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Modification of rotor modal properties based on structural changes in the stiffness of its bearings and shaft

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Abstract. The rotor structures are fundamental mechanical systems that realize the transfer of rotary motion in technical devices and systems. In some operating regimes, the rotor systems are often subjected to undesirable and damaging phenomena resulting in resonant state. The ability to prevent or reduce the level of unwanted rotor vibrations in the given operating mode is an important task in the process of structural design of rotor structures. In general, the dynamic properties of rotor systems depend on their spatial properties - geometric parameters and material properties, i.e. distribution of mass and stiffness properties of rotor structures. For the structural design of the rotor or its structural modification, which will allow the redistribution of its spatial properties (mass and stiffness), two approaches based on changing the bearing stiffness and also on the shaft stiffness are presented in this paper. The main goal for modifying the spatial properties of the rotor is to get the critical rotor speed out of the operating speed range.

1. Introduction

The rotor structures are basic systems that realize the transfer of rotary motion in technical devices and systems. The structure and the mass distribution of the rotor have a fundamental influence on the dynamic properties of the rotor. Improper geometry of the rotor structure causes the emergence of various unacceptable phenomena. These phenomena are usually caused by the rotational inertia effects of the rotor, as well as the stiffness of bearings [1] and they affect the rotor resonant states. Unfavorable phenomena arising during rotor operation [2] can be eliminated or minimized by appropriate modification of the rotor structure. The rotor modifications can be performed mainly by modifying the rotor's rotational mass, increasing the rotor's stiffness, or changing the stiffness of bearings. As a result of these modifications, it is possible to achieve such values of natural frequencies, i.e. critical rotor speeds that are outside the operating speed range. It is evident that in the case of a change in the operating regime, the undesirable conditions may occur, i.e. the rotor operates at operating speeds that were not taken into account in the design process of the rotor [3]. Under these operating conditions, the rotor exhibits inappropriate dynamic parameters, and therefore it is necessary to eliminate undesirable dynamic effects and their transmission to the production equipment, or to the work environment. One of the possibilities to eliminating these undesirable conditions is the design of such a rotor structure

[4, 5] which will be able to redistribute the mass and stiffness properties of the rotor during the design stage or even during operation.

The aim of the study is to analyze the influence of mass and stiffness parameters of the rotor on the dynamics of selected types and structures of rotors. The solution and analyzes of the modal properties of the rotors are performed for two rotor structures:

- rotor structure with a rigid disc between the bearings,
- rotor structure with an overhanging disc.

The study investigates the impact of the following modifications to the structural elements of the rotor on the modal properties of the considered types of rotors:

- geometrical parameters and material properties of the binding layers in which the bearings are inserted,
- length and position of the reinforcing core inserted into the rotor shaft.

2. Formulation of the problem

In general, it can be stated that the dynamic properties of the rotor are dependent on the rotor system structure [6, 7]. In this context, it is important what components (discs, shafts, bearings, couplings and other components) are used to create this rotor system, what are their shape and geometric parameters and what materials are used.

In general, the corresponding equations of motion describing the behavior of the rotor system can be formulated within theoretical considerations according to the following steps:

- expression of deformation energy (E_{def}), kinetic energy (E_{kin}), dissipative energy (E_{dis}) and virtual work of external forces (W) for each of the structural elements in the rotor system
- for the formulation of finite element equations of rotor system application of Lagrange's equations of the second kind expressed in the following general form

$$\frac{d}{dt}\left(\frac{\partial E_{kin}}{\partial \dot{q}_i}\right)-\frac{\partial E_{kin}}{\partial q_i}+\frac{\partial E_{def}}{\partial q_i}+\frac{\partial E_{dis}}{\partial \dot{q}_i}=Q_i\,, \tag{1}$$

where i - number of degrees of freedom (DOF) ($1 \le i \le N$), $q_i\,(\dot{q}_i)$ - generalized coordinate (velocity) for i^{th} DOF, Q_i - generalized force for i^{th} DOF.

Regarding the formulation of the finite element equation of motion for the rotor system [3], the basic calculation models for individual components, such as the disc, shaft, and bearing, are formulated as follows:

- *Computational model of the disc* - The basic assumption is a perfectly rigid disc. The geometry and position of the disc fixed to the shaft is shown in figure 1.

