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To cite this article: Elias Okon 2013 *J. Phys.: Conf. Ser.* **442** 012045

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Is the consistent histories approach to quantum mechanics consistent?¹

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Abstract. The Consistent Histories (CH) formalism attempts to construct a quantum framework which can be used without the need to introduce observers external to the studied system. The prime motivation in mind is the application of the formalism to the universe as a whole. In order to achieve this, CH maintains that a formulation of quantum mechanics should allow for the assignment of probabilities to alternative histories of a system. Therefore, it provides an observer-independent criterion to decide which sets of histories can be given probabilities and states rules to determine them. The framework establishes that each realm, that is, each set of histories to which probabilities can be assigned, provides a valid quantum-mechanical account of a system. Furthermore, the version of CH first presented in [1, 2] proposes an “evolutionary” explanation of our existence in the universe and of our preference for quasiclassical descriptions of nature. The present work critically evaluates claims to the effect that the formalism offered in [1, 2] solves many interpretational problems in quantum mechanics. In particular, it is pointed out that the interpretation of the proposed framework leaves vague two crucial points, namely, whether realms are fixed or chosen and the link between measurements and histories. The claim of this work is that by doing so, CH overlooks the main interpretational problems of quantum mechanics. Furthermore, we challenge the evolutionary explanation offered and we critically examine the proposed notion of a realm-dependent reality.

1. Introduction

The Consistent Histories (CH) program tries to develop a version of quantum mechanics, which can be used in the study of closed systems. That is, one that, in contrast to the standard interpretation, can be applied without the need to divide the world into a system and an observer. In particular, CH tries to develop a formulation of quantum mechanics appropriate for the universe as a whole. CH stresses the importance of histories for quantum mechanics, as opposed to measurements, and maintains that a satisfactory formulation of quantum mechanics allows one to assign probabilities to alternative histories of the universe. It claims however that not all histories can be assigned probabilities, therefore, it provides an observer-independent criterion for deciding which sets of histories can be so endowed and gives rules to tell what these probabilities are.

The CH approach was originally developed by Griffiths² but this paper will be concerned with a particular version of the formalism first developed in [1, 2]. Such interpretation proposes that

¹ A longer exposition of these ideas appears in [3].

² The interpretation of the formalism offered by Griffiths can be consulted in [4, 5].



each *realm*, that is, each set of histories to which probabilities can be assigned, provides a valid quantum-mechanical account of a given system. It insists, however, that different realms can be mutually incompatible, in the sense that quantum-mechanical statements are meaningful only relative to a particular realm. Moreover, it maintains that the standard measurement situation is no more than a special case of setting for which the CH formalism allows for probabilities to be assigned. Consequently, it claims that the standard interpretation of quantum mechanics must be seen as a limiting case of the CH framework. References [1, 2] also use the CH formalism to offer an “evolutionary” explanation for the existence in the universe of complex systems, like humans, and for their almost exclusive preference for quasiclassical descriptions of nature.

The purpose of this work is to critically evaluate arguments offered by CH proponents asserting that the CH formalism solves many of the interpretational problems of quantum mechanics. In particular, it is pointed out that the interpretation proposed in [1, 2] leaves vague two crucial points, namely, whether complex systems can choose or not a realm, and the link between measurements and histories, overlooking on the way the challenging interpretational problems involved. Furthermore, the evolutionary explanation offered is disputed and the proposed notion of a realm-dependent reality is critically examined. The plan for doing the above is as follows: section 2 presents the CH formalism and section 3 describes its interpretation developed in [1, 2]. Section 4 then presents four objections against the CH formulation and evaluates its overall viability. Finally, section 5 presents some conclusions.

2. Consistent Histories

As was mentioned in the introduction, the CH formulation of quantum mechanics aims at a quantum-mechanical framework which, in contrast with the standard interpretation, does not critically depend on the notion of measurement. This feature would allow such interpretation to be applied to closed systems and, in particular, to the entire universe. In order to achieve this, proponents of CH propose as the most general objective of quantum mechanics the prediction of probabilities for time histories of a system. The CH formalism then provides an observer-independent criterion to tell what sets of alternative histories of a given system can be assigned probabilities and allows for these probabilities to be computed.

