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CP Violation in B Decays at Belle

Masaya Iwabuchi

Department of Physics, Yonsei University, Seoul 120-749, Korea

E-mail: iwabuchi@post.kek.jp

Abstract.

I review the recent Belle measurements on CP violation in B decays. In this review, I cover the experimental constraints on the Cabibbo-Kobayashi-Maskawa (CKM) angles of the Unitarity Triangle by introducing several experimental approaches. Search for new physics beyond the Standard Model in $b \rightarrow s\overline{q}q$ transitions is also presented.

1. Introduction

The present status on the measurement of CP violation in the quark sector is mainly based on the results achieved by BaBar and Belle. Those *B*-factory experiments, already ended their operations, have been spectacularly successful at discovering and investigating CP violation in *B*-meson decays, thanks to the great success of PEP-II and KEKB asymmetric-energy $e^+e^$ colliders operating at the center of mass energy corresponding to the mass of $\Upsilon(4S)$. Those experiments have significantly improved our knowledge on CP-violation phenomena over a decade.

In the Standard Model (SM), CP violation arises from a complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [1, 2], where CP violation can be quantified in terms of three parameters, ϕ_1 , ϕ_2 and ϕ_3^1 , representing the three angles of the so-called Unitarity Triangle. Precise measurements of CKM angles are therefore of fundamental importance for the description of the weak interaction of quarks and the investigation for the new sources of CP violation.

In these proceedings, measurements of CKM angles at the Belle experiment are reviewed. This review also includes the measurement of effective ϕ_1 (ϕ_1^{eff}) in $b \to s\bar{q}q$ transitions for new-physics search.

2. ϕ_1

 $\sin 2\phi_1$ can be obtained using $b \to c\bar{c}s$ transitions, where the most precise measurement for ϕ_1 comes from $B^0 \to J/\psi K^0$ decays.

With 535 million $B\overline{B}$ pairs, Belle updated the measurement of $\sin 2\phi_1$ using $B^0 \to J/\psi K^0$ decays [3]. Figure 1 shows the results of a fit to the reconstructed candidates in the fit region. Several kinematic variables are used here; for $B^0 \to J/\psi K_S^0$ decay, the energy difference $\Delta E \equiv E_B^* - E_{\text{beam}}^*$ and the beam-energy constrained mass $M_{\text{bc}} \equiv \sqrt{(E_{\text{beam}}^*)^2 - (p_B^*)^2}$ are used in the fit, where E_{beam}^* is the beam energy, and E_B^* and p_B^* are the energy and momentum,

¹ The angles ϕ_1 , ϕ_2 and ϕ_3 are also sometimes known as β , α and γ , respectively.

respectively, of the reconstructed B candidate, all measured in the center-of-mass system. p_B^* is used in the fit for $B^0 \to J/\psi K_L^0$ candidates.



Figure 1. (a) $M_{\rm bc}$ distribution in the ΔE signal region for selected $B^0 \to J/\psi K_S$ candidates and (b) p_B^* distribution for selected $B^0 \to J/\psi K_L$ candidates.

The signal yields are determined to be 7484 \pm 87 for $B^0 \rightarrow J/\psi K_S$ and 6512 \pm 123 for $B^0 \rightarrow J/\psi K_L$, where errors are statistical only.

In the decay chain $\Upsilon(4S) \to B^0 \overline{B}{}^0 \to f_{CP} f_{\text{tag}}$, where one of the *B* mesons decays at time t_{CP} to a *CP* eigenstate f_{CP} and the other decays at time t_{tag} to a final state f_{tag} that distinguishes between B^0 and $\overline{B}{}^0$, the decay rate has a time dependence given by

$$\mathcal{P}(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} \bigg\{ 1 + q \cdot \Big[\mathcal{S}_{CP} \sin(\Delta m_d \Delta t) + \mathcal{A}_{CP} \cos(\Delta m_d \Delta t) \Big] \bigg\}.$$

Here S_{CP} and \mathcal{A}_{CP} describe mixing-induced and direct CP-violation, respectively, and τ_{B^0} is the B^0 lifetime, Δm_d is the mass difference between the two B^0 mass eigenstates, $\Delta t = t_{CP} - t_{\text{tag}}$, and the *b*-flavor charge q = +1 (-1) when the tagging *B* meson is a B^0 (\overline{B}^0). Here, the SM predicts $S_f = -\xi_f \sin 2\phi_1$ and $\mathcal{A}_{CP} = 0$ for both $b \to c\overline{cs}$ and $b \to s\overline{q}q$ transitions, where $\xi_f = +1$ (-1) corresponds to the *CP*-even (-odd) final states. Δt can be determined from the displacement in the electron beamline (*z*) between the f_{CP} and f_{tag} decay vertices: $\Delta t \simeq (z_{CP} - z_{\text{tag}})/(\beta\gamma c) \equiv \Delta z/(\beta\gamma c)$.

