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Coherent phonons as a new element of quantum computing and devices

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Abstract. We explore the possibility of strongly coupling semiconductor qubit states to nanomechanical resonators (phonons) in silicon. These systems may be relevant to qubit transduction schemes, as supporting technology for quantum information processing, for qubit characterization, and for quantum-enabled devices. Specifically, we consider systems where cavity phonons can interact with suitable qubit states in the 1-10 GHz (and higher) regime (tunable using strain/electric and/or magnetic fields). These results may be useful for several solid-state devices as well as being of interest to the optomechanics community.

1. Introduction

The purpose of this proceeding paper is to make a simple introduction to the use and appearance of phonons in nanomechanical structures in the context of the emerging field of quantum computing and quantum-enabled device applications. After re-capitulating essential features of cavity Quantum Electrodynamics (cQED)[1, 2], we introduce in an analogous way confined or propagating acoustic phonons and study the possibility to observe their quantum coherent nature in artificially created phononic cavities. Our main observation in further pursuing such an analogy is the finding that single impurities in an ideal crystal environment (e.g. in a silicon nanostructure, see below) may couple strongly to confined phonons, just as a single atom does with an optical cavity. By exploring recent achievements in optomechanics [3, 4], we propose a new way to probe/measure the quantum state of a phononic cavity, by using the phonon non-linearity introduced by an impurity (single donor or acceptor in a silicon phononic bandgap structure). The proposed scheme(s) can have applications related to conversion/transmission of quantum information between distant spin-qubit quantum nodes, using flying photonic qubits as transduction elements, while the qubit-photon interaction will be mediated by strongly coupled phonons.

2. Elements of quantum computing architecture

Quantum computing (QC) is commonly described in terms of qubits or two level systems (TLS) that can be coherently manipulated locally (so-called single qubit gates) or allowed to controllably interact with other TLSs (two-qubit or multi-qubit gates) [5]. Physically, the Hamiltonian (unitary) coupling between two qubits can be achieved by direct qubit-qubit interaction (dipole interaction in case of atoms, capacitive coupling in case of superconducting Josephson qubits, etc.), or by mediating the interaction via coupling to a common cavity (optical or microwave): so-called cavity bus. For a distributed quantum system, when a qubit (or qubits) are placed distantly from each other, a coherent coupling between the distant sub-systems is made via (ideally lossless) quantum channels/waveguides. While ideally, all these elements (qubits or cavities/waveguides) are lossless, in reality they are subject to relaxation/decoherence processes (e.g. due to spontaneous photon emission in case of atomic TLS) or intrinsic/extrinsic cavity losses (due to various photon scattering mechanisms). Thus, in the QC "race" the various physical approaches will complement each other rather than to compete: the photonic (in the optical frequency range, hundred THz) flying qubits will be suitable for long distance quantum state transfer, (due to very low-loss optical channels, even at room temperatures), superconducting Josephson junction (JJ) qubits and cavities (in the microwave range, MHz to GHz) will be suitable for quantum operations (due to potentially high qubit coherence times, very high qubit-cavity coupling), finally, semiconductor spin-qubits will be most suitable for quantum information storage (due to very high coherence times, of order of miliseconds to seconds). The need to inter-couple these various physical systems, eventually using tunable and strong coupling will dictate the use of phonons as mediators, as described below.

3. Cavity QED and circuit QED

Elements of the QC architecture (e.g., an atom strongly coupled to an optical cavity) were first realized in cavity QED [1, 2]. It was proposed to consider such a compound system as a node of a "quantum network", where different nodes are coupled via a photon waveguide and tunable couplings[6]. By "reducing" the atom to a two-level system the atom-cavity coupling comes into play in a famous Jaynes-Cummings Hamiltonian [7], which (in the rotating wave approximation) establishes a coherent exchange of TLS excitation σ^+ to an excitation of the confined optical cavity mode, $b^\dagger_{\boldsymbol{q},\sigma}$:

$$H_{\mathrm{g}} \approx \hbar g_{\boldsymbol{q}} \left(\sigma^+_{ge} b_{\boldsymbol{q},\sigma} + \sigma^-_{ge} b^\dagger_{\boldsymbol{q},\sigma} \right) \tag{1}$$