In order to achieve the above, the CH formalism takes as inputs an initial state $|\psi\rangle$ and a Hamiltonian \hat{H} , which are supposed to be given by an external theory. It then introduces the notion of an exhaustive and exclusive set of alternatives at a time. Such sets are represented (in the Heisenberg picture) by a set of projection operators:

$$\{P_\alpha(t)\}, \quad \alpha = 1, 2, 3, \dots \quad (1)$$

In order to implement the exhaustiveness and the exclusiveness, respectively, the projections of each set must satisfy the following equations:

$$\sum_\alpha P_\alpha(t) = 1, \quad \text{and} \quad P_\alpha(t)P_\beta(t) = \delta_{\alpha\beta}P_\alpha(t). \quad (2)$$

Employing these sets, it is possible to construct *sets of histories*. These are sequences in time of the above constructed sets of alternatives, and are represented by

$$\{P_{\alpha_1}^1(t_1)\}, \{P_{\alpha_2}^2(t_2)\}, \dots, \{P_{\alpha_n}^n(t_n)\}, \quad \text{at times} \quad t_1 < t_2 < \dots < t_n. \quad (3)$$

Individual histories are then collected by selecting one projection from each set: $(\bar{\alpha}_1, \bar{\alpha}_2, \dots, \bar{\alpha}_n)$. Such histories are represented by the corresponding chain of projections $C_{\bar{\alpha}} \equiv P_{\bar{\alpha}_n}^n(t_n) \dots P_{\bar{\alpha}_1}^1(t_1)$ and each history gets assigned a branch state vector: $|\psi_{\bar{\alpha}}\rangle = C_{\bar{\alpha}}|\psi\rangle$. Sets of histories are generally coarse-grained in the sense that sets of projections are only given at particular times

and individual projections are not required to be one-dimensional. Operations of fine- and coarse-graining can be defined by, for example, removing or adding sets of projections at additional times, or by compounding or splitting individual projections within a set.

The next step is to assign probabilities to individual histories within a set, but, according to the formalism, this cannot be done in general. Probabilities can be assigned to histories within a set only when there is *negligible* interference between its branches. That is, when the following relation holds:

$$\langle \psi_\alpha | \psi_\beta \rangle \approx 0. \quad (4)$$

The above condition guarantees that the assigned probabilities (approximately) satisfy the axioms of probability (sets satisfying the condition above are said to (medium) decohere). Then, according to CH, only for these sets it is possible to make predictions, and the corresponding probabilities are given by $p(\bar{\alpha}) = \|C_{\bar{\alpha}}|\psi\rangle\|^2$. A set of alternative (coarse-grained) histories which decoheres is called a *realm*.

Finally, in order to apply the formalism, the following prescriptions are postulated: given data d at time t_0 , represented by a projection operator $P_d(t_0)$, predictions for the probability of the future history α_f are given by

$$p(\alpha_f|d) = \frac{\|C_{\alpha_f}P_d(t_0)|\psi\rangle\|^2}{\|P_d(t_0)|\psi\rangle\|^2}, \quad (5)$$

with C_{α_f} an exhaustive set of alternative histories to the future of t_0 . Similarly, retrodictions for the past history β_p are given by

$$p(\alpha_f|d) = \frac{\|P_d(t_0)C_{\beta_p}|\psi\rangle\|^2}{\|P_d(t_0)|\psi\rangle\|^2}, \quad (6)$$

with C_{β_p} an exhaustive set of alternative histories to the past of t_0 .

3. An interpretation of the formalism

In this section we will explore four core aspects of the interpretation of the CH formalism presented in [1, 2]: i) the concept of incompatible realms, ii) the connection between quasiclassical realms and complex adaptive systems, iii) the way in which standard quantum mechanics is supposed to be contained in CH and iv) the notion of a realm-dependent reality.

3.1. Inconsistent realms

As was mentioned already, a realm is a set of histories which allows a consistent assignment of probabilities. It turns out that, given a generic system, many different realms can be constructed and that the theory does not distinguish between all these different realms. The problem is that not all realms are compatible, in the sense that two different realms of the same system may lead to *contrary inferences*. That is, it can be shown (see [6]) that using two realms, both compatible with the same given data, it is possible to arrive at inconsistent stories of what actually happened; i.e., it is possible to retrodict, with *certainty* in each realm, two inconsistent facts about the past. References [1, 2] are well aware of this complication and in order to avoid inconsistencies impose the following rule: *inferences may not be drawn by combining probabilities from incompatible realms*. Doing so is just something you are not allowed to do while using the formalism.