An unbinned maximum-likelihood fit to the observed Δt distribution is performed to determine CP-violation parameters. Figures 2 shows the Δt distributions and asymmetry. $\sin 2\phi_1$ is measured to be $0.642 \pm 0.031 \pm 0.017$ by the results of the fits to Δt distributions. Here, the first error is statistical and the second error is systematic.

 $\sin 2\phi_1$ will be updated soon at Belle using the full data set (772 million $B\overline{B}$ pairs). The magnitude of the statistical error would be comparable with that of the systematic error, where the statistical error is expected to be around 0.02.



Figure 2. Background subtracted Δt distributions and asymmetries for events with good tags for $B^0 \to J/\psi K^0$.

3. ϕ_2

 $b \to u \overline{u} d$ quark transitions such as $B^0 \to \pi \pi, \rho \pi, \rho \rho$ are sensitive to ϕ_2 .

The *CP*-violation parameters are expected to be $S_{CP} = \sin 2\phi_2$ and $\mathcal{A}_{CP} = 0$ by the SM with the tree diagram only, however, direct *CP* violation is possible, $\mathcal{A}_{CP} \neq 0$, due to a large contribution from the electroweak penguin-mediated decay processes. ϕ_2 can be determined by $S_{CP} = \sqrt{1 - \mathcal{A}_{CP}^2} \sin(2\phi_2 + \kappa)$, where κ is the shift due to the penguin contribution and determined by the SU(2) isospin analysis [4]. For the ϕ_2 measurements through $B^0 \to \pi^+\pi^-$ and $B^0 \to \rho^+\rho^-$ modes, it is possible to determine κ by considering the set of three $B \to hh$ decays where hh is either two pions or two longitudinally polarized ρ mesons. The $B \to hh$ amplitudes obey the complex triangle relations,

$$A_{+0} = \frac{1}{\sqrt{2}}A_{+-} + A_{00}, \quad \overline{A}_{-0} = \frac{1}{\sqrt{2}}\overline{A}_{+-} + \overline{A}_{00}.$$

The electroweak penguins can be neglected in the $B^+ \to h^+ h^0$, and thus these triangles share the same base, $A_{+0} = \overline{A}_{-0}$. κ can be determined from the difference between the two triangles. This method has an 8-fold discrete ambiguity in the determination of ϕ_2 .

3.1. $\phi_2 \text{ from } B^0 \rightarrow \pi^+\pi^-$

 ϕ_2 measurement is performed using $B^0 \to \pi^+\pi^-$ mode with 535 million $B\overline{B}$ pairs [5].

The signal yield is estimated by a fit to data, making use of ΔE , $M_{\rm bc}$ and the kaon identification probability. $B^0 \rightarrow \pi^+\pi^-$ signal yield is obtained to be 1464 ± 65 events. In the time-dependent analysis, direct CP violation is observed, $A_{CP} = +0.55 \pm 0.08(\text{stat}) \pm 0.05(\text{syst})$ with a significance of 5.5σ , while mixing-induced CP violation is also observed, $S_{CP} = -0.61 \pm 0.10(\text{stat}) \pm 0.04(\text{syst})$ with a significance greater than 5.3σ for any A_{CP} value.

To constrain ϕ_2 , the isospin relations are used with measured values of S_{CP} and A_{CP} , the world average (W.A.) branching ratios of $B^0 \to \pi^+\pi^-$, $\pi^0\pi^0$, and $B^+ \to \pi^+\pi^0$, and the W.A. direct CP-asymmetry for $B^0 \to \pi^0\pi^0$ [6]. χ^2 is constructed using the measured values and the six parameters, one of which is ϕ_2 . Four different solutions are found after the minimization of the χ^2 . With the expectation from other CKM measurements, $(97\pm11)^\circ$ is found. The ϕ_2 range $11^\circ < \phi_2 < 79^\circ$ is excluded at the 95% confidence level (C.L.). The resulting two-dimensional confidence regions in the S_{CP} and A_{CP} plane and the difference 1 - C.L. plotted for a range of ϕ_2 values are shown in Fig. 3.



