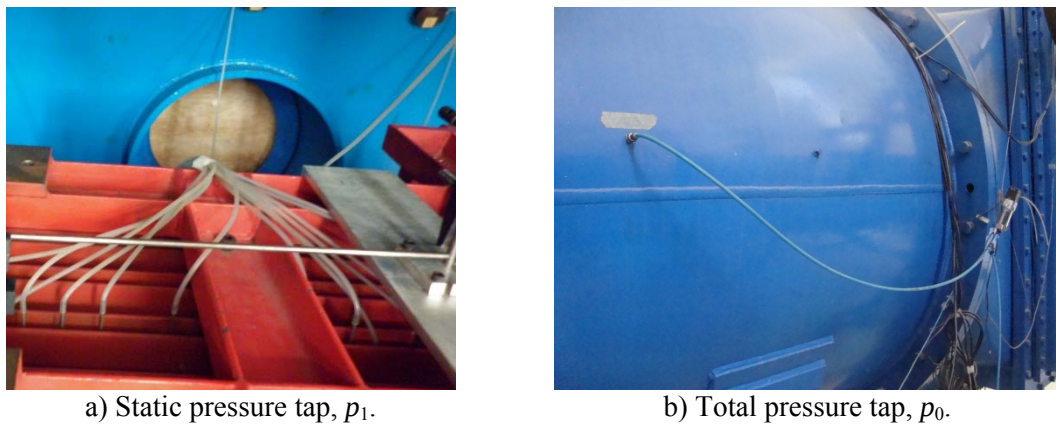


Figure 5. Pressure taps on the superior wall (a) and in the plenum chamber (b).

The static pressure behind the shock and expansion wave regions is estimated through the average value $\bar{p}_{plate}$ calculated from the six static pressure taps located on the flat plate surface.

The angle of the leading edge of flat plate is equal to $+8^\circ$. When positioning the plate at an angle of attack α equal to -4° , the airflow encounters the superior surface of the plate at $+4^\circ$.

In the following sections we will separate the aerodynamic phenomena which take place on the superior and inferior surfaces of the flat plate.

The uncertainties in the estimated parameters are evaluated by the Monte Carlo method. MatLab® codes were elaborated for data reduction. The number of trials employed is 500,000. The distribution assigned to the input quantities is the Gaussian.

4.1. Superior surface

In the superior surface of the plate, the airflow encounters a concave corner and the streamlines are deflected upward through the angle θ equal to $+4^\circ$. An oblique shock occurs in this case (figure 6).

The angle between the shock wave and the upstream flow direction is defined as the wave angle and is denoted by β .

Knowing the flow field parameters in region 1 ahead of the shock, one can estimate the parameters in region 2, behind the shock.

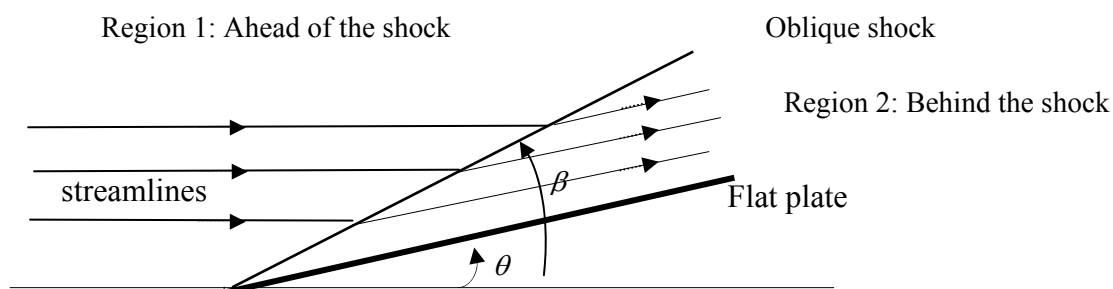


Figure 6. Deflection of a supersonic flow. θ : deflection angle, β : shock wave angle.