The so-called strong coupling regime refers to the situation when $g > \{\Gamma_{\mathrm{TLS}}, \kappa_{\mathrm{cav}}\}$, where Γ_{TLS} is the TLS relaxation rate to all modes (including a continuum of modes not confined to the cavity) and κ_{cav} is the leakage rate of the confined cavity mode (which, in the optical case, is mainly due to non-ideal mirrors of the cavity or scattering losses in case of a semiconductor photonic bandgap cavity). The strong coupling regime implies that coherent exchange (called a Rabi flop) between the atom and the cavity is possible before decoherence takes over via spontaneous emission process or cavity loss. In particular, this regime is essential in a seminal proposal[6] for quantum state transfer between distant atom-cavity optical nodes, using a time dependent coupling $g(t)$.

The "traditional" cavity QED[1, 2] was further implemented in different solid-state systems, including semiconductor microcavity polariton (an exciton pair coupled to a photon)[8, 9], as well as in a cQED with a single semiconductor quantum dot (QD) that is playing the role of an atom while the photonic cavity is realized in a (2D) photonic bandgap structure[10, 11].

More recently, in the so-called circuit QED[12], the field has been revolutionized. Here the TLS is based on a Josephson junction (JJ) transmon qubit[12, 13], which is strongly coupled to a microwave (stripline) resonator. The strong coupling here comes from the large dipole moment of the JJ qubit (essentially a macroscopic quantum system) and also due to the (quasi) 1D-geometry of the microwave (MW) resonator, so that the confined cavity MW mode occupies a volume[13] $\mathcal{V}_{\mathrm{mode}} \approx 10^{-6}\lambda^3$. Combined with a low loss qubit/cavity elements (e.g. the qubit life time reaches a range from a microsecond to tens of microsecond in the case of a 3D-transmon[14]) the number of Rabi flops (coherent exchanges) reaches $\sim 40-100$ and also the regime of the so-called strong dispersive coupling[12, 13] is reached, which is suitable for schemes for quantum non-demolition qubit readout[13].

4. Phonons instead of photons

In an ideal crystal environment, phonons may play a role analogous to photons, though they propagate with the much slower speed of sound. In fact, the photon-phonon analogy is well supported in semiconductors at low energies: the conduction/valence bands play a role similar to the QED vacuum: the quasi-free electrons/holes propagate according to a quadratic dispersion, scatter off acoustic phonons that possess linear dispersion, and can be bound to charged impurity potentials to establish hydrogen-like atoms (donors or acceptors). Thus, the pattern of QED (matter and photon interactions) repeats itself in the behavior of electronic/hole quasi-particles in the solid (the "matter") and the phonon vibrations in a crystal (the "photons").

Consider first the phonons alone (both propagating and confined one). For small atomic displacements $\boldsymbol{u}(\boldsymbol{r})$ an expansion in normal modes gives

$$\boldsymbol{u}(\boldsymbol{r}) = \sum_{\boldsymbol{q},\sigma} \left(\boldsymbol{u}_{\boldsymbol{q}\sigma}(\boldsymbol{r})\, b_{\boldsymbol{q}\sigma} + \boldsymbol{u}^*_{\boldsymbol{q}\sigma}(\boldsymbol{r})\, b^\dagger_{\boldsymbol{q}\sigma} \right), \tag{2}$$