Figure 3. The left plot shows confidence regions for S_{CP} and A_{CP} . The curves show the contour from 1σ to 6σ , from inside to outside. The point with error bars is the S_{CP} and A_{CP} measurement. The right plot shows difference 1 - C.L. plotted for a range of ϕ_2 values obtained with an isospin analysis. The solid and dashed lines indicate C.L. = 68.3% and 95%, respectively.

3.2. $\phi_2 \text{ from } B^0 \rightarrow \rho^+ \rho^-$

Possible relative angular momentum for the $B^0 \rightarrow \rho^+ \rho^-$ decay is L = 0, 1, 2. The *CP*violating parameters receive contributions from a longitudinally polarized state (*CP*-even) and two transversely polarized states (an admixture of *CP*-even and *CP*-odd states). However, measurements of the polarization fraction by Belle and BaBar show that the longitudinal polarization fraction is near unity. There is no need to elaborate the angular analysis, and pure *CP*-even state (100% longitudinal polarization) is assumed in this analysis.

Time-dependent analysis is performed for $B^0 \to \rho^+ \rho^-$ mode with 535 million $B\overline{B}$ pairs [7]. ΔE , $M_{\rm bc}$ and event shape variables for $e^+e^- \to q\overline{q}$ (q = u, d, s, c) continuum and $B\overline{B}$ events are used in a fit to estimate the signal yield. $B^0 \to \rho^+\rho^-$ signal yield is obtained to be 576 ± 53 events. Direct CP violation and mixing-induced CP violation are obtained to be $A_{CP} = +0.16 \pm 0.21(\text{stat}) \pm 0.08(\text{syst}), S_{CP} = +0.19 \pm 0.30(\text{stat}) \pm 0.08(\text{syst})$, respectively.

In the isospin relations to constrain ϕ_2 , the W.A. branching ratios of $B^0 \to \rho^+ \rho^-$ and $B^+ \to \rho^+ \pi^0$ are used [6], while the branching ratio of $B^0 \to \rho^0 \rho^0$ is obtained from the Belle measurement [8] to constrain ϕ_2 . ϕ_2 is determined to be $(91.7 \pm 14.9)^\circ$. Figure 4 shows the difference 1 - C.L. plotted for ϕ_2 .



Figure 4. 1 – C.L. vs. ϕ_2 obtained from the isospin analysis of $B \rightarrow \rho \rho$ decays.

3.3. $\phi_2 \text{ from } B^0 \rightarrow \rho \pi$

 $B^0 \to \rho \pi$ mode is also sensitive to ϕ_2 . Since this mode can be treated as an interference of $B^0 \to \rho^+ \pi^-$, $\rho^- \pi^+$ and $\rho^0 \pi^0$ modes, a time-dependent Dalitz plot analysis (TDPA) is employed, where measurements of interfering parameters must be included. Measurements for relative sizes and complex phases of the amplitudes are required to extract ϕ_2 , however, those amplitudes are related to ϕ_2 through an isospin relation. In this point, $B^0 \to \rho \pi$ channel has an advantage which can reduce discrete ambiguities, in contrast to the previous quasi-two-body modes for the ϕ_2 measurements.

With 449 million $B\overline{B}$ pairs, Belle performed a measurement of ϕ_2 using $B^0 \to \rho \pi$ [9]. The signal yield is determined from a four-dimensional fit to the $\Delta E - M_{\rm bc}$ and Dalitz plot distribution. The fit yields $971 \pm 42 \ B^0 \to \pi^+ \pi^- \pi^0$ events in the signal region. To constrain ϕ_2 , measurements with results on the branching fractions for $B^0 \to \rho^{\pm} \pi^{\mp}$ and $B^+ \to \rho^+ \pi^0$, $\rho^0 \pi^+$, and flavor asymmetries of the latter two [6] are all combined. χ^2 scans are shown in Fig. 5 where ϕ_2 can be constrained to be $68^\circ < \phi_2 < 95^\circ$ at 68.3% C.L.



Figure 5. 1 – C.L. vs ϕ_2 . Dotted and solid curves correspond to the result from the TDPA only and that from the TDPA and an isospin combined analysis, respectively.

