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TURBULENT SKIN-FRICTION DRAG ON A SLENDER BODY OF REVOLUTION AND GRAY'S PARADOX

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Summary The boundary layer on a slender body of revolution differs considerably from that on a flat plate, but these two cases can be connected by the Mangler-Stepanov transformations. The presented analysis shows that turbulent frictional drag on a slender rotationally symmetric body is much smaller than the flat-plate concept gives and the flow can remain laminar at larger Reynolds numbers. Both facts are valid for an unseparated flow pattern and enable us to revise the turbulent drag estimation of a dolphin, presented by Gray 74 years ago, and to resolve his paradox, since experimental data testify that dolphins can achieve flow without separation. The small values of turbulent skin-friction drag on slender bodies of revolution have additional interest for further experimental investigations and for applications of shapes without boundary-layer separation to diminish the total drag and noise of air- and hydrodynamic hulls.

1. THE FLAT PLATE CONCEPT AND EMPIRICAL DATA FOR THE DRAG OF ROTATIONALLY SYMMETRIC BODIES

Bodies of revolution are of great practical interest in fluid dynamics, being not only a good approximation to animal bodies but also to aircraft and submarine hulls. The experimental data concerning the total drag of rotationally symmetric bodies show a great deal of scatter, e.g., Hoerner [1] noted that the drag coefficients at the same Reynolds number of two rotationally symmetric bodies having the same fineness ratio might differ by 100%. He explained this scatter by the influence of the body support and the boundary-layer transition, which is a complex function of shape, Reynolds number and stream turbulence. Nonetheless, he put forward two well-known empirical formulae for the total drag coefficient C_{dS} in the case of a purely laminar boundary layer

$$C_{dS} = C_{f1} [1 + 1.5(D/L)^{1.5}] + 0.11(D/L)^2, \quad (1.1)$$

where

$$C_{f1} = \frac{1.328}{\sqrt{\text{Re}_L}} \quad (1.2)$$

is the flat-plate skin-friction coefficient obtained by Blasius for laminar flow (see, e.g., [7]), and for a turbulent boundary layer

$$C_{dS} = C_{f1} [1 + 1.5(D/L)^{1.5} + 7(D/L)^3], \quad (1.3)$$

where

$$C_{f1} = \frac{0.0307}{\text{Re}_L^{1/7}} \quad (1.4)$$

is the flat-plate skin-friction coefficient obtained by Falkner for turbulent flow (see, e.g., [7]). C_{dS} is based on the wetted area S , and D and L are the maximum body diameter and its length.

The main term in (1.3) (for slender bodies $D/L \ll 1$) corresponds to the flat-plate frictional drag coefficient, the second term takes into account higher velocities on a body surface and the third is the residual, calculated statistically to fit the experimental data and attributed to the pressure drag [1]. Relation (1.3) yields a good coincidence with experimental data for standard shapes, but overestimates greatly the

frictional drag of a dolphin. Gray used the flat-plate concept in his famous paper [2] and concluded that either the dolphin's "muscles must be capable of generating energy at a rate at least seven times greater than that of other types of mammalian muscle", or that dolphins' tale movements prevent turbulence in the flow. Discussion of the question has continued over the following seven decades; after Gray, further ideas have been put forward involving possible drag reduction properties of dolphin skin ranging from dermal ridges to skin compliance to dermal secretions [3]. In addition to these exotic attempts, some special rigid bodies of revolution were designed and tested [4, 5], which revealed less than half the drag in the turbulent range in comparison with (1.3). These facts do not resolve the paradox, but perhaps better shapes could enable a further diminishing of drag? In particular, the Hansen & Hoyt body [5] revealed a separated flow pattern, whereas a dolphin shape ensures a flow without separation [6].

If the real turbulent frictional drag is probably smaller than (1.3) yields, than the real turbulent pressure drag of standard bodies must be greater to have the same total drag and to fit the experimental data. Some support for this fact can be found by comparing the laminar and the turbulent pressure drags of the body "Dolphin" [4] and by the measurements of the pressure drag on the shape "Akron". It should be noted that, according to the flat-plate concept, turbulent pressure drag is very small for slender bodies and can be neglected when $D/L < 0.2$ (see (1.3)), i.e. there is no need to improve the shape to avoid separation, it is enough only to increase the fineness ratio L/D . This paradigm leads to evident contradictions; e.g., body "Dolphin" [4] has a sharp trailing edge in comparison with a standard cone-cylinder torpedo, which causes a severe separation bubble at its base zone. Nevertheless, according to the relation (1.3), the turbulent pressure drag of the standard torpedo ($L/D=8$) is approximately 14 times smaller than the same value for "Dolphin" ($L/D=3.33$).

