Principal properties of the velocity distribution of dark matter particles near the Solar System

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Principal properties of the velocity distribution of dark matter particles near the Solar System

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Abstract.   The velocity distribution of the dark matter particles on the outskirts of the Solar System remains unclear. We show that under very common assumptions it should be highly anisotropic and have a sharp maximum near $v \sim 500$ km/s. The distribution is totally different from the Maxwell one. We analyze the influence of the distribution function on the results of dark matter detection experiments. It is found that the direct detection signal should differ noticeably from the one calculated from the Maxwell distribution with $\langle v \rangle \simeq 220$ km/s, which is conventional for direct detection experiments. Moreover, the sharp distinction from the Maxwell distribution can be very essential to the observations of dark matter annihilation.

1. Introduction

One of the most evident manifestations of the dark matter existence is the detection of huge invisible halos (with density profiles $\rho \sim r^{-2}$) surrounding galaxies. Our Galaxy also has such a halo. We symbolize the orbital radius of the Solar System, the average velocity of the Galaxy rotation at this radius, and the escape velocity at it by $r_\odot$, $v_\odot$, and $v_{\rm esc}$, respectively. We also denote the radial and tangential components of a dark matter particle velocity by v_r and $v_\rho \equiv \sqrt{v_\phi^2 + v_\theta^2}$. Velocity distribution of the dark matter particles (hereafter DMPs) inside the halo is poorly known; it is usually supposed to be Maxwell with a cut-off when $v > v_{\rm esc}$ [1].

$$f(v) = \frac{4N}{\sqrt{\pi} v_\odot} \left(\frac{v}{v_\odot} \right)^2 \exp\left(-\frac{v^2}{v_\odot^2} \right), \quad v < v_{\rm esc} \tag{1}$$

We accept $r_\odot = 8$ kpc, $v_\odot = 220$ km/s, $v_{\rm esc} = 575$ km/s. N is a normalizing constant, and for the chosen parameters $N \simeq 1.003$. It is worthy of noting that $v_{\rm orb}$ remains almost constant throughout the halo.

Distribution (1) faces with difficulties. In fact, in the framework of collisionless dynamics (1) can be naturally obtained from profile $\rho \sim r^{-2}$ subject to the condition, however, that function f is isotropic, i.e. f depends only on $|v|$ (so called isothermal model). This assumption seems highly improbable. Indeed, if (1) is true, the majority of the dark matter particles has large specific angular momentum $\mu \equiv \mathfrak{M}/m_\chi = [v \times r]$. The average angular momentum of the particles (and, consequently, of the halo) is zero. However, the root-mean-square momentum is $\sqrt{\langle \mu^2 \rangle} \simeq 1800$ kpc $\cdot$ km/s at $r_\odot$. Moreover, since in this model $\mu \sim r v_{\rm orb}$ and $v_{\rm orb}$ is constant in the halo if $\rho \sim r^{-2}$, the root-mean-square angular momentum reaches an incredibly huge value $\sqrt{\langle \mu^2 \rangle} \sim 2 \cdot 10^4$ kpc $\cdot$ km/s at the edge of the halo ($r \sim 100$ kpc). Meanwhile, according to
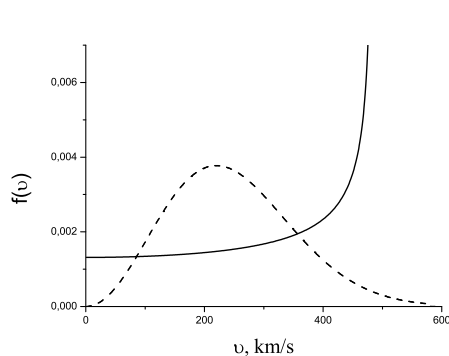
Figure 1. Anisotropic (6) (solid line) and Maxwell ((1), dashed line) distributions of dark matter particles over the velocity absolute value.
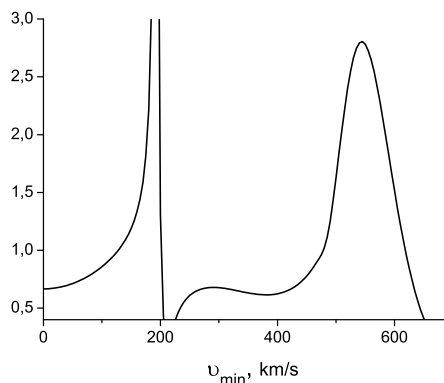

Figure 2. The ratio between the double amplitudes $2A = I_{\max} - I_{\min}$ of direct detection signals (9) calculated for anisotropic distribution (8) and Maxwell distribution (1).

modern cosmological conceptions, not only the total angular momentum of the halo but also the momentum of each particle should have been negligibly small on the linear stage of the structure formation. The halo could gain some angular momentum later, as a result of tidal perturbations or merging of smaller halos; numerical simulations show, however, that it cannot be large [2].