which approximately diagonalizes the phonon Hamiltonian: $H_{\text{ph}} = \sum_{\boldsymbol{q},\sigma} \hbar\omega_{\boldsymbol{q}\sigma} \left(b^\dagger_{\boldsymbol{q}\sigma} b_{\boldsymbol{q}\sigma} + \frac{1}{2} \right) + H_{\text{anh}}$. Higher terms correspond to small anharmonicity: $H_{\text{anh}} = c\, b^\dagger_{\boldsymbol{q}\sigma} b_{\boldsymbol{q}'\sigma} b_{\boldsymbol{k}\sigma} + \cdots$, and are related to phonon self-interaction. The mode normalization in Eq. 2 is $\int d^3\boldsymbol{r}\, \boldsymbol{u}^*_{\boldsymbol{q}\sigma}(\boldsymbol{r}) \boldsymbol{u}_{\boldsymbol{q}\sigma}(\boldsymbol{r}) = \frac{\hbar}{2\rho\,\omega_{\boldsymbol{q}\sigma}}$, so that $b^\dagger_{\boldsymbol{q}\sigma}$ creates a phonon in the mode $\boldsymbol{q},\sigma$ with energy $\hbar\omega_{\boldsymbol{q}\sigma}$ in a material with mass density ρ. The vector $\boldsymbol{q}$ denotes a collective index of the discrete phonon mode defined via the phonon cavity boundary conditions and quantization volume $\mathcal{V}$. In particular, the plane wave expansion corresponds to rectangular periodic boundary conditions and $\boldsymbol{u}_{\boldsymbol{q}\sigma}(\boldsymbol{r}) = \left(\frac{\hbar}{2\rho\mathcal{V}\omega_{\boldsymbol{q}\sigma}} \right)^{1/2} \boldsymbol{\xi}_{\boldsymbol{q},\sigma}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}$ with wave vector $\boldsymbol{q}$, polarization $\boldsymbol{\xi}_{\boldsymbol{q},\sigma}$, and phonon branch σ.

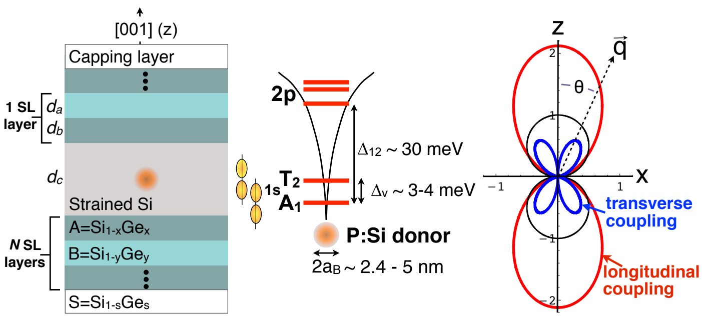

Figure 1. Ref.[15]:(a) A cavity-phoniton can be constructed in a Si/Ge heterostructure cavity as a hybridized state of a trapped single phonon mode and a donor TLS placed at a maxima of the phonon field. (b) The P:Si donor lowest $1s$ valley states, A_1, T_2, and upper levels ($1s$/ $2p$); their energy splittings can be controlled by the applied strain in the Si cavity. (c) Angular dependence of the coupling $g_{\boldsymbol{q}}(\theta)$, Eq.(2) of [15], for the deformation potentials of Ref. [16] vs. dipole $\sim \cos\theta$-dependence (thin circles).

As an example of a phonon cavity, we have theoretically proposed[15] a (quasi) 1D-phononic bandgap micro-pillar silicon cavity (figure 1a). Here a Si cylindrical cavity is sandwiched between two acoustic distributed Bragg reflectors (DBR): the cavity mirrors. The boundary conditions are as for a free-standing cylinder, i.e. the stresses T_{zz} and T_{zr} are zero on the cylinder surface of the cavity. In the z (growth) direction of the Si cavity the displacement field is chosen in such a way, that (for a λ-cavity) one has nodes for the strain field on the Si-cavity z-boundaries. (This was our special choice for a donor:Si [15], we briefly comment below). In our prototype example, the DBR mirrors are made from SiGe layers of two different contents of Ge, thus forming a periodic (Bragg) structure (figure 1a). For an optimized DBR structure[15] we have estimated that the main phonon loss in the cavity is due to phonon leakage through the mirrors (extrinsic loss); however one can reach a high quality factors of $Q_{\text{ext}} \approx 10^5 - 10^6$ using sufficient number of layers (e.g., for $N = 33$ gives $Q_{\text{ext}} \simeq 10^6$ for the transition frequency of $\omega_{P:Si}/2\pi \simeq 730\,\text{GHz}$, related to phosphorous donor in Si, see below). The corresponding cavity loss was estimated as $\kappa_{\text{ext}} = \omega_{P:Si}/Q_{\text{ext}} \simeq 2.8 \times 10^6\,\text{s}^{-1}$. The intrinsic phonon cavity loss comes from phonon anharmonicity loss, Γ_{anh} and loss from scattering off isotopic (impurity) mass fluctuations, Γ_{imp} of which the latter loss rate is two orders of magnitude higher than anharmonicity loss rate (for low temperatures). This rate amounts to $\Gamma_{\text{imp}} \simeq 7 \times 10^5\,\text{s}^{-1}$, however it can be made one order of magnitude smaller for isotopically purified silicon. One should mention, that recent experiments on 1D and 2D phononic bandgap cavities shows possibility to achieve similar phononic cavity Q-factors of $10^5 - 10^6$ in silicon membrane nanostructures[17, 3], which will be discussed below.