2. SKIN-FRICTION DRAG ON AN UNSEPARATED SLENDER BODY OF REVOLUTION

According to (1.3) the total drag of slender rotationally symmetric bodies ($D/L \ll 1$), must coincide with the flat plate skin-friction drag. It was shown in [8] that this is not necessarily the case, because the boundary layer on a slender body of revolution differs from the flat plate one. This fact was supported with the use of the Mangler-Stepanov transformations [7], which reduce the axisymmetric boundary-layer equations to a 2D case and yield the following relationship between the coordinates x , y for the rotationally symmetric boundary-layer and the corresponding two-dimensional coordinates \bar{x} , \bar{y} (all the coordinates are dimensionless based on the body length):

$$\bar{x} = \int_0^x R^2(\xi) d\xi; \quad \bar{y} = R(x)y \quad (2.1)$$

where $R(x)$ is the dimensionless radius of the rotationally symmetric body based on its length L . The flow velocity at the outer edge of the boundary layer, the displacement thickness and the skin-friction coefficient are related as follows (see, e.g., [7]):

$$\bar{U} = U; \quad \delta^* = \frac{\bar{\delta}}{R(x)}; \quad \tau_w = \bar{\tau}_w R(x) \quad (2.2)$$

All these relationships are dimensionless, based on velocity of the ambient flow U_∞ , body length L and $0.5\rho U_\infty^2$ respectively. These equations are valid for an arbitrary rotationally symmetric body provided the thickness of the boundary layer is small in comparison with the radius, which implies that the flow should be unseparated. For a slender body, the coordinate x can be calculated along the body's axis and the velocity U can be supposed to be equal to unity, neglecting the thickness of the boundary layer and the pressure distribution peculiarities. From the first equation of (2.2) the value of U will be also equal to unity, i.e., the rotationally symmetric boundary layer on a slender body can be reduced to the flat-plate one [8].

According to the Blasius expression for a laminar flow, $\bar{\tau}_w = 0.664(\bar{x})^{-1/2} \text{Re}_L^{-1/2}$ (see, e.g., [7]). Introducing the variable x , using (2.1) and (2.2) the laminar skin-friction drag coefficients of a slender rotationally symmetric body may be obtained as

$$C_{dfl} = \frac{2X}{\rho U_\infty^2 L^2} = 2\pi \int_0^1 R(x) \tau_w(x) dx = \frac{4.172}{\sqrt{\text{Re}_L}} \int_0^1 R^2(x) \left[\int_0^x R^2(\xi) d\xi \right]^{-1/2} dx = 8.344 \sqrt{\frac{V}{\pi L^3 \text{Re}_L}} \quad (2.3)$$

and using the volumetric drag coefficient and Reynold number

$$C_{dV} = \frac{4.708}{\sqrt{\text{Re}_V}} \quad (2.4)$$

Note that the volumetric friction drag coefficient C_{dV} does not depend on the slender body shape. To calculate C_{dS} , information about the shape is necessary, for which, for slender bodies like elliptical bodies of revolution, the empirical Hoerner equations, [1]

$$V \approx 0.65L\pi D^2 / 4, \quad S \approx 0.75L\pi D \quad (2.5)$$

may be used. Then the laminar skin-friction drag coefficient based on the wetted area can be calculated with (2.3) and (2.5) as

$$C_{dS} \approx \frac{1.43}{\sqrt{\text{Re}_L}} \approx 1.075C_{fL} \quad (2.6)$$

According to (2.6), a slender rotationally symmetric body with a laminar boundary layer has only 8% greater drag coefficient than a flat plate (1.2), i.e., the flat-plate concept may be used for the laminar flow.

