Journal of Physics: Conference Series

A Level-Set Approach for Radar Imaging From Ramp Response in Arbitrary Directions

A Level-Set Approach for Radar Imaging From Ramp Response in Arbitrary Directions

Y Wen[1], N de Beaucoudrey[1], J Saillard[1], J Chauveau[2] and P Pouliguen[3]
[1] IETR UMR 6164 CNRS, Université de Nantes, Lunam Université, Nantes, France
[2] DGA Maîtrise de l'Information, Bruz, France
[3] DGA DS/MRIS, Bagneux, France

E-mail: yan-hua.wen@etu.univ-nantes.fr, nicole.de-beaucoudrey@univ-nantes.fr

Abstract. Low frequency Radar imaging can be used to reconstruct the global shape of targets using the ramp response technique, which only needs no more than 3 observing angles. We have developed a new algorithm permitting to generate promising shapes of targets for mutually orthogonal directions, but giving distorted results otherwise. To solve this ill-posed inverse problem, we use the level set method, which iteratively deforms the shape of the target under a velocity field, and we analyze the property of the desirable velocity. Numerical results obtained by this method in arbitrary directions are given as well.

1. Introduction

Among inverse scattering methods for radar imaging, the ramp response technique [1] only needs no more than 3 observing angles. The ramp response, which is the transient scattering response from an incident wave in the form of ramp, is related to the cross-sectional area of the target perpendicular to the observing direction. This area, named profile function, can be used to reconstruct the global shape of the target. The reconstruction algorithm originally proposed by Young [2] has been applied for years to image scattering [3-5], as well as to underwater acoustic imaging [6]. This algorithm is limited to convex and single objects, that is why we developed a new algorithm [7] able to overcome this limitation. Both algorithms work well for mutually orthogonal directions, while they produce distorted shapes of the target in non-orthogonal cases.

Therefore, an optimization procedure is required to evolve towards a promising estimate by minimizing the cost functional. The level set method, devised by Osher and Sethian [8] [9], represents the evolving object as the zero level of an implicit higher dimensional function. This allows topology changes, such as splitting and merging of connected components during the deformation of the object, in a completely automatic and implicit manner. This advantage is of great importance for inverse problem since it requires no a priori assumption about the object geometry. It has been proved to be effective in retrieving separated objects from a single initial guess and has been illustrated in the inverse scattering problem [10-12]. Consequently, we apply this method to obtain a satisfactory shape of the target.

2. Image reconstruction from profile functions

The ramp response of a target, $h_r(t)$, is defined by the far zone transient scattered wave resulting from illumination by a plane electromagnetic wave with a temporal ramp wave shape. In time domain, it is the second integral of its impulse response, $h_i(t)$ and it can be expressed as the Inverse Fourier Transform (IFT) of the weighted transfer function, $H_r(j\omega)$:

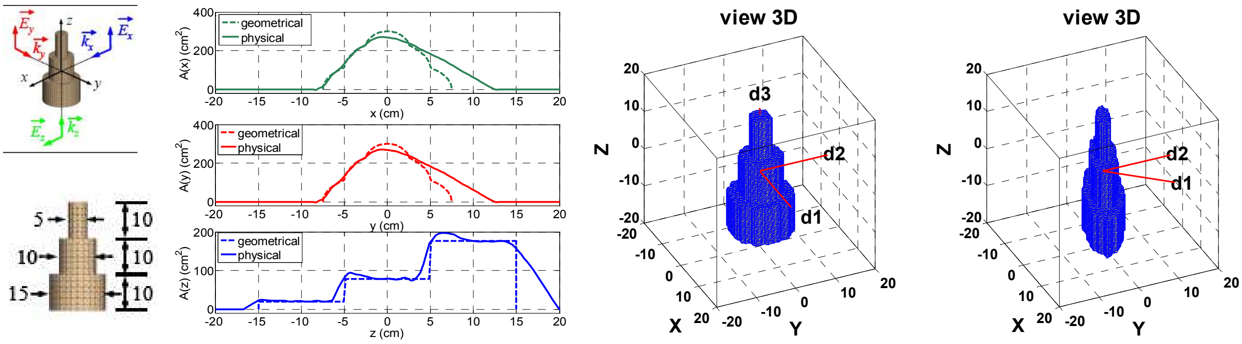
$$h_r(t) = \int_{-\infty}^{t}\int_{-\infty}^{t'} h_i(t'')dt''dt' = IFT[H_r(j\omega)] = IFT\left[\frac{H(j\omega)}{(j\omega)^2}\right] \tag{1}$$

The monostatic ramp response is directly related to the profile function of the target, $A(u)$, which is the transverse cross-sectional area of the target as a function of the distance along the line-of-sight [1]

$$h_r(t) \approx -\frac{1}{\pi c^2}A(u) \quad with \quad u = \frac{ct}{2} \tag{2}$$

where c is the speed of light in free space, t the time variable, and u the space variable.

