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Abstract. Controllers for feedback substitution schemes demonstrate a trade-off between noise power gain and normalized response time. Using as an example the design of a controller for a radiometric transduction process subjected to arbitrary noise power gain and robustness constraints, a Pareto-front of optimal controller solutions fulfilling a range of time-domain design objectives can be derived. In this work, we consider designs using a loop shaping design procedure (LSDP). The approach uses linear matrix inequalities to specify a range of objectives and a genetic algorithm (GA) to perform a multi-objective optimization for the controller weights (MOGA). A clonal selection algorithm is used to further provide a directed search of the GA towards the Pareto front. We demonstrate that with the proposed methodology, it is possible to design higher order controllers with superior performance in terms of response time, noise power gain and robustness.

1. Introduction

In its simplest form, a feedback substitution scheme (FSS) comprises a transducer, followed by a filter and controller as well as a second transducer re-converting the energy back to the original domain as illustrated in Fig. 1. $W_r(s)$ is the external signal to be nulled through the first transduction process, $G(s)$ is the transfer function of the transducer that measures the external input, $F(s)$ represents a low pass filter related to a lock-in amplifier involved in the demodulation of the incoming signal, which would be normally modulated above the $1/f$ noise floor, $K(s)$ is the controller to be designed and $W_f(s)$ is the transfer function of the second transducer, which aims to oppose the effect of $W_r(s)$ in the first transduction process $\left(W_f(s) = -W_r(s)\right)$. The output $W_f(s)$ is the combination of the input and noise transfer functions. The feedback signal W_f is generated to counteract the external input. This signal incorporates the low-pass characteristics of the detection process as well as the dynamics of the controller. The entire system operates within certain specifications in terms of response time and overshoot. The main objective in this system is to optimize the two conflicting requirements; response time t_r and noise power gain $\mathcal{G}$. The response time t_r is defined as the time required for the system to produce a signal $W_f = -(W_r - \varepsilon_T)$ where $\pm\varepsilon_T$ is a tolerance level [1].

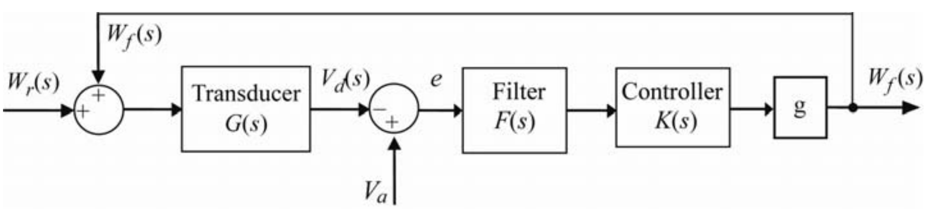

Figure 1. Block diagram for a generic feedback substitution scheme

In the simplest implementation of the system, the transfer function of the transducer is of first order $G = \mu_d / (1 + \tau s)$ with time constant τ and gain μ_d. This is a part of the system that may not be changed. The filter $F(s) = \mu_f \left(1 + \tau_f s\right)$ and the controller $K(s) = (1 + (\tau_i s)^{-1})\mu_i$ have transfer functions that can be changed accordingly (by changing $\tau_i, \mu_i, \tau_f, \mu_f$) to obtain the desired response.

2. Loop shaping design procedure and analysis of robustness

The system described in section 1 can be converted into the system shown in Figure 2a where a Loop Shaping Design Procedure (LSDP) and an H-infinity controller can be synthesized as described by McFarlane and Glover [2]. A co-prime factorization of the shaped plant G_s can also be performed as shown in Figure 2b. The introduction of an explicit co-prime factorization of the system permits the design of a controller with explicit parametrization of robustness. This is of interest as different design procedures such as the one described by Clare and White as well as the new LSDP formulation have different robustness.