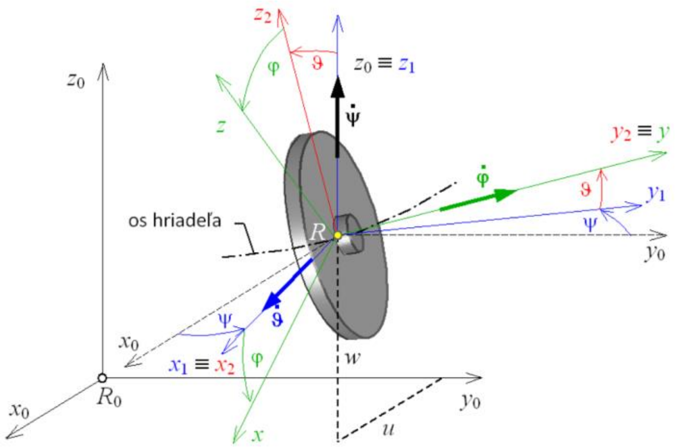

Figure 1. Model of disc structure.

The coordinate system $R_0(x_0, y_0, z_0)$ is regarded as a fixed, non-moving coordinate system, while $R(x, y, z)$ is a moving coordinate system rigidly connected to the disk center. The position of the disk on the shaft depends on the mutual configuration of the coordinate systems R and R_0, defined by two displacements u and w, and three rotations (Euler angles) using three angles ψ, ϑ, φ.
The instantaneous angular velocity of the moving coordinate system R rigidly connected to the disk with respect to the fixed coordinate system R_0 is expressed in the form

$$\boldsymbol{\omega}_{R/R_0} = \dot{\psi}\mathbf{k}_{z_0} + \dot{\vartheta}\mathbf{i}_{x_1} + \dot{\varphi}\mathbf{j}_y \,, \tag{2}$$

where $\mathbf{k}_{z_0}$, resp. $\mathbf{i}_{x_1}$, resp. $\mathbf{j}_y$ are the unit vectors in z_0, resp. x_1, resp. y axis directions.
The angular velocity vector is expressed as follows

$$\boldsymbol{\omega}_{R/R_0} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} -\dot{\psi}\cos\vartheta\sin\varphi + \dot{\vartheta}\cos\varphi \\ \dot{\varphi} + \sin\vartheta \\ \dot{\psi}\cos\vartheta\cos\varphi + \dot{\vartheta}\sin\varphi \end{bmatrix}. \tag{3}$$

The disc mass is m_d. For the rotationally symmetric disc shape and the assumption that the x, y and z axes are the principal axes of inertia, the inertia moments around the x and z axes must be equal, i.e. $I_{d,x} = I_{d,z}$. If it is possible to assume that the values of the angles ψ and ϑ are very small and the angular velocity of the disc rotation is constant (i.e. $\dot{\varphi} = \omega_d$), the formulation of the kinetic energy of the disc can be simplified and expressed in the form

$$E_{kin,d} = \frac{1}{2} m_d (\dot{u}^2 + \dot{w}^2) + \frac{1}{2} I_{dx} (\dot{\vartheta}^2 + \dot{\psi}^2) + \frac{1}{2} I_{dy} (\omega_d^2 + 2\omega_d \dot{\psi}\vartheta) \,. \tag{4}$$

The term $I_{d,y}\omega_d\dot{\psi}\vartheta$ has a specific meaning and represents a gyroscopic (Coriolis) effect.

- *Computational model of the shaft* - From a structural point of view, the shaft can be considered as a beam structure with an axisymmetric circular cross-section ($J_x = J_z = J_s$), which is characterized by strain and kinetic energies [3].
  The equation (4) expressing the kinetic energy of the disc is applicable in an expanded form to the general formulation of the *kinetic energy* of the shaft

$$E_{kin,s} = \frac{\rho S_s}{2} \int_0^{L_s} (\dot{u}^2 + \dot{w}^2) dy + \frac{\rho J_s}{2} \int_0^{L_s} (\dot{\vartheta}^2 + \dot{\psi}^2) dy + \rho J_s L_s \omega_d^2 + 2\rho J_s \omega_d \int_0^{L_s} \dot{\psi}\vartheta dt \,, \tag{5}$$

where ρ - specific density of the shaft material, S_S - cross-sectional area of the shaft, supposed to be constant, J_S - second moment of the shaft cross-section.

Generally, the strain energy of the shaft with axisymmetric circular cross-section is expressed

$$E_{def,s} = \frac{EJ_s}{2} \int_0^{L_s} \left[\left(\frac{\partial^2 u}{\partial y^2} \right)^2 + \left(\frac{\partial^2 w}{\partial y^2} \right)^2 \right] dy \,. \tag{6}$$

- *Computational model of the bearings* - When creating the bearing calculation model, the stiffness of the connection between the stationary and moving parts of the rotor system must be taken into account. If the bearings exhibit high stiffness (e.g. rolling bearings) and the frame structure in which the bearings are placed is also sufficiently rigid, then the model of rotor system assume rigid, inelastic bearings. In case these rotor components lack sufficient rigidity (e.g. sliding or magnetic bearings, seals), and the frame structure is not rigid, they are considered as viscoelastic supports (figure 2) in the rotor system model. It is assumed that the width of the bearings is small compared to the shaft length. Hence, only the radial stiffness of the bearings in the x and z axes directions is considered, while the rotational stiffness around the x and z coordinate axes is neglected [3].