3.2. Quasiclassical domains and IGUSes

A *quasiclassical domain* is defined in [1] as a realm that is maximally refined and that contains individual histories exhibiting as much patterns of classical correlation in time as possible. The

world we perceive is supposed to be a great example of such a domain. In addition, humans are taken to be complex adaptive systems, and, in particular, special types of IGUSes (i.e., information gathering and utilizing systems). The foremost characteristic of an IGUS is taken to be the fact that it uses a (possibly primitive) physical theory in order to navigate, and make predictions, about its environment.

Given these two notions, references [1, 2] claim that the existence of IGUSes is to be explained in *evolutionary* terms. Concretely, the proposal is that:

- (i) IGUSes evolved to make predictions because it is adaptive to do so;
- (ii) they focus on quasiclassical domains because these present enough regularity to permit predictions by primitive methods.

As a result, the formalism explains the existence of IGUSes, and their preference for quasiclassical realms.

3.3. Recovering the standard interpretation

In order to display the fact that the CH formalism contains, as a special case, standard quantum mechanics, reference [1] first defines a *measurement* as a correlation between values of operators of a quasiclassical domain. Then, it states that this definition implies that the standard measurement situation (i.e., one with a system, a measuring apparatus and an observer) is only a special case of situation in which CH allows for an assignment of probabilities. Furthermore, it is claimed that the standard quantum-mechanical probabilities coincide with the ones supplied through the CH formulation. The conclusion is that the standard interpretation of quantum mechanics is no more than the application of the CH formalism to the particular setting of a standard measurement situation.

3.4. Reality relativism

Another element of the interpretation in [1, 2] that deserves our attention is the proposal of what could be called a *reality relativism*, i.e., the ontological claim that the notion of reality, or what is real, is meaningful only relative to a realm. Furthermore, [7] holds that our problem for accepting such an idea arises from shortcomings in our everyday language, i.e., from tension between colloquial language and the language of physics.

The idea of a reality relativism is of course a strong and controversial assertion, one that needs to be evaluated carefully. The next section scrutinizes this and other components of the interpretation of the formalism proposed in [1, 2].

4. Objections

This section presents four objections³ against the formulation of CH developed in [1, 2, 7]. The first one asserts that the fact that the formalism does not provide a mechanism for realm selection represents a serious omission that greatly hinders its viability. The second objection arises from the fact that the formalism seems to deny the empirical possibility of determining the Hamiltonian and initial state of a system, both of them necessary for making predictions. Next, the claim that standard quantum mechanics is contained in CH is challenged and, finally, the evolutionary explanation for the existence of IGUSes, and their relation to quasiclassical realms, is disputed.

4.1. Realm-dependent reality

As was seen in the previous section, references [1, 2, 7] quite explicitly propose a reality relativism. They are asking us to consider the idea that reality, or what is real, is relative to a

³ An early thorough evaluation of conceptual issues in CH is contained in [8].

realm. There is no doubt that the mere idea that reality is relative is controversial. However, the qualm considered here is not about the idea itself but about the self-consistency of the realm-dependent reality proposed in [1, 2, 7]. However, before developing the objection, it will be useful to examine the assertion in [7] regarding the connection between problems with colloquial language and problems for accepting a reality relativism.

The history of science is full of concepts that at some time are considered absolute but that latter turn out to be relative. A standard example of this, specifically mentioned in [7], is the time order of events, which was considered absolute in Newtonian mechanics but turns out to be relative (on the frame of reference) according to Special Relativity (SR). Therefore, if true, the reality-relativism proposal would surely not be the first time science discovers something to be relative. Furthermore, human language seems perfectly capable of dealing with relative concepts; we do it all the time with notions like big, far, cold, etc. Therefore, it is not clear how the uneasiness with the idea of a realm-dependent reality could have anything to do with shortcomings or excess baggage in human language; it seems more like an ontological problem than a language-related one.