Using as target the example of figure 1(a), two types of profile functions (PF) are compared in figure 1(b): 1/ the ideal PF, "geometrical", equal to the cross-sectional area $A(u)$, for $u = x,y,z$; 2/ the real PF, "physical", calculated by (2) from the ramp response. To ensure that the physical PF is a valid estimate of the geometrical PF, it is necessary to match the frequency band to target dimensions, choosing the upper Rayleigh region and the resonance region of the target.



(a) (b) (c) (d)

Figure 1. (a) Configuration of a radar target; (b) geometrical and physical profile functions in 3 orthogonal directions: x, y, z; (c) reconstruction with 3 mutually orthogonal directions; (d) reconstruction with arbitrary directions.

The original algorithm, proposed by Young [2] for 3D reconstruction from ramp responses, is limited to convex and single object. We have therefore developed a new algorithm [7], which is able to overcome this limitation by exploiting more effectively the information contained in the profile functions. With 3 geometrical profile functions of the object in figure 1(b), the resulting reconstruction with mutually orthogonal directions, figure 1(c), shows a high similarity with the original object. Considering now the case of arbitrary directions (with one direction at 50° in plane xoy, and the other two directions unchanged along y and z), the reconstructed result is distorted, figure 1(d).

3. Optimization using the level-set method

To improve the performance of this algorithm for arbitrary directions, it is necessary to use an iterative process to obtain an optimal estimate of the target by minimizing the mismatch between the data, i.e. profile functions of the unknown object, and the profile functions of the updated object.

Our problem can be formalized in the form of

$$A^d u = g^d \tag{3}$$

where g^d(M,1) is the observed data, the PF in direction d, u(N,1) the vector representing the unknown binary object we are looking for, A^d(M, N) the observing matrix (or mapping matrix) and d the index number of direction. d=1, 2, 3 in our case with three observing directions.

3.1. The forward problem

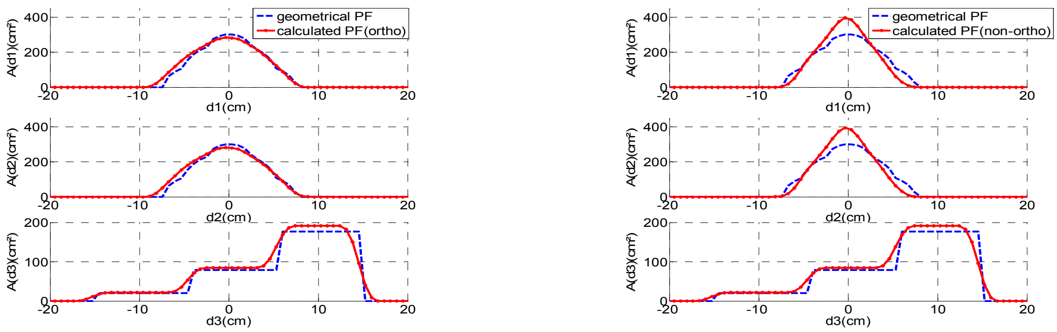
The forward problem, at step *k*, consists in calculating the profile functions of the evolving object with the following equation:

$$g_k^d(i) = \sum_{j=1}^{N} A^d(i,j)\,u_k(j) \quad \text{with} \quad i = 1, 2 \cdots M \tag{4}$$

For this, we developed an algorithm to calculate the profile function of a 3D object in an arbitrary direction: its principle is to distribute the pixels of the object in successive cutting planes perpendicular to the chosen direction. With the definition of profile functions, $A^d(i,j)$ represents the contribution that the pixel $u(j)$ gives to the slice i, and the profile function is obtained by summing individual contributions from all pixels. At iteration k, we define the cost functional, F_k , as the norm of the error between the PF calculated from the evolving object, g_k, and the PF observed from the real object, g:

$$F_k = \sum_d \frac{1}{2}\left\| g_k^d - g^d \right\|^2 \quad \text{with} \quad d = 1, 2, 3 \tag{5}$$

Applying the proposed algorithm, the three geometrical profile functions are compared in figure 2 with those calculated from the initial guess reconstructed from orthogonal (figure 1(c)) as well as arbitrary directions (figure 1(d)). In the orthogonal case, the cost function is negligible (F_0 = 0.02), whereas it is much more significant in the non-orthogonal case (F_0 = 0.16). Therefore, the cost functional behaves as a quantitative indicator for the evolution and quality evaluation of the estimate.