4. ϕ_3

 $B \to D^{(*)0}K$ modes provide a good probe for the ϕ_3 measurement, where the interference between $B \to D^{(*)0}K$ and $B \to \overline{D}^{(*)0}K$ decays is utilized. The magnitude of the amplitude ratio of these decays (r_B) can be evaluated to be ~ 0.1, using the CKM matrix elements with an additional color suppression factor. The weak phase ϕ_3 arises from the complex phase of V_{ub} included in the $B^+ \to D^{(*)0}K^+$ decay. There is no contribution from the penguin diagram in these decays.

Among the several approaches proposed for the ϕ_3 measurement, the current most precise measurement of ϕ_3 is based on the so-called GGSZ method [10]. In this method, amplitude analysis of the multibody decay of the neutral D meson is required. However, complex phase variations and imperfect assumption of the D-decay model potentially includes the irreducible uncertainties for the ϕ_3 measurement. In other approaches, the so-called ADS method [11] is also considered to be promising. The ADS method can be free from the model uncertainty, while the measurements based on this method have been statistically limited.

4.1. ϕ_3 from the GGSZ method

In the $B^{\pm} \to DK^{\pm}$ interference, D can be either D^0 or \overline{D}^0 , and the amplitude of D can be expressed as

$$|\tilde{D}^{0}\rangle = r_{B}e^{\pm i\phi_{3} + i\delta_{B}}|D^{0}\rangle + \overline{D}^{0}\rangle.$$

Here δ_B is the strong phase difference of the *B* decays. It is possible to distinguish D^0 from \overline{D}^0 if the multibody *D* decays are used.

With 657 million $B\overline{B}$ pairs, Belle updated the ϕ_3 measurement using the GGSZ method with $B^+ \to D^{(*)}K^+$, $D \to K_S \pi^+ \pi^-$ decays [12]. This measurement is based on the model-dependent approach, including the model determination of *D*-meson decay using the $e^+e^- \to q\overline{q}$ continuum processes.

The signal yield is estimated by a two-dimensional fit for the ΔE and $M_{\rm bc}$ distribution for each decay mode. Signal yields of $B^+ \to DK^+$, $B^+ \to D^*K^+$ followed by $D^* \to D\gamma$ or $D^* \to D\pi^0$ are obtained to be 756, 149 and 141 events in each signal region, respectively.

To obtain ϕ_3 , Dalitz distributions of the B^+ and B^- are fitted separately, using Cartesian parameters $x_{\pm} = r_{\pm} \cos(\pm \phi_3 + \delta)$ and $y_{\pm} = r_{\pm} \sin(\pm \phi_3 + \delta)$, where the indices "+" and "-" correspond to B^+ and B^- decays, respectively. Here, r_+ and r_- are not constrained to be equal. The advantage of this approach is for the low bias and simple distribution of the fitted parameters. Several kinematic variables are used in a fit; Dalitz plot distribution, ΔE , $M_{\rm bc}$ and event shape variables that separates the B decays from the $e^+e^- \rightarrow q\bar{q}$ continuum processes. The results of the separate B^+ and B^- data fits are shown in Fig. 6. The separation of the B^- and B^+ positions in the (x, y) plane indicates direct CP violation and is measured to be $2r_B |\sin \phi_3|$.



Figure 6. Results of signal fits with free parameters $x = r \cos \theta$ and $y = r \sin \theta$ for $B^+ \to DK^+$ (a) and $B^+ \to D^*K^+$ (b) samples, separately for B^- and B^+ data. Contours indicate 1, 2, and 3 (for $B^{\pm} \to DK^{\pm}$) and 1 standard deviation regions (for $B^{\pm} \to D^*K^{\pm}$).

A frequentist technique is used to evaluate the statistical significance of the measurements. In order to improve the sensitivity to ϕ_3 , event samples of $B^+ \to DK^+$ and $B^+ \to D^*K^+$ are combined. ϕ_3 for the combination of these modes is obtained to be $\phi_3 = 78.4^{\circ} + 10.8^{\circ} + 10.8^{\circ} + 10.6^{\circ} + 10.8^{\circ} + 10.8^$

Model uncertainty is dominated by the uncertainty due to the complex phase. To reduce this model uncertainty, model-independent approach is proposed [10, 13, 14]. Based on this method, CLEO recently updated the results of relative phase between D^0 and \overline{D}^0 by studying the $\psi(3770) \rightarrow D^0 \overline{D}^0$ decays [15]. The model error in the ϕ_3 measurement will be replaced by a statistical error of about a few degrees due to the finite $\psi(3770) \rightarrow D^0 \overline{D}^0$ sample, while the statistical error associated with the *B* data sample should increase by 10%-20% due to the binned fit procedure.

