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Numerical Relativity in D dimensional space-times: Collisions of unequal mass black holes

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Abstract. We present unequal mass head-on collisions of black holes in $D = 5$ dimensional space-times. We have simulated BH systems with mass ratios $q = 1, 1/2, 1/3, 1/4$. We extract the total energy radiated throughout the collision and compute the linear momentum flux and the recoil velocity of the final black hole. The numerical results show very good agreement with point particle calculations when extrapolated to this limit.

1. Introduction

Black holes (BH) in four or higher dimensional generic space-times play a key role in fundamental physics; reaching from astrophysics to high energy physics. In high energy collisions of particles, the center of mass energy is beyond the Planck scale and gravity becomes dominant. Then, due to Thorne’s hoop conjecture [1] and “no-hair” theorem type arguments, the internal structure of these particles are not important to understand these processes. Therefore, the trans-Planckian scattering of particles is well described by scattering of highly boosted BHs [2]. This becomes especially interesting in the context of TeV gravity scenarios, i.e., theories of gravity with extra dimensions, which have been proposed in order to solve the so-called hierarchy problem [3]. The fact that the effective Planck scale decreases to the order of TeV offers the exciting possibility that BHs could be produced, e.g., at the Large Hadron Collider (LHC) [4]. The LHC thus might provide an opportunity to experimentally test predictions by such higher dimensional theories of gravity. In order to compute the amount of energy that is radiated in form of gravitational radiation and to determine the cross-section rate it is mandatory to evolve BH collisions in full general relativity. Due to the complex structure of Einstein’s equations this goal can only

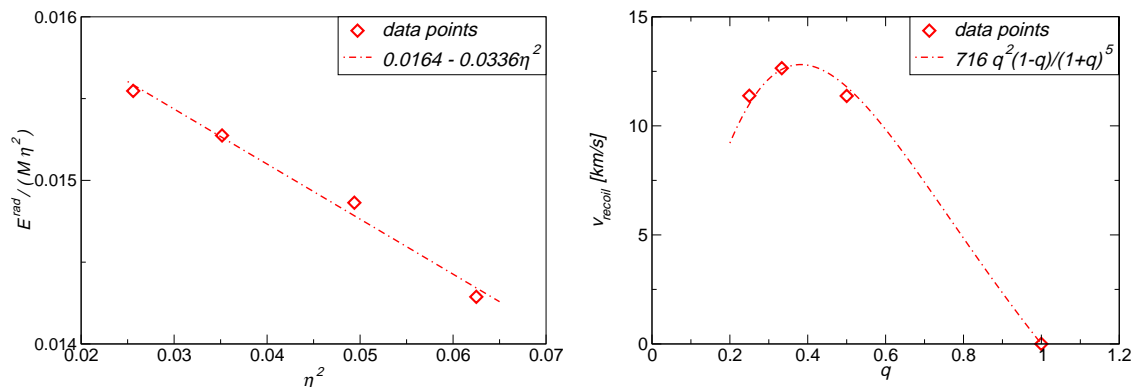


Figure 1. (Color online) Total integrated energy as function of $\eta^2 = [q/(q+1)^2]^2$ (left) and recoil velocity as function of the mass ratio q (right) for a head-on collision in $D = 5$ (red symbols). The curves are the corresponding fitting functions.

be achieved by using numerical methods. Recently, succesful numerical methods have been developed to evolve higher dimensional BH space-times [5, 6, 7].

Here, we intend to extend our previous work [6, 7] to study collisions of unequal mass BH binaries colliding head-on in $D = 5$ space-time dimensions. We calculate the energy and linear momentum emitted throughout this process. We will see that our numerical results, extrapolated to the point particle (PP) limit, are in very good agreement with recently presented PP calculations [8].

2. Numerical setup and results

We have performed numerical simulations of unequal mass BH binaries using the LEAN code [9] extended along the lines of [6]. The LEAN code is based on the **Cactus** computational toolkit [10] and the **Carpenter** mesh refinement package [11]. In order to evolve higher dimensional, sufficiently symmetric space-times we perform a dimensional reduction on a $SO(D-4)$ sphere [6]. Thus, the D dimensional vacuum Einstein's equations are cast into an effectively $3+1$ system of gravity which is coupled to a scalar field. The system of BHs, initially at rest, is set up using Brill-Lindquist type initial data in the form presented in Eq. (2.15) in [7]. We evolve this system using the BSSN [12] formulation, together with the moving puncture approach [13]. We extract information about gravitational radiation using the Kodama-Ishibashi (KI) formalism [14]. The radiated energy and energy flux are calculated from the KI gauge-invariant wave function $\Phi_{,t}^l$ by Eqs. (2.56), (2.57) in [7]. The momentum flux is given by an infinite series coupling the different multipoles, but the following lowest-order expansion is sufficient for our accuracy purposes [15]

$$\frac{dP}{dt} = \frac{\Phi_{,t}^{l=3}}{4\pi} \left(5\Phi_{,t}^{l=2} + 21\Phi_{,t}^{l=4} \right). \quad (1)$$