Some results of stellar dynamics are frequently used in order to show that the dark matter particles could gain a large angular momentum during the Galaxy evolution. The parallels between DMP and stellar dynamics, however, are not universally true. The point is that stars are compact objects, and their gravitational field can be strong, at least, locally. On the contrary, the small-scale gravitational field of the dark matter is always small. Therefore important relaxation mechanisms of stellar systems like close pair approaches or an interaction with the interstellar medium are completely ineffective for DMPs. So called violent relaxation [3] is perhaps the only way to impart significant angular momentum to the dark matter particles. However, it acts also on the halo stars; moreover, its efficiency decreases with radius, and it should stronger affect the star distribution since the stellar halo is more compact.

To summarize: all halo objects initially had $v_\rho \simeq 0$ and later gained some angular momentum because of various processes like relaxation or tidal effects. All the mechanisms increased $\langle v_\rho^2 \rangle$ of the halo stars, at least, as much as of the DMPs, while some mechanisms affected only the stars, and not the DMPs. Consequently, velocity distribution of the oldest stellar halo population should be closer to the Maxwellian one, than the distribution of the dark matter particles. In particular, the tangential velocity dispersion $\sigma(v_\rho)$ of the DMPs cannot be larger than that of the halo stars at the same radius.

Modern observations of the oldest halo stars – subdwarfs – confirm the above reasoning [4]. Their tangential dispersion $\sigma(v_\rho) \equiv \sigma_0 \simeq 80$ km/s, which corresponds to $\sqrt{\langle \mu^2 \rangle} \simeq 900$ kpc·km/s, is two times lower than in (1). Moreover, the distribution widely differs from the Maxwellian: $\sigma(v_r)$ is much larger than $\sigma(v_\rho)$. Consequently, $\sigma(v_\rho)$ of the dark matter particles on the outskirts of the Solar System does not exceed $\sigma_0 = 80$ km/s and can be even smaller. Second, the observations show that the halo stars have not yet relaxed and their orbits are rather prolate. On the other hand, if distribution (1) is correct, the ellipticity of the majority of DMP orbits is small, and then the dark matter is the only class of halo objects that move almost circularly. It seems much more natural to assume the opposite.

2. Calculations

For our Galaxy we accept $R = 90$ kpc, which corresponds to the total mass of the Galaxy $M = 10^{12} M_\odot$. The module of gravitational potential on the edge of the halo is equal $\Phi = GM/R$. It is easy to see that $\Phi = v_\odot^2$. Since $\rho \sim r^{-2}$, the mass inside some radius can be found as $\frac{r}{R} M$, gravitational field is equal $\vec{g} = G\frac{M}{rR}$, and we obtain the gravitational potential inside the halo:

$$\phi = -\Phi \left(1 + \ln \frac{R}{r} \right) \tag{2}$$

Let us start our consideration from the case when the DMPs have no angular momentum at all. Then their trajectories are radial ($v = |v_r|$), and the task becomes one-dimensional. Therefore the particle distribution in the halo can be entirely described by a single function $\psi(r, v)$, so that $\psi(r, v)drdv$ gives the total mass of dark matter in the element of phase space $drdv$. In our case $r \propto r^{-2}$, and the halo density is bound with the function ψ by a trivial relation

$$4\pi r^2 \rho(r) = \int_0^\infty \psi(r, v)dv = \frac{M}{R} \tag{3}$$

Each dark matter particle executes a radial oscillation around the galactic centre. We denote by r_0 the maximum distance it moves off the centre. We introduce a distribution function ξ of the particles throughout parameter r_0, so that $\xi(r_0)dr_0$ is the total mass of DMPs which apoapsis lies in the interval $[r_0; r_0 + dr_0]$. The r-coordinate of these particles varies between 0 and r_0, and they give a yield into the halo density over all this interval. We can easily find function ξ if we suppose that the halo boundary is sharp, i.e. the density obeys the law $\rho \sim r^{-2}$ up to radius R and is equal to zero just after it. Then the maximum velocity the halo particles may have at radius r is

$$v_{\max} = \sqrt{2\Phi \ln \frac{R}{r}} = v_\odot \sqrt{2 \ln \frac{R}{r}} \tag{4}$$