4.2. ϕ_3 from the ADS method

In the ADS method, measurements of CP asymmetry (\mathcal{A}) and partial rate (\mathcal{R}) play an important role for the ϕ_3 measurement. In the decay chain $B^{\pm} \to DK^{\pm}$, $D \to K^{\pm}\pi^{\mp}$, where D can be either D^0 or \overline{D}^0 , \mathcal{A} and \mathcal{R} are described by

$$\mathcal{A} \equiv \frac{\mathcal{B}(B^{-} \to [K^{+}\pi^{-}]_{D}K^{-}) - \mathcal{B}(B^{+} \to [K^{-}\pi^{+}]_{D}K^{+})}{\mathcal{B}(B^{-} \to [K^{+}\pi^{-}]_{D}K^{-}) + \mathcal{B}(B^{+} \to [K^{-}\pi^{+}]_{D}K^{+})} \\ = 2r_{B}r_{D}\sin(\delta_{B} + \delta_{D})\sin(\phi_{3})/\mathcal{R}, \\ \mathcal{R} \equiv \frac{\mathcal{B}(B^{-} \to [K^{+}\pi^{-}]_{D}K^{-}) + \mathcal{B}(B^{+} \to [K^{-}\pi^{+}]_{D}K^{+})}{\mathcal{B}(B^{-} \to [K^{-}\pi^{+}]_{D}K^{-}) + \mathcal{B}(B^{+} \to [K^{+}\pi^{-}]_{D}K^{+})} \\ = r_{B}^{2} + r_{D}^{2} + 2r_{B}r_{D}\cos(\delta_{B} + \delta_{D})\cos(\phi_{3}).$$

Here δ_D is the strong phase difference of the *D*-meson decays, and r_D is the amplitude ratio of $D^0 \to K^+\pi^-$, doubly Cabibbo suppressed mode, to $\overline{D}^0 \to K^+\pi^-$, Cabibbo favored mode. In the $B^- \to [K^+\pi^-]_D K^-$ interference, either *B* or *D* decay is suppressed, while the $B^- \to [K^-\pi^+]_D K^-$ decay process is dominated by the favored modes of *B* and *D* decays.

With 772 million $B\overline{B}$ pairs, Belle updated the measurement for the ADS method using the above decay chain, where preliminary results are obtained on \mathcal{A} and \mathcal{R} . In this measurement, a tool for multi-variate analysis named NeuroBayes is employed. In order to separate the $B \to DK$ signal from backgrounds, ten kinematic variables are used as inputs for NeuroBayes, such as a likelihood ratio of Fisher discriminant of modified Fox-Wolfram moments, the distance of vertices between the reconstructed B and the other B, and the decay angle of $D \to K\pi$.

The signal yield is extracted by a two dimensional fit on ΔE and NeuroBayes output \mathcal{NB} , and determined to be $56.0^{+15.1}_{-14.2}$ for $B^- \rightarrow [K^+\pi^-]_D K^-$, where the charge-conjugate mode is included. Figure 7 shows the results of the fit to the reconstructed candidates in the fit region. The ΔE distributions for $[K^+\pi^-]_D K^-$ and $[K^-\pi^+]_D K^+$ in the \mathcal{NB} signal region are shown.



Figure 7. ΔE distributions for $[K^+\pi^-]_D K^-$ (left) and $[K^-\pi^+]_D K^+$ (right). Curves represent the various components from the fit: DK signal (thick dashed line), $D\pi$ backgrounds (dashed line), $B\overline{B}$ backgrounds (dash-dotted line), continuum (dotted line) and sum of all components (solid line).

The first evidence of the DK signal is obtained with a significance of 4.1σ including systematic uncertainties. The obtained results for the CP asymmetry and the ratio of the suppressed to favored decay rate are, $\mathcal{A} = -0.39^{+0.26}_{-0.28}(\text{stat})^{+0.04}_{-0.03}(\text{syst})$, and $\mathcal{R} = [1.63^{+0.44}_{-0.44}(\text{stat})^{+0.07}_{-0.13}(\text{syst})] \times 10^{-2}$. To constrain ϕ_3 , r_D , δ_D and other relevant observables will be used.