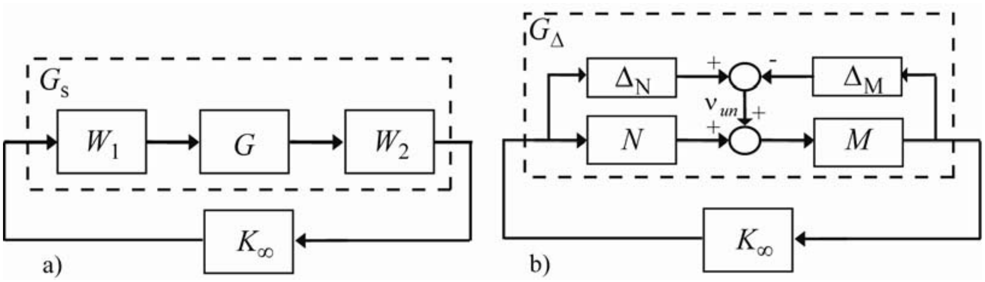

Figure 2. a) LSDP formulation and b) co-prime factorization of the system.

In the original systems shown in Figure 1, the optimization parameters are an overall gain of the system as a single parameter $\mu = \mu_d \mu_f \mu_i$, a normalized time constant of the filter with respect to the detector time constant τ denoted as x and a normalized integration time constant of the controller, y. In the loop shaping design procedure, the design parameters μ, $x = \tau_f / \tau$, and $y = \tau_i / \tau$ are substituted by weighting functions W_1 and W_2 to shape the plant G to achieve the desired response. A normalized co-prime factorization of the shaped plant $G_s = NM^{-1}$ satisfying $MV + NU = 1$ is used to model uncertainty as perturbations Δ_N and Δ_M to the co-prime factors of the shaped plant.

The perturbations Δ_N and Δ_M are stable transfer functions with uncertain dynamics and parameters, the only thing known about this transfer functions is that their H^∞ norm is less than a value ε_{sm} which is interpreted as the stability margin of the system. The resulting perturbed plant equation is:

$$G_\Delta = \left\{ \left(M + \Delta_M\right)^{-1} \left(N + \Delta_N\right) : \left\| \begin{bmatrix} \Delta_N & \Delta_M \end{bmatrix} \right\|_\infty < \varepsilon_{sm} \right\} \tag{1}$$

From a robust analysis perspective, to maximize the stability margin ε_{sm}, a controller is calculated to minimize the function

$$\gamma := \left\| \begin{bmatrix} K(1-KG)^{-1} M^{-1} \\ (1-KG)^{-1} M^{-1} \end{bmatrix} \right\|_\infty \le \frac{1}{\varepsilon_{sm}} \tag{2}$$

where γ is the H_∞ norm of the system in Fig. 2b and $\left(1 - KG\right)^{-1}$ is the sensitivity function S.

In Figure 3, we consider 3rd order systems composed of a 1st order plant, a 1st order filter and a 1st order controller fulfilling an overshoot requirement of a tolerance of 0.001 using the Clare and White procedure [1]. We plot the normalized response time t_r / τ as a function of the system gain $\mu = \mu_d \mu_f \mu_i$ for different filter time constants $x = \tau_f / \tau$ when the value of y is adjusted to meet the overshoot requirement (tolerance 0.001) and evaluate the resulting system noise power gain and robustness. The resulting systems that demonstrate low robustness margin as well as those with an amplification of noise are shown. Furthermore, the possible systems that result with an unacceptable undershoot are also shown.