It is a matter of direct verification to prove that function

$$\xi(r_0) = \frac{M}{\sqrt{\pi} R \sqrt{\ln \frac{R}{r_0}}} \tag{5}$$

satisfies condition (3). This function has a peculiarity at $r_0 = R$. So the main part of DMPs comes to us from the very edge of the halo. Normalized velocity distribution on the outskirts of the Solar System is equal to

$$f(v) = \frac{2}{\pi \sqrt{v_{\max}^2 - v^2}} \simeq \frac{2}{\pi \sqrt{(2.2 v_\odot)^2 - v^2}} \tag{6}$$

The angular momentum of the dark matter particles, however, hardly can be exactly equal to zero [5]. If a particle possesses some specific momentum μ, its velocity in gravitational field (2) is equal to:

$$v_\rho = \frac{\mu}{r}; \quad |v_r| = \sqrt{2\Phi \ln \frac{r_0}{r} - \mu^2 \left(\frac{1}{r^2} - \frac{1}{r_0^2} \right)} \tag{7}$$

As we have already discussed in the Introduction, specific angular momentum of the majority of the dark matter particles hardly can be larger than 900 kpc · km/s. Substituting $r_0 = R$, $r = r_\odot$, we can see that the second term under the root is equal to 110^2 km^2/s^2, while the first one is $\sim 500^2$ km^2/s^2. Hence near the Solar System the influence of the angular momentum on the radial dynamics is still negligible for the majority of DMPs, and we can use (5) up to $r_\odot$

as before. Thus the DMP distribution throughout v_r in this approximation coincides with (6). However, the particles have also some distribution throughout v_ρ. For simplicity we will suppose that $f \propto \exp(-v_\rho^2/2\sigma_0^2)$ where $\sigma_0 = 80$ km/s, though the distribution can be much narrower. Then the normalized DMP distribution near the Solar System can be closely approximated by

$$f(v) = \frac{\exp\left(-\dfrac{v_\rho^2}{2\sigma_0^2}\right)}{2\pi^2\sigma_0^2\sqrt{v_{\max}^2 - v_r^2}} \tag{8}$$

where $v_r \in [-v_{\max}; v_{\max}]$, $v_{\max} = 484$ km/s. Distribution (8) is strongly anisotropic and actually describes two colliding beams of particles.

3. Discussion

Fig. 1 represents distributions (6) and (1) (solid and dashed lines, respectively). One can see that (6) is much narrower and has much higher average velocity. The physical reason of it is obvious: in the case of Maxwell distribution (1) the particles move almost circularly, which is why only a few of DMPs from the edge of the halo reach the Solar orbit. On the contrary, in the case considered in this article the majority of DMPs come from the halo edge and thus are much more accelerated by the gravitational field. Consequently, the question of what of the distributions, (1) or (6), is correct, can be reduced to whether the particles from the halo edge can reach the Solar orbit or not. In addition to the arguments presented in the Introduction we note that, according to (7), a particle falling from $r = R$ should have a specific angular momentum $\mu \sim 4000$ kpc $\cdot$ km/s, lest the particle can reach $r = 8$ kpc. This value is huge, it far exceed not only the characteristic momentum of halo objects, but even the momentum of the disk, and thus looks very unlikely. So particles from the edge of the halo freely reach the Earth, and their spectrum should be closer to (6).

The difference between (1) and (6) is important for various aspects of the dark matter physics. In the case of direct dark matter search the signal, roughly speaking, can be represented as a product of a part depending almost not at all on the DMP distribution and an integral [6]

$$I(v) = \int_{v_{\min}}^{\infty} \frac{\tilde{f}(v)}{v}\, d\vec{v} \tag{9}$$

Here $v_{\min}$ is the minimal DMP speed, to which the detector is sensitive, $\tilde{f}(v)$ is the distribution in the Earth's frame of reference, obtained from (1) or (6) by a Galilean transformation (see details in [1], section 3.3). Because of the Earth's orbital motion I varies with a year period, and it is this variation that is observed in the direct detection experiments. Fig. 2 shows the ratio between the double amplitudes $2A = I_{\max} - I_{\min}$ of direct detection signals calculated for anisotropic distribution (8) and Maxwell distribution (1), as a function of $v_{\min}$. One can see a very significant difference. The difference between distributions (1) and (6) can also be important for the indirect dark matter search.

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