What about the self-consistency of the proposal for a realm-dependent reality developed in [1, 2, 7]? The proposal is that in order to make sense of the notion of reality, and in order to use the formalism to make predictions, etc., it is necessary to fix a realm. The problem is that it isn't clear how IGUSes are supposed to select realms (among all the possible options offered by the formalism) because the formalism does not offer any mechanism for doing so. Actually, the sources are ambiguous about it:

“...we could adopt a subjective point of view... and say that the IGUS “chooses” its coarse graining of histories and, therefore, “chooses” a particular quasiclassical domain... It would be better, however, to say that the IGUS evolves to exploit a particular quasiclassical domain or set of such domains.” [1].

It is not clear, then, if an IGUS chooses a realm or if it is the realm that limits or constrains the possibility of existence and characteristics of IGUSes living in it. In any case, there are only two options: either IGUSes can or cannot “choose” realms. The problem is that neither option is satisfactory. On the one hand, if selecting a realm is beyond our capacities as IGUSes, talk of multiple realms seems extravagant and serves no real purpose in the theory (other realms being empirically inaccessible). On the other hand, if an IGUS can choose a realm, proponents of the formalism owe us an explanation of how this could be so, especially after noticing that it involves fixing projections everywhere in the universe, and at all times, and considering that this projections might radically change our current experience, or even our present existence! The problem then is not only that a mechanism for selecting a realm is missing; the problem is that the formalism seems to lack the resources for providing it.

The standard reply of CH proponents with respect to a realm-choosing mechanism within CH is that *realms must be chosen according to the questions one is trying to answer and the predictions one is interested in obtaining*. Nevertheless, that reply misses the point considered here because the issue is not about a recipe for applying the formalism in standard measurement situations. That is, a recipe for coming up with predictions for experiments in which we can take for granted a myriad of things, like a distinct system, a measuring apparatus, well defined observables, observers, etc. The issue is rather about evaluating the formalism as a theory of all this things together, which of course is the central motivation for taking CH seriously in the first place. The point is that there are two different levels of discourse that get entangled, one about how IGUSes use the theory to make predictions and the other about what the theory tells us with respect to reality. Maintain these two levels of discourse separate is essential for an adequate assessment of the CH approach.

4.2. Initial conditions

The second objection we will consider has to do with the (seemingly innocuous) fact that the formalism takes as inputs an initial state $|\psi\rangle$ and a Hamiltonian \hat{H} . The problem is that these objects, which are of course essential for applying the formalism (see for example equations 5 and 6), are supposed to be fixed and absolute, i.e., *realm-independent*. However, if the idea of a realm-dependent reality is taken seriously, it is far from clear how one could have access to this absolute elements. In other words, how are we supposed to choose initial conditions for a theory that holds that the past is relative? As a way out of this state of affairs, [1, 2] insist that we need to use a separate and external theory for choosing the initial conditions. However, if the past really is relative, nothing at all that we observe can ever count as evidence for such an external theory. That is, we cannot, even in principle, test these theories.

4.3. Measurements

We turn next to two related issues, both having to do with the treatment of the concept of measurements in CH. The first is the debatable way in which the formalism is supposed to contain standard quantum theory and the second is the fact that [1, 2] fail to determine the connection between actual measurements and the projection operators of the formalism.

As was remarked in section 3, references [1, 2] sustain that the CH formalism incorporates Copenhagen quantum theory as a limiting scenario. If this is true, it of course implies that CH is consistent with experiments (to the extent that standard quantum mechanics is). However, the situation is a bit more complicated than what [1, 2] suggest. The first problem is that the proposal remains silent about what is the ontological status of the different histories within a given realm. Once again, two options are available.

- (i) The first one is that *only one of the histories within the realm is actual*, in which case we would have to conclude that the formalism is descriptively incomplete since, as it lacks a projection postulate, does not offer a mechanism to explain the preference for the chosen history from among all of the available choices.
- (ii) The remaining option is that *all the histories within the realm are actual*, in which case there are two problems. First, it is not possible to interpret as probabilities the numbers generated by the framework since all options are realized. Second, it sharply conflicts with our everyday experience of obtaining determined outcomes when we perform measurements, (these of course are the standard objections against many-world scenarios).