(a) (b)

Figure 2. Comparison between the 3 geometrical PF with those calculated from the reconstruction of figure 1(c) and (d): (a) orthogonal case; (b) non-orthogonal case.

3.2. The inverse problem

Most of iterative methods evolve the explicit function of object shape during the iteration. On the contrary, the level set method represents the shape as the zero level of a higher order level set function, which requires no a priori assumption on the object geometry or structure. In a computational domain D enclosing the object Ω, the level set function $\varphi(\boldsymbol{x})$ is defined as:

$$\begin{cases} \varphi(\boldsymbol{x}) > 0 & \text{if} \quad \boldsymbol{x} \notin \Omega \\ \varphi(\boldsymbol{x}) < 0 & \text{if} \quad \boldsymbol{x} \in \Omega \\ \varphi(\boldsymbol{x}) = 0 & \text{if} \quad \boldsymbol{x} \in \mathrm{C} \end{cases} \tag{6}$$

where $\boldsymbol{x}$ is the position variable, *(x, y)* and *(x, y, z)* for 2D and 3D case respectively, and C is the contour of the object. The deformation of the object is formalized as a Hamilton-Jacobi equation for the level set function φ under a normal velocity field V [8]:

$$\frac{\partial \varphi}{\partial t} + V\left|\nabla \varphi\right| = 0 \tag{7}$$

Following the formulation in [13], a common choice of velocity is:

$$V = -J(u)^T F(u)(u_i - u_o) \tag{8}$$

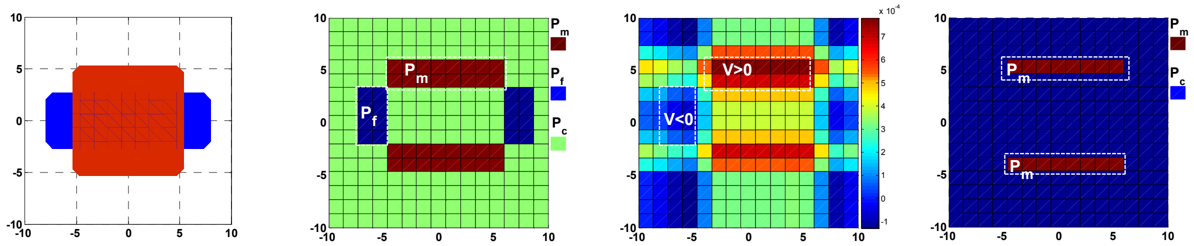
where $J(u)$ is the Jacobian of $F(u)$ at u. u_i *and* u_o are the prescribed values to represent the inside and outside of the desired unknown. In our case, $u_o = 0$ and $u_i = 1$ represent the binary unknown.

Shape deformation, ‘expand’ or ‘shrink’, is directly related to the change of the level set function, which is mainly determined by the sign of the velocity. From (7), when V is positive, φ decreases, therefore the shape expands in the normal direction of the contour; whereas, when V is negative, the shape shrinks. In the computational domain, as compared with the real object, the evolving object might have “correct pixels”, P_c, “missing” pixels, P_m, which belong to the real object but are not selected by the evolution, and “false” pixels, P_f, which are in the opposite situation. The aim of the shape optimization is to iteratively fill up P_m and remove P_f. These two “actions” are directly related to the change of the level set function, which is determined by the sign of the velocity. Therefore, an effective velocity should have the properties shown in Table 1, where the subscript ‘e’ represents the evolving object and ‘r’ the real object. For P_m, the level set function φ should decrease, so V is positive, while for P_f, φ should increase, so V is negative.

Table 1. Properties of desirable velocity.