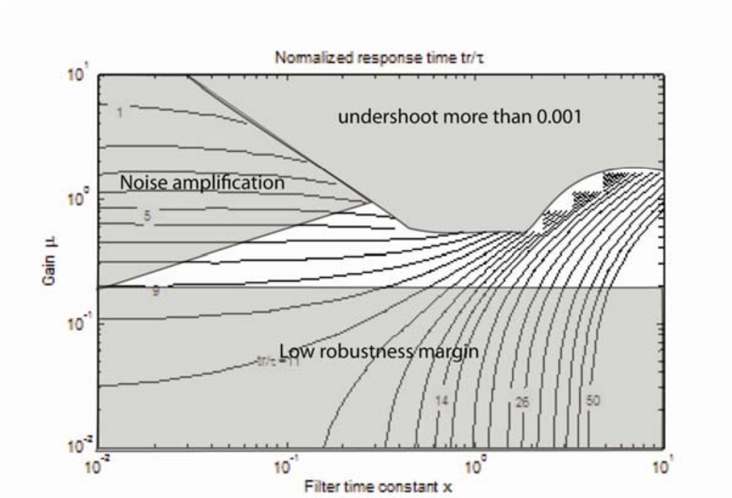

Figure 3. Possible 3rd order systems composed of a 1st order plant, a 1st order filter and a 1st order controller fulfilling an overshoot requirement of a tolerance of 0.001 using the Clare and White procedure. All possible systems that can result by tuning a first-order controller are shown.

Because of the precision of the system's overshoot requirement, a systematic and iterative strategy is needed to design the weighting functions W_1, W_2. The problem specifications are casted out as inequalities [3]. A genetic algorithm is then used to map the surface of all possible H^∞ controllers that can be designed with the specifications provided through the inequalities defined by the designer [2-8].

First the system requirements are specified using a representation similar to that used in the method of inequalities. These inequalities define a goal attainment problem which is translated into a minimization problem. A Multi-objective Optimization with Genetic Algorithms (MOGA) from MATLAB's Genetic Algorithm and Direct Search toolbox V2.4.1 (R2009a) the function '*gamultiobj*' is used to find the parameters and structure of the weighting functions that lead to an optimum solution.

The design requirements for the system are as follows: The output $W_f(t)$ for a unitary step input $W_r(t) = 1$ must have an overshoot $\varepsilon_T = 0.001$ which is specified to a numerical precision of $\varepsilon_T / 500$; after the first overshoot, the output $W_f(t)$ must remain within the range $1 \pm \varepsilon_T$; the noise power gain $\mathcal{G}$ in the newly designed system must be minimized for all normalized response times considered. Additional requirements are a zero steady state and a robust stability margin ε_{sm} >0.25. Systems with a normalized response time t_r/τ in the range 1 to 20 are only considered. Using inequalities, the above specifications are casted as follows:

$$\varphi_1 = \left| W_h\left[t, W_r(t)\right] - \frac{\varepsilon_T}{500} \right| \le W_r(t) + \varepsilon_T, \quad t > 0 \tag{3}$$

$$\varphi_2 = \left| W_h\left[t, W_r(t)\right] - \frac{\varepsilon_T}{500} \right| \le W_r(t) - \varepsilon_T, \quad t > t_r \tag{4}$$

$$\varphi_3 = \min \sqrt{\frac{\tau}{4\pi j} \int_{-\infty}^{\infty} \left| Q(s) \right|^2 ds} \,\Big|_{s=j\omega} \tag{5}$$

$$\phi_4 = \left\| \begin{bmatrix} K(1-KG)^{-1} M^{-1} \\ (1-KG)^{-1} M^{-1} \end{bmatrix} \right\|_\infty \le \frac{1}{0.25} \tag{6}$$

A vector of designer objective functions $\Phi = \left[\varphi_1 \;\; \ldots \;\; \varphi_4\right]^T$ with p_j design parameters and objectives ε_i each, with $\varphi_i\left(p_j\right) \le \varepsilon_i$ is constructed with an admissible set composed of all admissible points. The minimization problem is then solved using a MOGA. An auxiliary vector $\boldsymbol{\lambda}$ is required to translate the goal attainment problem into a minimization MOGA problem. Each element of the vector is given by:

$$\lambda_i\left(p_j,\varepsilon_i\right)=\begin{cases}0 & \text{if} \quad \varphi_i\left(p_j\right)\le\varepsilon_i \\ \varphi_i\left(p_j\right)-\varepsilon_i & \text{if} \quad \varphi_i\left(p_j\right)>\varepsilon_i\end{cases} \tag{7}$$

The purpose of this auxiliary vector is to provide the MOGA with a vector of fitness functions where the fitness value is a minimum (zero) if the goal is attained. The difference between the actual value and the goal is returned if the goal is not attained.