5. Optomechanical cooling experiments

The fact that acoustic phonons can be quantum coherent (phonon coherence can be understood similar to photons, e.g. via the ability for interference and/or coherent quantum transfer) has been explored in a number of novel architectures, allowing seminal experiments in nanomechanical cooling via optomechanics [18, 19, 20, 3], trapping of phonons in artificial phononic bandgap cavities [21, 3], photon translation via phonons [22, 23], and indirect qubit-phonon coupling [24, 25].

We briefly discuss here the qualitative features of the optomechanical cooling[18, 19, 20, 3], that is closely related to the proposal of coherent photon-to-phonon conversion[4], we discuss later. For the cooling of the mechanical (phononic) cavity one couples it to an optical cavity via the photon optical pressure Hamiltonian[26, 27]: $H_{OM} = \hbar g_{OM} \hat{a}^\dagger \hat{a}\,(\hat{b}^\dagger + \hat{b})$; here the optical pressure force is proportional to the photon number operator $\hat{a}^\dagger \hat{a}$ in the cavity and H_{OM} can be simply related to the work to move a mechanical boundary[21] of an effective mass m_{eff}; notice that $\hat{b}^\dagger + \hat{b}$ is essentially the position operator $\hat{x}$ associated with the mechanical degree of freedom: a generic example of what we described could be a standard Fabry-Perot cavity with one of the mirrors being movable and coupled to a spring, representing a mechanical cavity of frequency ω_m. Then the optomechanical coupling g_{OM} can be related to the work performed by a single photon to move the mirror (mechanical boundary) at a distance given by zero-point mechanical fluctuations: $g_{OM} = \omega_c \frac{x_{ZPF}}{L} = (\omega_c/L)\sqrt{\frac{\hbar}{2m_{\text{eff}}\omega_m}}$.

The non-linear interaction, H_{OM}, leads to the presence of optical sidebands, most prominent being the closest one: the so-called blue and red sidebands at frequencies $\omega_c \pm \omega_m$, where ω_c is the cavity resonant optical frequency. It can be shown that in the sideband resolved limit $\omega_m \gg \kappa_{\text{opt}}$ higher sidebands are well suppressed. Thus, in a good approximation, if the optical input field is a red sideband photon of frequency $\omega_c - \omega_m$, it will scatter in the cavity to produce an output photon of higher frequency, ω_c, which "needs" a phonon from the mechanical cavity to "get out". The higher frequency optical channel is empty, which is plausible for, e.g., few hundred THz photons even at not very low temperatures, see below. Thus, this scattering process leads to cooling, and the effective phonon temperature will go down inversely proportional to the number of pumped red sideband photons in the optical cavity.