The Mach number in region 1 is evaluated by the expression [1]:

$$M_1^2 = \frac{2}{\gamma - 1} \left[\left(\frac{p_0}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \quad (1)$$

where:

p_1 : freestream static pressure;

p_0 : freestream total pressure; and

γ : ratio of specific heats.

The equation relating the deflection angle, θ , the Mach number, M , in region 1 and the shock wave angle, β , is [1]:

$$\tan \theta = 2 \cot \beta \frac{M_1^2 \sin^2 \beta - 1}{M_1^2 (\gamma + \cos 2\beta) + 2} \quad (2)$$

where γ is equal to 1.4 for air at normal conditions.

Knowing the deflection angle, θ , and the Mach number M , in region 1, one obtains the shock wave angle, β , using equation (2) or the θ - β - M diagram [4].

The normal component of the of the upstream Mach number, $M_{n,1}$, is:

$$M_{n,1} = M_1 \sin \beta \quad (3)$$

The pressure ratio across the shock wave is:

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1} (M_{n,1}^2 - 1) \quad (4)$$

The normal component of the Mach number behind the shock, $M_{n,2}$, is:

$$M_{n,2}^2 = \frac{1 + [(\gamma - 1)/2] M_{n,1}^2}{\gamma M_{n,1}^2 - (\gamma - 1)/2} \quad (5)$$

The Mach number behind the shock, M_2 , is calculated by the expression:

$$M_2 = \frac{M_{n,2}}{\sin(\beta - \theta)} \quad (6)$$

The total pressure in region 2, $p_{0,2}$, is evaluated using the equation:

$$p_{0,2} = p_2 \left(1 + \frac{\gamma - 1}{2} M_2^2 \right)^{\frac{\gamma}{\gamma - 1}} \quad (7)$$

4.2. Inferior surface

On the inferior surface of the plate, the airflow encounters a convex corner and an expansion wave is formed. Such wave is denoted as Prandtl-Meyer expansion wave (figure 7).

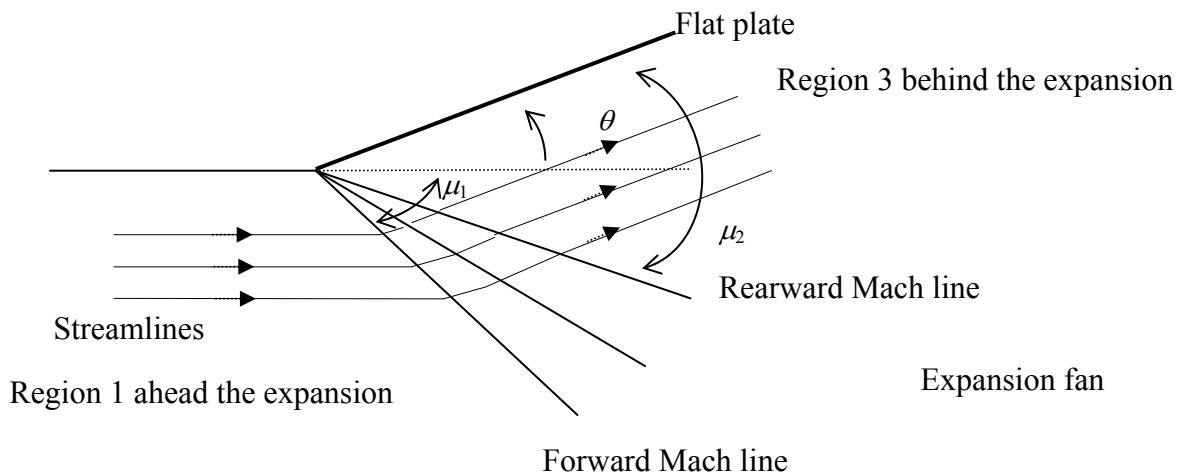


Figure 7. Prandtl-Meyer expansion. θ : deflection angle $= +4^\circ$. μ_1 : angle of the forward Mach line. μ_2 : angle of the rearward Mach line.