We assume the weighting function W_1 to be composed of several lead-lag structures of first or second order. This approach ensures that we can have complex poles and zeros. The coefficients for the lead-lag structures are provided from the design parameters p_j. In order to explore different parameters as well as different structures with the same MOGA with the purpose of finding optimum solutions, the MOGA has to work in a hierarchical structure as proposed in [8]. Here the parameter vector p_j is divided into two different chromosomes (Figure 4). One chromosome corresponds to the coefficients of the weighting function. The other one controls the activation or suppression of elements in the lead-lag structure.

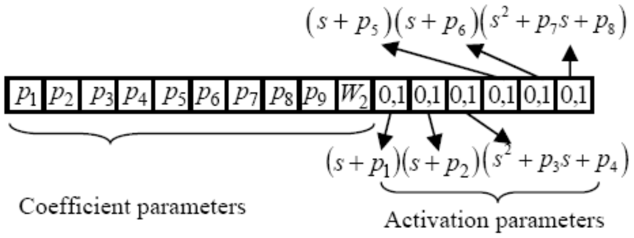

Figure 4. Chromosome separation for hierarchical GA.

The activation parameters p_{11-16}, $p_j\in[0\quad 1]$ change the structure of the lead-lag filters in W_1, whereas the filter coefficients are given from p_{1-9}, $p_j\in\mathbb{R}$. The weighting function W_2 is assumed to have a structure of a simple gain given from p_{10}. In addition, we are including one more integrator with a very small value $(s+\upsilon)^{-1}$ (with $\upsilon=10^{-6}$) to avoid poles in the origin that lead to internal instability when the controller is synthesized. W_1 is calculated from:

$$W_1=p_9\left[\frac{(s+p_5)(s+p_6)\left(s^2+p_7s+p_8\right)}{(s+\upsilon)(s+p_1)(s+p_2)\left(s^2+p_3s+p_4\right)}\right] \tag{8}$$

Each parameter that forms part of W_1 and W_2 in the MOGA is initially randomly initiated. For each individual in the population, a controller is synthesized for the system as shown in Fig. 4. That system is simulated using a step response so that from the simulation, the objectives (response time, overshoot and noise power gain) can be measured. Then the MOGA uses these objectives as fitness functions to rank the produced solution according to how close they are to the Pareto-optimal front. The fittest solutions are selected for recombination and mutation. The stopping criterion is based on meeting the overshoot objective while at the same time minimizing the noise power gain. Instead of finding directly the minimum normalized response time, our strategy was to minimize the noise over small bands (± 0.15) of set values of response time.

The algorithm starts with random weights which correspond to transfer functions translated into state space and two Riccati equations are solved to finally design the controller. The GA mapped the surface of all possible H^∞ controllers that can be designed with the specifications provided using the defined inequalities. The proposed methodology permits us to analyze designed plants of up to 7th order assuming an open structure.

As discussed in [2], once the stabilizing controller K_∞ is designed, the implementation of the final controller requires the movement of the two shaping functions at the input and output of the K_∞ to form a new transfer function block $K = W_2 K_\infty W_1$ as shown in Figure 3.

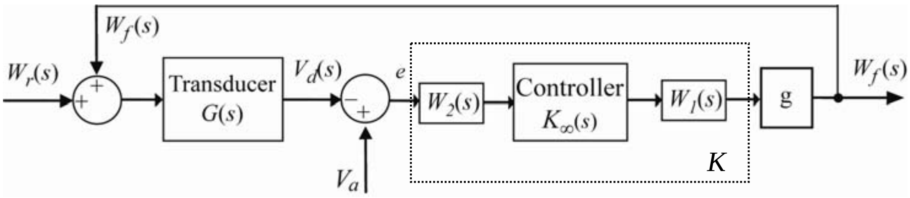

Figure 5. Final LSDP implementation of the system once the K_∞ controller is designed.