In the remarkable experiment of Ref.[3] the authors achieved an optomechanical cooling of a phononic bandgap nanostructure (similar to the 1D structure on a patterned Si membrane, shown on figure 2c). The "defect" in the 1D periodicity, as in the previous example of the micro-pillar DBR cavity, forms a phononic cavity, that can confine phonons (of frequency $\approx 4\,$GHz). Since it is simultaneously a photonic bandgap structure, it also forms a photonic cavity of the same wavelength, $\lambda_{\text{opt}} = \lambda_m \approx 1500\,$nm (corresponding to frequencies of $\omega_c \approx 200\,$THz, i.e. telecom photons). Thus, it establishes a typical OM-structure, discussed above. For the base temperature of $T = 20\,$K the OM-cooling process leads to an effective temperature of $T_{\text{eff}} = 0.2\,$K with some $n_c \simeq 2000$ (red sideband) pumped photons in the cavity. Further cooling, by increasing n_c, is prevented by the noise entering the phononic cavity.

6. Phonon-to-photon translator proposal as a key element to probe phonon physics

The photon-to-phonon translator, proposed in Ref.[4], works in a similar fashion to the sideband cooling (a similar idea for quantum transduction was also discussed, e.g., in Ref.[28]). Just as for the cooling, the cavity confines two optical modes and a mechanical one, where the optical modes are designed to be sidebands to each other to resonantly increase the three-wave OM-coupling[4]: $H_{OM} = \hbar \tilde{g}_{OM}(\hat{a}_c^\dagger \hat{a}_p \hat{b} + \text{h.c.})$., Now, the higher frequency optical channel, $\hat{a}_c$ is not "empty" but is used to input/output an optical coherent signal, that is coherently translated to the phonon. The coherent nature of the translator is described by an effective beam-splitter type Hamiltonian[29]

$$H_{OM}^{\text{trans}} = \hbar G_{\text{eff}}\,(\hat{a}_c^\dagger \hat{b} + \hat{a}_c \hat{b}^\dagger) \tag{3}$$

The effective coupling is of the order of $G_{\rm eff} \sim \sqrt{n_c}\tilde{g}_{OM}$, i.e. it is enhanced by $\sqrt{n_c}$ (since the incoming photon or phonon "does not know" to which of the n_c red pump photons to scatter). Notice, that the translator works in both directions. The necessary conditions for this is the weak coupling regime[4], $G_{\rm eff} < \kappa_{opt}, \kappa_m$ (to avoid total internal reflection)

7. Non-linearity introduced by a single donor/acceptor impurity for Si nanostructures

Up to this point we demonstrated the coherent properties of phonons alone, and in particular, that optomechanical coupling allows probing of phononic structures by quantum optical means. To complete the analogy with photons one needs a non-linear element similar to an atom in cavity QED. Such an element has been proposed recently[15] where a single donor in silicon (P:Si or Li:Si) couples *directly* to confined phonons to form a hybrid state: a phoniton (direct analog of the polariton in cQED).

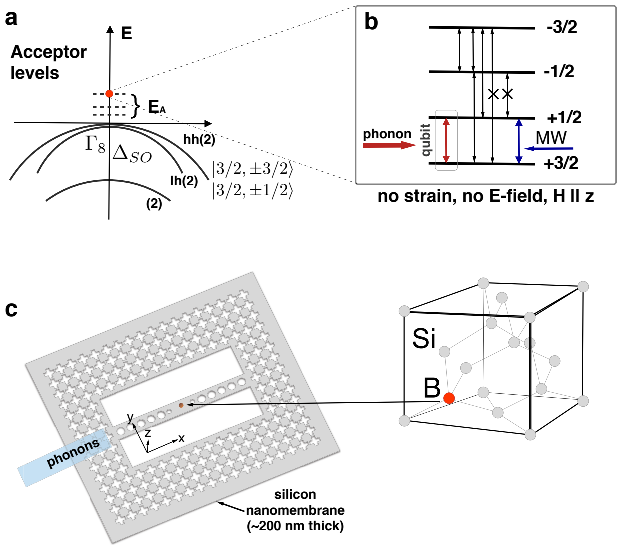

Figure 2. Ref.[30]: (a) Hole valence bands in Si; 4-fold degeneracy at the band top (and of lowest acceptor states) corresponds to particles of spin $J = 3/2$ (Γ_8 representation of cubic symmetry, see, e.g. Ref.[16]). (b) Ground state splitting is via external magnetic field along $[0,0,1]$ direction; allowed (forbidden) phonon transitions and qubit phonon driving are shown. Level rearrangement is via additional strain. System manipulation is via electric static/microwave fields. (c) Examples of a nanomechanical 1D phonon bandgap cavity reminiscent of already fabricated high-Q cavities in a patterned Si membrane [21, 3]. The acceptor is enclosed in the cavity and an on-chip phonon waveguide allows phonon coupling in/out of the system.