In order to obtain the properties of the flow in region 3 behind the expansion wave, knowing the values of p_1 , M_1 and θ , one first estimates the value of the upstream Prandtl-Meyer function, $\nu(M_1)$, using the equation:

$$\nu(M_1) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1} (M_1^2 - 1)} - \tan^{-1} \sqrt{M_1^2 - 1} \quad (8)$$

or using the table “Prandtl-Meyer Function and Mach Angle”, presented in Appendix C of reference [1].

One evaluates the Prandtl-Meyer function in region 3 by employing the following equation:

$$\theta = \nu(M_3) - \nu(M_1) \quad (9)$$

The table of Appendix C [1] supplies the Mach number M_3 , from the value $\nu(M_3)$ given by equation 9.

To obtain the static pressure value behind the expansion wave, p_3 , one uses the expression:

$$p_3 = p_1 \left(\frac{1 + [(\gamma-1)/2] M_1^2}{1 + [(\gamma-1)/2] M_3^2} \right)^{\gamma/(\gamma-1)} \quad (10)$$

Expansion is an isentropic phenomenon and therefore the total pressure is constant across the expansion.

Figure 8 summarizes the phenomena which occur on the flat plate at an angle of attack equal to -4° in a supersonic flow, for the freestream Mach number $M_1 = 1.21$. Flow is left to right. The results were obtained employing Computational Fluid Dynamics, CFD.

At the leading edge, one observes shock waves on the top surface and expansion waves on the bottom surface.

At the trailing edge, the flat plate experiences the opposite behavior. Mach number distribution of the airflow is shown. Only the leading edge region was investigated in this first part of the wind tunnel test campaign.

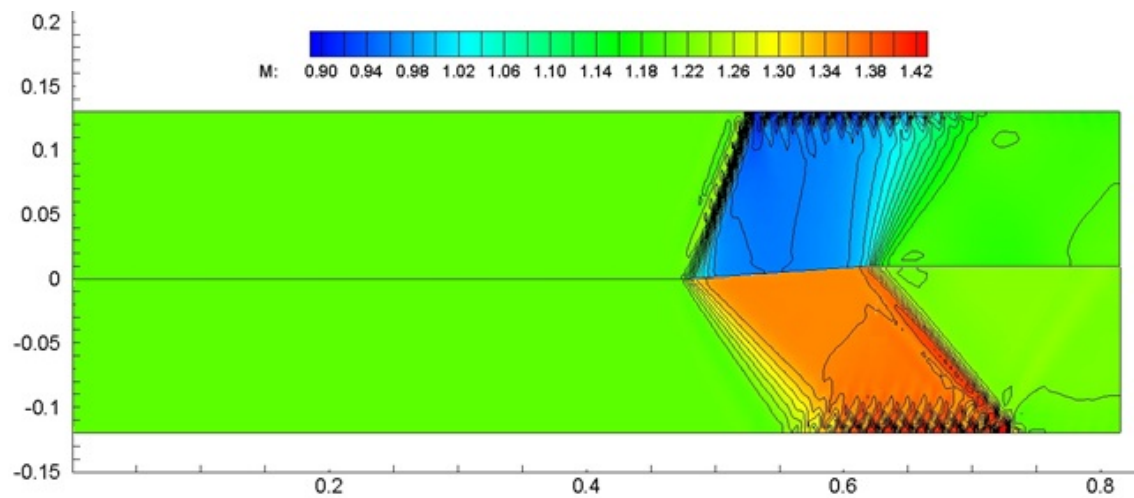


Figure 8. CFD results for the flat plate in a supersonic flow ($\alpha = -4^\circ$). Axis unit: meter.