5. ϕ_1^{eff}

Precise measurement of ϕ_1 also provides a useful comparison between decays mediated by $b \rightarrow s\overline{q}q$ and $b \rightarrow c\overline{c}s$ transitions. A new physics may enter into the loop diagram, and to search

for the new physics effects, various approaches have been tested by investigating deviations of CP violating parameter ϕ_1^{eff} from ϕ_1 determined by $b \to c\overline{c}s$ decays.

With 657 million $B\overline{B}$ pairs, Belle performed a measurement of ϕ_1^{eff} using $B^0 \to K^+K^-K_S$ decay with a time-dependent Dalitz approach [16]. Several decay modes are included in the signal model: $\phi(1020)K_S$, $f_0(980)K_S$, f_XK_S , $\chi_{c0}K_S$ and nonresonant $K^+K^-K_S$.

The signal yield is estimated by a fit to data, making use of ΔE , $M_{\rm bc}$ and the flavor-tag quality interval. $B^0 \to K^+ K^- K_S$ signal yield is obtained to be 1176 ± 51 events. In the time-dependent analysis, four solutions are found from a fit. A single solution is preferred from all the solutions using the branching fractions of $f_0(980) \to \pi\pi$ and $f_0(1500) \to \pi\pi$ as external information. Figure 8 shows the Δt distributions and asymmetry of the preferred solution.



Figure 8. (a) Δt distribution and (b) raw asymmetry for the $B^0 \to K^+ K^- K_S$ candidates in the $\phi(1020)K_S^0$ region, with good tags. In (a), the dotted black curve shows the background component. In (b), the solid curve shows the fit projection, while the dashed curve shows the SM expectation from the time-dependent CP asymmetry measurement in $b \to c\overline{cs}$ decays.

In the preferred solution, ϕ_1^{eff} is measured to be $(32.2 \pm 9.0 \pm 2.6 \pm 1.4)^\circ$ for $\phi(1020)K_S$ mode, and $(31.3 \pm 9.0 \pm 3.4 \pm 4.0)^\circ$ for $f_0(980)K_S$ mode. The results are still consistent with the SM expectation like other results from Belle, where measurements of ϕ_1^{eff} are all consistent with the value of ϕ_1 obtained from the decay $B^0 \to J/\psi K^0$ within the two standard deviations.

6. Summary

Various studies have been performed to investigate CP violation in the *B* decays at Belle. The upgraded Belle II experiment will allow us to constrain the CKM angles with much higher statistics, and also help us to explore and clarify the nature of CP violation.

References

- [1] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963).
- [2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
- [3] K.-F. Chen et al. (Belle Collaboration), Phys. Rev. Lett. 98, 031802 (2007).
- [4] M. Gronau and D. London, Phys. Rev. Lett. 65, 3381 (1990).
- [5] H. Ishino et al. (Belle Collaboration), Phys. Rev. Lett. 98, 211801 (2007).
- [6] Heavy Flavor Averaging Group, http://www.slac.stanford.edu/xorg/hfag.
- [7] A. Somov *et al.* (Belle collaboration), Phys. Rev. D **76**, 011104 (2007).
- [8] C.-C. Chiang et al. (Belle collaboration), Phys. Rev. D 78, 111102(R) (2008).
- [9] A. Kusaka et al. (Belle collaboration), Phys. Rev. Lett. 98, 221602 (2007).
- [10] A. Giri, Yu. Grossman, A. Soffer and J. Zupan, Phys, Rev. D 68, 054018 (2003).
- [11] D. Atwood, I. Dunietz and A. Soni, Phys. Rev. Lett. 78, 3357 (1997).
- [12] A. Poluektov et al. (Belle collaboration), Phys, Rev. D 81, 112002 (2010).
- [13] A. Bondar and A. Poluektov, Eur. Phys. J. C 47, 347 (2006).
- $[14]\,$ A. Bondar and A. Poluektov, Eur. Phys. J. C ${\bf 55},\,51$ (2008).
- $[15]\,$ J. Libby et al. (CLEO collaboration), Phys. Rev. D 82, 112006 (2010).
- [16] Y. Nakahama et al. (Belle collaboration), Phys. Rev. D 82, 073011 (2010).