Pixel	$\mathbf{P_m}$	$\mathbf{P_f}$
u_e	0	1
u_r	1	0
φ_e	>0	<0
φ_r	<0	>0
φ	↓	↑
V	>0	<0

An example is given to explain how the velocity works during the evolution. Figure 3(a) compares the real object (red part) and the estimate (blue part) in a cut-plane of the computational domain. Therefore, there are some “missing” pixels, P_m, and “false” pixels, P_f, (figure 3(b)). According to the property of desirable velocity in table 1, figure 3(c) shows that the velocity is positive for P_m and negative for P_f. Therefore, after one iteration, all “false” pixels are removed well and some “missing” pixels vanish (figure 3(d)). Remaining “missing” pixels will be corrected after a few iterations with the positive velocity.

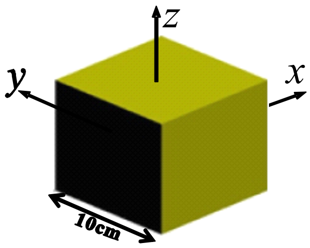


(a) (b) (c) (d)

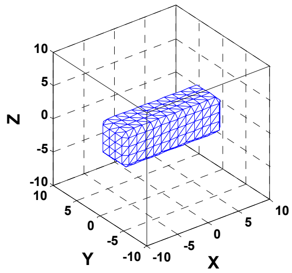
Figure 3. (a) Comparison between the real object and the estimate; (b) the initial cut-plane; (c) the velocity for each pixel in this cut-plane; (d) the cut-plane after one iteration.

4. Numerical results

Now we consider to reconstruct a PEC cube (figure 4(a)) with the level set method. The computational domain is distributed into $N=16^3$ cells, with 16^2 pixels in each cut plane. To apply this method, an initial guess is required. It can be arbitrary, and we choose here a rectangular box to test the performance or this method, since it needs either to "expand" or to "shrink" for obtaining the shape of the original cube. Once the initial contour C_0 is given, the initial level set function φ_0 is defined as the signed distance between each point and C_0, negative inside and positive outside [10]. The gradient of the level set function, $|\nabla\varphi|$, is approximated by a Hamiltonian scheme [8]. Combining (5) with (8), the velocity can be easily obtained by the algorithm we proposed in the forward problem.



(a)



(b)

Figure 4. (a) Configuration of a PEC cube; (b) the initial guess: a rectangular.

Here, we use the geometrical profile function as the observed data of the real object. With 3 mutually orthogonal directions and 20 iterations, the reconstructed result (figure 5) shows high similarity with some acceptable errors comparing to the real object. For the non-orthogonal case (d1 = 45°,y,z), the estimate (figure 6) has more "missing" and "false" pixels, but still it is not severely distorted. Now if we add another direction (d4 = 60°), the result (figure 7) shows greater agreement with the real shape. It is worth mentioning that more accurate images can be obtained with a larger number of samples.

5. Conclusion and perspectives

Radar imaging from ramp responses can reconstruct the shape of a target with only three directions. Our reconstruction algorithm overcomes the limitation of previous algorithms for orthogonal directions, but it produces distorted estimate in non-orthogonal case. The calculation of the profile function of a 3D object is used to compute the cost function for the evolution process. The level set method, which has been proved to be effective in shape optimization, is used. The property of an effective velocity for this method is analyzed. This method gives good results with 3 orthogonal directions and promising results with 4 non-orthogonal directions without restrictive requirement of the initial guess. Our further work is to construct an adapted cost functional so that the velocity satisfies desirable properties. The regularization of the evolution, for example uniqueness and convergence, should be considered as well.

References

[1] Kennaugh E M and Moffatt D L 1965 Transient and impulse response approximations *Proc IEEE* 53 893-901
[2] Young J D 1976 Radar imaging from ramp response signatures *IEEE Trans. Antennas Propagat.* AP-24 276-282
[3] Moffatt D L 1981 Ramp response radar imagery spectral content *IEEE Trans. Antennas Propagat.* 29 399-401
[4] Nag S and Peters L 2001 Radar images of penetrable targets generated from ramp profile functions *IEEE Trans. Antennas Propagat.* 49 32-40