The turbulent case, however, is rather different. The Falkner equation $\bar{\tau}_w = 0.0263(x)^{-1/2} \text{Re}_L^{-1/2}$ (see, e.g., [7]) yields for the turbulent skin-friction drag coefficient C_{dL}

$$C_{dL} = \frac{2X}{\rho U_\infty^2 L^2} = 2\pi \int_0^1 R(x)\tau_w(x)dx = 0.166\text{Re}_L^{-1/2} \int_0^1 R^2(x) \left[\int_0^x R^2(\xi)d\xi \right]^{-1/2} dx = 0.0726 \frac{V^{6/7}}{L^{18/7} \text{Re}_L^{1/7}} \quad (2.7)$$

Other skin-friction drag coefficients can be calculated as follows:

$$C_{dV} = 0.073 \frac{V^{5/21}}{L^{15/21} \text{Re}_V^{1/7}}, \quad C_{dS} \approx 0.564C_{fL} \left(\frac{D}{L} \right)^{5/7} \quad (2.8)$$

(2.8) indicates that C_{dS} should be $0.564(D/L)^{5/7}$ times smaller than the value of (1.4), i.e., that using the turbulent skin-friction drag of a flat plate greatly overestimates the turbulent drag on a rotationally symmetric body, if the flow is unseparated. From the curves computed in Fig. 1 it can be seen that there is not a great difference between the laminar and the turbulent skin-friction drag for slender rotationally symmetric bodies with unseparated flow.

According to Fig.1 there are ranges of the Reynolds numbers where the turbulent friction of slender axisymmetric bodies is less than the laminar one. This contradiction can easily be resolved by taking into account the fact that expressions (2.7) and (2.8) were obtained for the fully turbulent boundary layer, which develops for rather high Reynolds numbers. The same situation applies with classical expressions (1.2) and (1.4), e.g., the turbulent flat-plate friction (1.4) is smaller than laminar one (1.2) at $\text{Re}_L < 3.8 \cdot 10^4$. Furthermore, the laminar-turbulent transition in the axisymmetric boundary layer takes place at larger Reynolds number in comparison with a flat plate and the critical values of Reynolds number increase with increasing fineness (or aspect) ratio L/D (see the next Section).

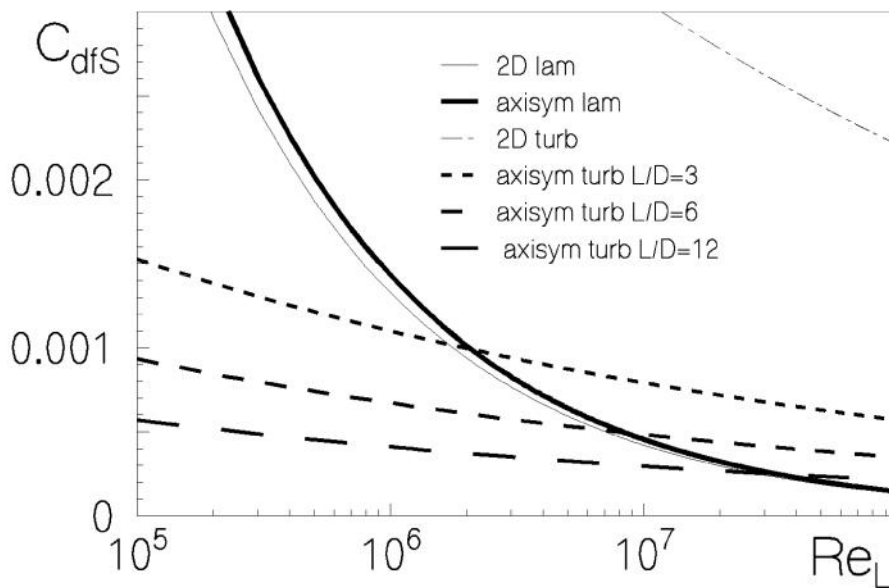


Figure 1. Laminar and turbulent skin-friction drag coefficients computed for the flat plate from (1.2), *2D lam*, and (1.4), *2D turb*, and for rotationally symmetric bodies with unseparated flow from (2.6), *axisym lam*, and (2.8) with various values of the fineness ratio L/D , *axisym turb*, show that both laminar coefficients and the rotationally symmetric turbulent one are very similar in magnitude, but the flat-plate turbulent coefficient is much greater.