By considering low-energy acoustic phonons for electrons close to the band minimum the electron-phonon interaction can be written in a form similar to e.m. interactions in QED:

$$H^{\rm ac}_{\rm e,ph}(\boldsymbol{r}) = \sum_{ij} D_{ij}\,\varepsilon_{ij}(\boldsymbol{r}). \tag{4}$$

The operator $D_{ij} = -\hat{p}_i\hat{p}_j/m + V_{ij}(\boldsymbol{r})$ (with $\hat{\boldsymbol{p}} = -(i/\hbar)\nabla$ and a crystal model dependent $V_{ij}(\boldsymbol{r})$) coincides with the constant-strain deformation potential [16] and the strain $\varepsilon_{ij}(\boldsymbol{r}) = \frac{1}{2}\left(\frac{\partial u_i}{\partial r_j} + \frac{\partial u_j}{\partial r_i}\right)$ causes transitions between electronic states, just as e.m. fields do in QED. Despite the fact that this is a single impurity system, the impurity-phonon interaction may be large due to large deformation potential matrix elements:[16] $\langle\psi_{s'}|\hat{D}_{ij}|\psi_s\rangle \sim$ eV. In our proposal[15], the Si cavity was constantly strained, figure 1a, via suitable SiGe substrate in order to make the lowest $1s \to 1s$ transition in the P:Si ground state as small as possible (here it approaches 3 meV or 730 GHz for P:Si, and 0.586 meV or 146 GHz for Li:Si). This corresponds to a cavity length $d_c \sim \lambda_m \approx \{$few tens nm$\}$ in principle amenable to crystal growth engineering. We skip here calculational details[15] and mentioned two features differing our system from cQED. First, is the directionality of the coupling, figure 1c, related to the Si crystal symmetry under undirectional strain (along the growth z-direction). Also the impurity is "always there" vs. the finite traverse time for an atom in cQED. Estimations of the coupling strength shows that a strong coupling regime ($g > \{\Gamma_{\rm TLS}, \kappa_{\rm cav}\}$) (discussed above in the context of cQED) is well established; particularly the number of Rabi flops, $2g/(\kappa+\Gamma)$ can reach ≈ 100, comparable to circuit-QED[12]. This means that, in principle, experiments with single phonons could be as accessible as analogous experiments already performed with single MW photons, e.g. in Ref.[13].

The proposed donor system, however, requires very high frequencies[15] and can be difficult to integrate with other phonon components, which are now developed in the GHz range. While other impurities such as colour centers in diamond[25] or in III-V semiconductors are possible, a practical system in silicon would be highly desirable given recent demonstrations of high-Q cavities in silicon nanostructures [3], silicon's investment in materials quality, and compatibility with CMOS technology and silicon photonics.

In a recent paper [30] we propose a new quantum circuit element based on a single acceptor (such as B, Al, In, etc.) embedded in a patterned silicon nano-membrane, (figure 2), that would be compatible with established optomechanical components[21, 3]. The acceptor qubit (lowest two levels in the split acceptor ground state), figure 2b, is easily tunable in the range of $1-50\,$GHz by external magnetic field, and also by additional electric field or strain. We show that the regime of strong resonant and also strong dispersive coupling of the acceptor qubit to a confined acoustic phonon is possible. Variation of the coupling and frequencies in time, as well as direct manipulation of the acceptor qubit via microwaves (figure 2b) would provide possibilities for state manipulation, readout, and quantum state transfer. We give explicit measurement protocols to observe the phonon vacuum Rabi splitting and readout of the phonon number state fine structure [30].

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