The numerical code was based on finite difference and structured mesh with first-order implicit time discretization and second-order spatial central discretization. The code follows the main characteristics of the Alternating Direction Implicit scheme, ADI, and linear artificial viscosity. The mesh is composed by two juxtaposed blocks. Each block has 191 points in the longitudinal direction and 51 points in the vertical direction and the Δy minimum is equal to 0.002 m. Inlet boundary conditions were imposed, outlet boundary conditions were extrapolated by zero-th order, slip condition were imposed on wall surfaces together with adiabatic wall consideration and pressure extrapolation. The test section slotted walls were simulated alternating far field static pressure and solid wall configurations.

4.3. Forces in the flat plate

A flat plate of length c at an angle of attack α in a supersonic flow causes oblique shock and expansion waves (figure 9). The plate surfaces are subjected to different pressures, p_3 and p_2 , which generates a resultant aerodynamic force R .

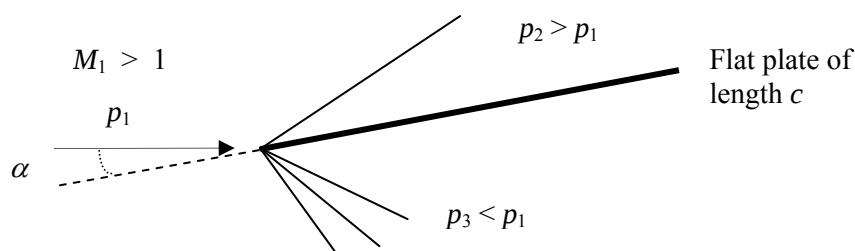


Figure 9. An inclined flat plate in a supersonic flow.

The resultant force for a unit of span, R' , and its components lift, L' , and drag, D' , are expressed by[1]:

$$\begin{aligned} R' &= (p_3 - p_2)c \\ L' &= (p_3 - p_2)c \cos \alpha \\ D' &= (p_3 - p_2)c \sin \alpha \end{aligned} \quad (11)$$

where c is the length of the plate.

The force coefficients, C_F , are calculated using the equation:

$$C_F = \frac{F}{q_\infty S} \quad (12)$$

In equation (13), F represents the lift force, L , or the drag force, D , S is the reference area and q_∞ is the dynamic pressure of the freestream flow, given by:

$$q_\infty = \frac{\gamma}{2} p_1 M_1^2 \quad (13)$$

5. Results

5.1. Superior surface

The mean value of the static pressures, measured by the tap pressures located on the flat plate, is $\bar{p}_{plate} = 66491.3$ Pa, with a standard deviation equal to 2506.7 Pa.

Deflection angle: $\theta = 4$ degrees.

Through the measurement of the total pressure, p_0 , and the static pressure ahead of the shock, p_1 , we obtain the Mach number M_1 using equation 1. The value of p_1 was obtained in the first pressure tap on the superior wall of the tunnel and the measurement of p_0 was taken in the plenum chamber. The mean value and standard deviation of the pressure signals, p_0 and p_1 , are calculated and used to estimate the M quantity and the standard uncertainty associated with this estimate, u_M . Results are: $M_1 = 1.21$, $u_M = 0.01$ for the measured values $p_1 = 50853.0$ Pa and $p_0 = 124348.7$ Pa. The standard deviation of p_1 and p_0 are 418.3 Pa and 949.8 Pa, respectively.

The angle of the shock wave with respect to the freestream flow is calculated manipulating equation 2: $\beta = 69.13^\circ$, or using the table θ - β - M .

Equation 3 and Monte Carlo supplies $M_{n,1} = 1.13$ and associated uncertainty $u_{M_{n,1}} = 0.01$. Using equation 1 and propagating distributions of $M_{n,1}$ and the measured p_1 , we obtain the static pressure which should occur behind the shock wave: $p_2 = 66857.0$ Pa, and $u_{p_2} = 1186.6$ Pa. There is a difference equal to 365.7 Pa between the measured static pressure value ($\bar{p}_{plate} = 66491.3$ Pa) and that obtained algebraically ($p_2 = 66857.0$ Pa).