We have evolved head-on collisions of BHs, initially at rest, with mass ratios $q \equiv M_1/M_2 = r_{S,1}^{D-3}/r_{S,2}^{D-3} = 1, 1/2, 1/3, 1/4$, where M_i is the mass of the i -th BH, in $D = 5$ dimensions. The proper initial separation has been set to $L/r_S = 6.33$. For the details of the initial setup of the simulations in we refer the reader to Table 1 in [15]. The mass parameters of the smaller BH are given by $r_{S,1}^{D-3}/r_S^{D-3} = 0.5, 0.33, 0.25, 0.2$. The values of the second BH are adapted accordingly.

By integrating the energy flux computed from the KI master function, we obtain the total radiated energy. In Table 1 we summarize the radiated energy in units of total mass M . The

| q | $E/M(\%)$ | $E_{l=2}(\%)$ | $E_{l=3}(\%)$ | $E_{l=4}(\%)$ |
|-----|-----------|---------------|---------------|---------------|
| 1 | 0.089 | 99.9 | 0.0 | 0.1 |
| 1/2 | 0.073 | 97.7 | 2.2 | 0.1 |
| 1/3 | 0.054 | 94.8 | 4.8 | 0.4 |
| 1/4 | 0.040 | 92.4 | 7.0 | 0.6 |

Table 1. We show the total radiated energy E/M and the fraction of energy E^l excited in the l -th mode as compared to the total radiated energy, measured from the energy flux at r_{ex} for unequal mass, head-on collisions of BHs

maximum amount of energy is emitted in the equal mass case $E^{\text{rad}}/M = 0.089\%$ ($D = 5$) as found previously in Ref. [7]. As reported in [7, 15], the error in the radiated energy is estimated to be about 5 %. We also show the fraction of energy excited in the higher multipoles in Table 1. As expected from calculations in the PP limit, higher multipoles are enhanced as the mass ratio decreases. Post-Newtonian calculations, extended to general dimension, suggest a dependence of the total radiated energy $E^{\text{rad}}/M \propto \eta^2$, where $\eta = q/(1+q)^2$ is the dimensionless reduced mass [16]. In the left panel of Fig. 1 we show the ratio $E^{\text{rad}}/(M\eta^2)$ which can be seen to depend very weakly on η^2 . We make this argument more precise by fitting our numerical results to $E^{\text{rad}}/M\eta^2 = A_0 + A_1\eta^2$. We find

$$\frac{E^{\text{rad}}}{M\eta^2} = 0.0164 - 0.0336\eta^2. \quad (2)$$

Linearized, PP calculations presented in [8] show that in the limit of zero mass ratio one obtains

$$\frac{E_{\text{PP}}^{\text{rad}}}{M\eta^2} = 0.0165, \quad (3)$$

which agrees with the extrapolation of our numerical results within less than 1%. We observe, that the contributions of higher multipoles for $q < 1$ become stronger with increasing dimensionality of the space-time. These results are consistent with the observation in [17] that higher multipoles contribute more to the radiation than in $D = 4$, where for instance the $l = 3$ mode contributed roughly 10% of the total energy in the PP limit. Linearized, point-particle calculations show that the trend is consistent and continues in higher- D [8].

In unequal-mass collisions, the asymmetric emission of radiation along the collision axis causes a net momentum to be carried by gravitational waves. As such, the final BH will “recoil”, according to $v_r = \left| \int_{-\infty}^{\infty} dt \frac{dP}{dt} \right|$. The recoil velocities for different mass-ratios are shown in the right panel of Fig. 1. We estimate the errors in the recoil velocity to be $\approx 5\%$. The general functional dependence of the momentum on the mass ratio is given by $v_r = A_0 \frac{q^2(1-q)}{(1+q)^5}$ [16]. By fitting this function to our numerical data, we obtain $A_0 = 716$ km/s. In the PP limit these results are, again, in good agreement with PP calculations [8]. We note that momentum emission is given by a non-trivial interference between different multipoles, so this is a non-trivial agreement.

3. Conclusions and Outlook

We have simulated head-on collision of BHs, initially at rest, with mass ratios $q = 1, 1/2, 1/3, 1/4$ in $D = 5$ space-time dimensions. We have computed the total energy radiated throughout the collision, as well as the fraction of energy excited in the different multipoles. We observe that this fraction is enhanced as the mass ratio decreases. We have extrapolated the numerical results

for the total radiated energy to the PP limit and find agreement within 1% with PP calculations [8]. Furthermore, we have computed the recoil velocity of the final BH. We find a maximum of $v_{r,\max} = 12.8\text{km/s}$ ($D = 5$) at $q \simeq 0.38$. Again, we extrapolate our results to the PP limit and find agreement within less than 10% with PP calculations [8, 15].

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