Estimates (2.7) and (2.8) are valid only for an unseparated flow pattern. Taking (2.8) as an estimation of the total drag coefficient for the unseparated shape of a bottlenose dolphin ($L/D = 5.2$), we obtain a value 5.8 times smaller than (1.4) yields, and so resolve Gray's paradox. Why are such low values of the turbulent drag not achieved in technological applications? The answer is evident: the hulls developed in ship and aircraft design have separated flow, in contrast with the ideal unseparated biological shapes. Two popular fluid-dynamical paradigms – that “separation is inevitable” (e.g., [10]) and the flat-plate concept – have not stimulated the improvement of shapes in order to avoid separation.

3. BOUNDARY LAYER STEADINESS ON AN UNSEPARATED SLENDER BODY OF REVOLUTION

The laminar-to-turbulent flow transition in the boundary layer influences the skin-friction drag and depends on many parameters such as pressure gradient, surface roughness, pulsations in the ambient flow and so on (e.g., [7]). Nevertheless, according to Tollmin-Schlichting-Lin theory (e.g., [11]) the boundary-layer on the flat plate remains laminar at any wavelength of disturbances, if

$$\text{Re}^* = \frac{\overline{U} \delta^*}{\nu} < 420 . \quad (3.1)$$

Inequality (3.1), taking into account the Blasius expression for displacement thickness (e.g., [7])

$$\delta^* = 1.721(x)^{1/2} \text{Re}_L^{-1/2} \quad (3.2)$$

can be rewritten as follows:

$$\sqrt{x} \operatorname{Re}_L < 244.04. \quad (3.3)$$

It was shown in Section 2 that the boundary-layer on a slender body of revolution can be reduced to that on the plate with the use of the Mangler-Stepanov transformation. Thus with the use of (2.1) and (3.3), the condition for the axisymmetrical boundary-layer to remain laminar can be written as follows, [9]:

$$\operatorname{Re}_L \int_0^x R^2(\xi) d\xi < 59558. \quad (3.4)$$

If we wish the boundary layer to remain laminar over the entire surface we can substitute the integral in (3.4) as follows and use (2.5):

$$\int_0^1 R^2(\xi) d\xi = \frac{V}{\pi L^3} \approx 0.163 \frac{D^2}{L^2}. \quad (3.5)$$

Therefore, inequality (3.4) acquires the appearance:

$$\operatorname{Re}_L < 3.7 \cdot 10^5 \frac{L^2}{D^2} \quad \text{or} \quad \operatorname{Re}_V < 2.9 \cdot 10^5 \frac{L^{4/3}}{D^{4/3}} \quad (3.6)$$

Estimations (3.6) testify that the boundary-layer remains laminar on slender bodies of revolution at rather large Reynolds numbers. Specifically, for $L/D = 12$ the critical values are $\operatorname{Re}_L \approx 5.3 \cdot 10^7$ and $\operatorname{Re}_V \approx 7.9 \cdot 10^6$; for $L/D = 5.2$ (e.g., a bottlenose dolphin) $\operatorname{Re}_L \approx 10^7$ and $\operatorname{Re}_V \approx 2.6 \cdot 10^6$. Experiments with live dolphins and fish show that the boundary layer on the fore-body remains laminar and near the tail become turbulent [6].

Thus slender bodies of revolution can delay the laminar-turbulent transition on their surfaces and reduce the skin-friction drag. It must be recalled that the presented estimations (3.4) and (3.6) are valid only for a flow pattern without separation. That is why the effect of the turbulization delay was not achieved on the standard (separated) slender bodies of revolution.

4. TECHNOLOGICAL APPLICATIONS OF UNSEPARATED SHAPES

The presented analysis allows one to draw a significant conclusion: **the main opportunity for drag reduction consists in using shapes providing an unseparated flow pattern.** For example, equations (2.4) and (2.8) yield the values $C_{dV} = 0.0032$ (laminar boundary layer) and $C_{dV} \approx 0.0038$ (turbulent case) at $\operatorname{Re}_V = 2.2 \cdot 10^6$ for the body of Hansen & Hoyt ($L/D = 4.5$). The experimental value $C_{dV} = 0.007$ (see [5]) demonstrates that with the use of an unseparated shape, the total drag might be halved.