The normal Mach number estimated using equation 5, $M_{n,2} = 0.89$, leads to the Mach number $M_2 = 0.98$, behind the shock wave (equation 6). Both associated uncertainties $u_{M_{n,2}}$ and u_{M_2} are equal to 0.01.

Using equation 7, the total pressure $p_{0,2}$ is equal 124100.5 Pa and $u_{p_{0,2}} = 2425.3$ Pa, for p_2 calculated algebraically. This value becomes $p_{0,2} = 123319.0$ Pa and $u_{p_{0,2}} = 4738.6$ Pa if the chosen p_2 in equation 7 is the mean value obtained in flat plate pressure taps. The difference of the experimental value in respect to the theoretical value is 781.5 Pa.

5.2. Inferior surface

For $M_1 = 1.21$, $p_1 = 50853.0$ Pa and associated uncertainties, and $\theta = +4^\circ$, the upstream Prandtl-Meyer function, calculated using expression (8) is $v(M_1) = 3.7^\circ$, with negligible uncertainty, as obtained through the Monte Carlo method. This results in $v(M_3)$ in region 3 equal to 7.7° , obtained through equation 9. Consulting the Appendix C [1], the Mach number behind the expansion is $M_3 = 1.35$.

Equation 10 supplies the static pressure $p_3 = 41905.5$ Pa. The uncertainty value is $u_{p_3} = 583.4$ Pa. As the tests for measuring p_3 have not yet been carried out, there is no comparison between experimental and theoretical values in this paper.

The angle between the forward Mach line and the upstream airflow direction is $\mu_1 = 56.0^\circ$ and the angle between the rearward Mach line and the flat plate direction is $\mu_2 = 47.8^\circ$. The angle between the rearward Mach line and the upstream airflow direction is $\mu_2 - \theta = 43.8^\circ$ (figure 7).

Numerical results for Mach number distribution of the flow field using CFD were also obtained and were presented in figure 8.

5.3. Aerodynamic forces

One obtains the aerodynamic forces acting on the flat plate positioned at an angle of attack $\alpha = -4^\circ$, multiplying the results obtained employing equations 11 by the width of the plate: $R = -450.2$ N, $L = -449.1$ N and $D = 31.4$ N, where the theoretical values of p_2 and p_3 were used. Uncertainties are 24 N, 24 N, and 17 N, for the resultant force, lift and drag, respectively.

This leads to the drag force coefficient, C_D , due to the wave and lift force coefficient, C_L , equal to:

$$\begin{aligned} C_D &= 0.03 \\ C_L &= -0.48 \end{aligned}$$

estimated using equations 12 and 13.

Applying Monte Carlo supplies $u_{CL} = 0.03$ and negligible uncertainty in C_D .

These results are compatible with figures 4.20 and equation 4.48b of reference [5], which presents the value 0.17 for the rate C_D/C_L^2 . The value obtained in this study is 0.15.

6. Conclusions

This paper presents the results obtained in the tests with the flat plate in the transonic wind tunnel TTP, in the supersonic regime. In this range of airflow speed, shock and expansion waves are expected. The parameters static pressure, p_1 , and total pressure, p_0 , of the freestream flow field were measured. From the measurement of these parameters, it was possible to evaluate the changes in the airflow properties due to the presence of the plate.

The six pressure taps on the superior flat plate surface supply the experimental data for the comparison between the measured static pressure and that predicted by the shock wave theory. The drag and lift forces acting on the plate were also estimated experimentally. The uncertainties in the estimated flow field parameters were estimated through the Monte Carlo method.

In both cases, static pressure and aerodynamic forces, the experimental results comply with literature.

Measurements of parameters such as temperature of the freestream airflow and total pressure behind the shock, as well as static pressure behind the expansion, will be performed in future tests.

Acknowledgment

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