3. Comparing results from the two procedures

Figure 6 shows time domain responses for systems designed using the Clare and White procedure as well as the LSDP procedure which uses the MOGA to design the optimal controllers. Both systems have been selected to have the same normalized response time specifications so a comparison could be performed. Both graphs are considering a structured uncertainty by adding a 10% parametric uncertainty to the detector time constant τ as well as to the detector gain μ_d. In terms of Robust Stability both systems can tolerate the modeled uncertainty, The Clare and White system is 76% more sensitive than the LSDP one to variations of μ_d, but 7% less sensitive to variations of τ. In terms of robust performance, the Clare and White system is 5% less sensitive than the LSDP one to variations of μ_d, for variations of τ both systems have the same robust performance. With regard to unstructured uncertainty the new methodology had an improvement of 98% in the robust stability margin as calculated with equation (2) and a reduction of 10.8% in noise power gain using the LSDP MOGA mehtod.

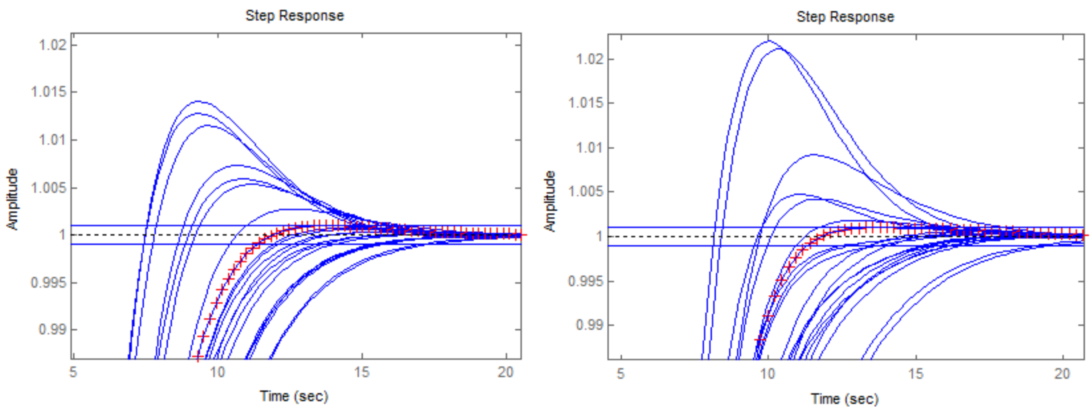

Figure 6. Time domain responses of systems to a step input, with optimal controllers designed using a) the Clare and White method which results in a 3rd order system and b) the LSDP MOGA mehtod which results in systems of higher order. The two horizontal lines close to unity amplitude indicate the allowed overshoot and undershoot tolerance specification for the designed systems and the crossed line indicates the nominal system response time whereas the other lines indicate step responses for systems with parametric perturbation.

4. Conclusion

Optimal feedback substitution schemes having a controller of first or second order can be analytically found satisfying precise overshoot characteristics, while at the same time minimizing response time and noise power gain. The design of higher-order controllers is possible using LSDP and an H∞-

MOGA procedure. In this work we have handled constrains using MOGA converting them into objectives. Because the MOGA has a very limited domain of feasible solutions, we plan to use particle swarm optimization combined with GAs in the future. This is so, because the derived MOGA population often is outside the feasible solution space and another constrain handling algorithm needs to be used to move this population within the feasible solutions space. The current work shows that there is a need to adopt alternative computational intelligence algorithms that will tune the parameters of higher order systems. An advantage of the proposed approach is that parameters W_1, W_2 could also be implemented assuming a non-rational function, so that a fractional order controller could be designed.

The currently developed controller design procedure is applicable to all radiometric detectors governed by first order dynamics and may be extended to 2nd order transducers or higher order transducers which are not currently employed for absolute measurements.

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