[5] Chen C and Peters L 2007 Radar scattering and target imaging obtained using ramp-response techniques *Antennas and Propagation Magazine* 49 13-27
[6] Zhang X Lib W and Liu G 2002 A new technique in ramp response for acoustic imaging of underwater objects *Applied Acoustics* 63 453-465
[7] Chauveau J and de Beaucoudrey N 2011 A new algorithm of 3D image reconstruction of radar targets from ramp responses in low frequency *PIER* M16 1-18
[8] Osher S and Sethian J 1988 A front propagating with curvature-dependent speed: algorithms based on Hamiton-Jacobi formulations *J. Comput. Phys.* 7 912-49
[9] Sethian J A 1999 *Level set methods and fast marching methods* (Cambridge: Cambridge University Press)
[10] Litman A, Lesselier D and Santosa F 1998 Reconstruction of a two-dimensional binary obstacle by controlled evolution of a level-set *Inverse Problems* 14 685-706
[11] Dorn O and Lesselier D 2006 Level set methods for inverse scattering *Inverse Problems* 22 R67–R131
[12] Hajihashemi M and El-Shenawee M 2011 Inverse scattering level set algorithm for retrieving the shape and location of multiple targets *Antennas and Propagation (EUCAP)* (Rome, Italy, 11-15 April *2011*) pp.3819-3823
[13] Santosa F 1996 A level set approach for inverse problems involving obstacles *Control Optimizat. Calculus Variat* 1 17-33

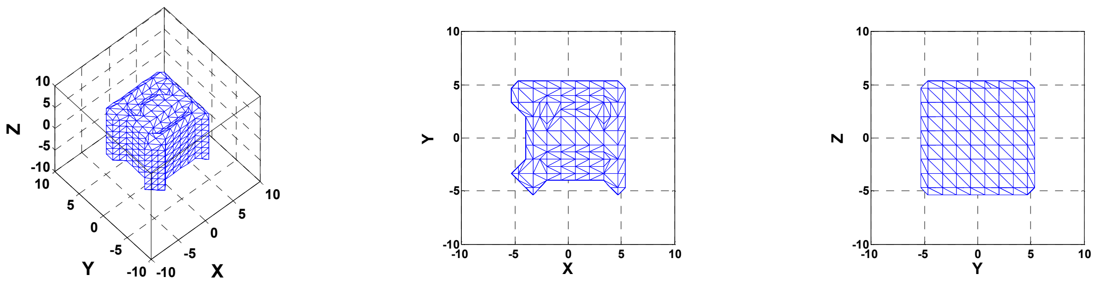


Figure 5. Result obtained with 3 mutually orthogonal directions after 20 iterations

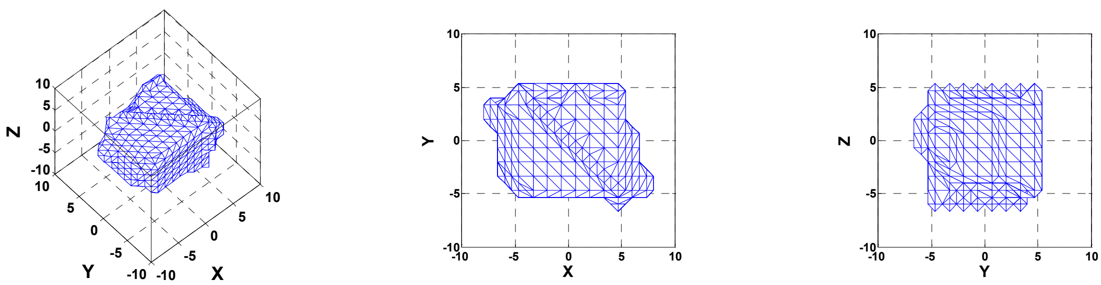


Figure 6. Result obtained with 3 non-orthogonal directions (d1=45°) after 20 iterations

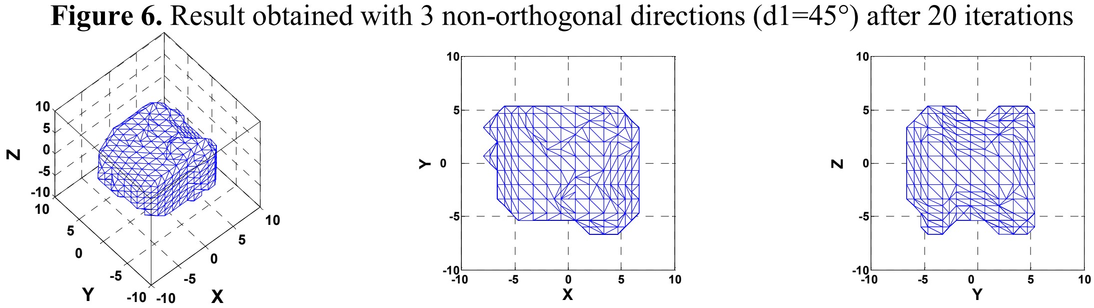


Figure 7. Result obtained with 4 non-orthogonal directions (d1=45°, d4=60°) after 20 iterations