Examples of rigid bodies with special pressure distributions calculated and tested in order to remove the boundary layer separation without any active control (such as suction, shape transformations etc.) can be found in [11-13]. To diminish the total drag of hulls, unseparated shapes similar to a dolphin body, UA-2 or U-1 (see [11-13]) could be used. The existence of shapes like UA-2c (see [13]) whose form approaches that of a bottlenose dolphin (Fig. 2) and is similar to the unseparated body UA-2 (see [13]) demonstrates that even as a rigid body it would be possible for a dolphin to achieve unseparated flow, as appears to be the case from observations [6]. The obtained shapes can be applied both to subsonic aerodynamics and in marine technologies. For example, in a new generation of aircraft or in SWATH ships in order to reduce drag and noise and to improve the cavitation inception characteristics.

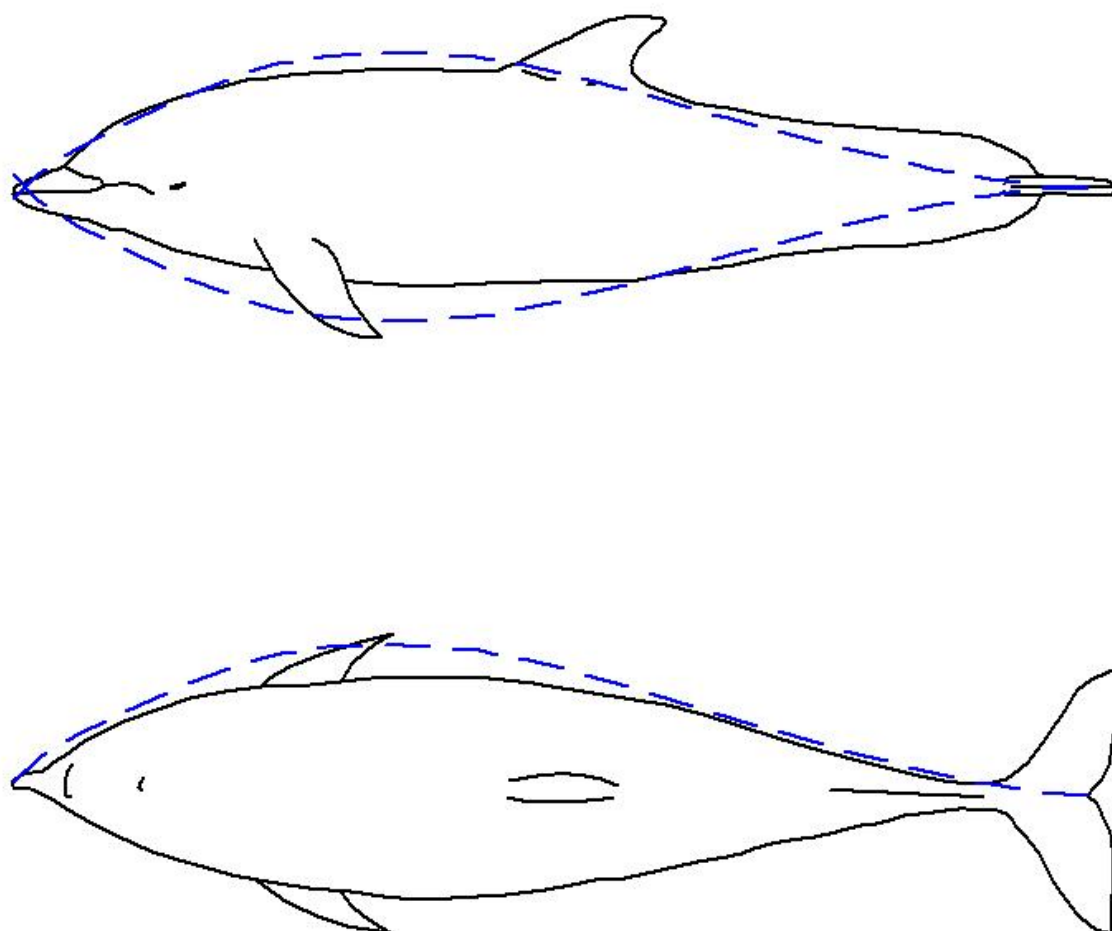


Figure 2. Comparison of the shape UA-2c with the body of a bottlenose dolphin. Note that UA-2c is a member of a whole family of shapes having this property of unseparated flow [13].

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