As was mentioned before, the other important omission in [1, 2] with respect to measurements is that the proposal does not make explicit what is the relation between actual measurements performed by IGUSes and the projection operators of the formalism. That is, it does not specify how are we to connect the mathematical formalism provided, with experimental practices. The only rule in this respect that the formalism provides is the following: “realms are to be chosen according to the questions one is trying to answer.” But, is this rule useful in practice? That is, given a standard measurement situation, would it help us select a realm? Which one would it be? An initial (and partial) response could be that the chosen realm would contain, as a minimum, a projection corresponding to the measurement to be realized. This, however, is deeply problematic because *the CH formalism does not specify under what conditions one is allowed to conclude that a measurement is taking place*. In other words, it does not solve the measurement problem.

The situation is even worse for the rest of the projections that comprise the realm (remember that realms contain sets of projections at various times). Clearly, these cannot be associated with measurements performed by IGUSes, even if that link could be established, regardless of what was said above. That is because for a measurement situation to arise, with an IGUS, and experimental apparatus, etc., specific projections must have occurred early on in the history of

the universe (those that permitted the formation of galaxies, for example), before any IGUS was around to perform measurements. The remaining option is to disassociate the projections of the formalism with measurement but this solution is also unacceptable. The reason is that, without a connection between our measurements and the formalism, we would lose all predictive power. A possible last resource would be to hold that free will is an illusion: that whenever we (falsely) believe that we decide to perform a measurement, a projection occurs. In this case the issue of the two different levels of discourse mentioned above resurfaces because it would turn out that IGUSes are in fact *incapable* of using the formalism to make predictions. The discussion would then have to be entirely in terms of CH as seen as a the theory of everything, but as we stressed already, that issue is far from being satisfactorily addressed by its proponents.

4.4. IGUSes and quasiclassicality

The last critique we will consider questions the viability of the explanation for the existence of IGUSes, and their preference for quasiclassical realms, offered in [1, 2]. As was mentioned in section 3, such explanation is an evolutionary one: IGUSes exist because i) they are good at making predictions and ii) it is evolutionarily advantageous to be able to do so. Furthermore, they prefer quasiclassical realms because such environments present enough regularities for predictions to be generated.

The explanation displayed above might seem reasonable at first. Nevertheless, when examined with some care, starts looking less promising. To see why, it is useful to start by recalling the basic mechanism behind evolutionary theory, which, in a nutshell, can be stated as follows: *impact of the environment on reproductive success*. Therefore, the essential elements in an evolutionary scenario include: a varied initial population, an external environment, heredity and selection. However, none of these elements seem to be present in the CH context. In particular, there is no environment for IGUSes to evolve since, according to the proposed explanation, such environment (i.e., the realm in which the IGUSes dwell) is an essential part of what is supposed to be adaptively selected (the result purportedly being a quasiclassical realm). In other words, one cannot argue that evolution takes place unless the scenario in which it is supposed to happen permits the operation of mechanisms such as the fact that failure to obtain resources results in death, or that systems that are unfit do not reproduce. Those rules presuppose a quasiclassical realm, and it is thus clear that they cannot be used to argue that they play a role in selecting one such realm over something else.

Getting back to the explanation offered in [1, 2], we notice that, in effect, what it says is the following: we experience a quasiclassical realm because it is the only one that allows for IGUSes like ourselves to exist. Written this way, such explanation can easily be recognized as an instance of the, so called, *anthropic principle*. That is, of the principle that holds that *features of the world are what they are because, otherwise, we wouldn't be here to remark on it*. Therefore, it seems clear that what [1, 2] consider to be an evolutionary argument is, essentially, no more than an invocation of the anthropic principle.

5. Conclusions

The CH program aims at a generalization of the standard quantum formalism that does not employ the notions of measurements or observers in any fundamental way. It also attempts to construct a purely unitary version of quantum mechanics, where temporal evolution is implemented solely in terms of Schrödinger's equation, with no mention of a projection postulate. Certainly, these two goals are worthy and interesting. Unfortunately, the formulations of CH presented in [1, 2] suffer from too many conceptual problems in order to be considered satisfactory.

On the other hand, it is very likely that the problems encountered while trying to apply quantum theory to the universe as a whole, arise not from our inability to interpret quantum

theory, as advanced by CH proponents, but from the (bold) assumption that quantum theory is universally valid.

Acknowledgements

This work has been partially supported by project PAPIIT IA400312-2.

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