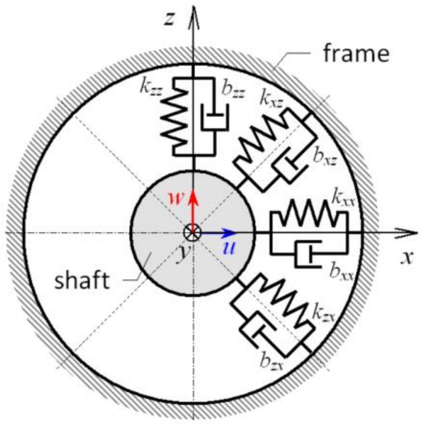

Figure 2. Model of bearing.

The generalized forces arising due to non-rigid bearings having certain stiffness and damping properties can generally be expressed in the form

$$\begin{bmatrix} F_u \\ F_w \end{bmatrix} = \begin{bmatrix} k_{xx} & k_{xz} \\ k_{zx} & k_{zz} \end{bmatrix} \begin{bmatrix} u \\ w \end{bmatrix} + \begin{bmatrix} b_{xx} & b_{xz} \\ b_{zx} & b_{zz} \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{w} \end{bmatrix}. \tag{7}$$

3. Finite element model of the rotor system

Formulations and expressions of kinetic and strain energies are applied in Lagrange equations of the second kind. After the discretization process of the rotor system components into finite elements, the finite element equation of motion of the rotor system [3] is derived in the form

$$\mathbf{M}\ddot{\mathbf{q}} + (\mathbf{B} + \mathbf{B}_{gyr})\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{Q}\,. \tag{8}$$

where $\mathbf{M}$ - mass matrix, $\mathbf{B}$ - damping matrix, $\mathbf{B}_{gyr}$ - matrix of gyroscopic effects, $\mathbf{K}$ - stiffness matrix, $\mathbf{Q}(t)$ - vector of generalized force effects, $\mathbf{q}$ - vector of nodal displacements, $\dot{\mathbf{q}}$ - vector of nodal velocities, $\ddot{\mathbf{q}}$ - vector of nodal accelerations.

Rotor systems are exposed to various loading conditions during operation, which often result in undesirable behaviour of the rotors. The occurrence of critical speeds in rotor systems can be listed among the most unfavorable effects. Each rotor system is characterized by critical speeds, the values of which depend on the structure of the rotor system. In the process of designing the rotor system, it is important to create its structure so that undesirable states do not occur or they are minimized to an acceptable level. If operating conditions causing a resonance state of the rotor system occur, it is necessary to design the rotor system structure to enable a change in the distribution of its mass, stiffness and damping properties. In the event of a structural change in the rotor system resulting in the redistribution of mass, damping, and stiffness, a modification in the equation of motion (8) is required [4]. After the structural modification, the equation of motion of the rotor system has the following form

$$(\mathbf{M} + \Delta\mathbf{M})\ddot{\mathbf{q}}_m(t) + (\mathbf{B} + \Delta\mathbf{B} + \mathbf{B}_{gyr} + \Delta\mathbf{B}_{gyr})\dot{\mathbf{q}}_m(t) + (\mathbf{K} + \Delta\mathbf{K})\mathbf{q}_m(t) = \mathbf{Q}(t)\,. \tag{9}$$

where $\Delta\mathbf{M}$ - modifying mass matrix, $\Delta\mathbf{B}$ - modifying damping matrix, $\Delta\mathbf{B}_{gyr}$ - modifying gyroscopic effects matrix, $\Delta\mathbf{K}$ - modifying stiffness matrix, $\mathbf{q}_m$ - modified vector of nodal displacements, $\dot{\mathbf{q}}_m$ - modified vector of nodal velocities, $\ddot{\mathbf{q}}_m$ - modified vector of nodal accelerations.

4. Determination of bearing stiffness

The shafts of the considered rotor systems are fixed in rolling bearings, which are assumed to be rigid in the radial direction. In the case of bearings placed in flexible layers, it is necessary to consider the radial stiffness of the bearings.

The value of the radial stiffness depends on the thickness and material of the layer, but also on the dimensional parameters of the bearing. The stiffness properties of the bearings embedded within the layers serve as input parameters for the simulation models of rotor systems solved in the ROTORINSA program. The deformation of the layer under the considered load (figure 3) is determined using the model created by the finite element method in the ANSYS program.

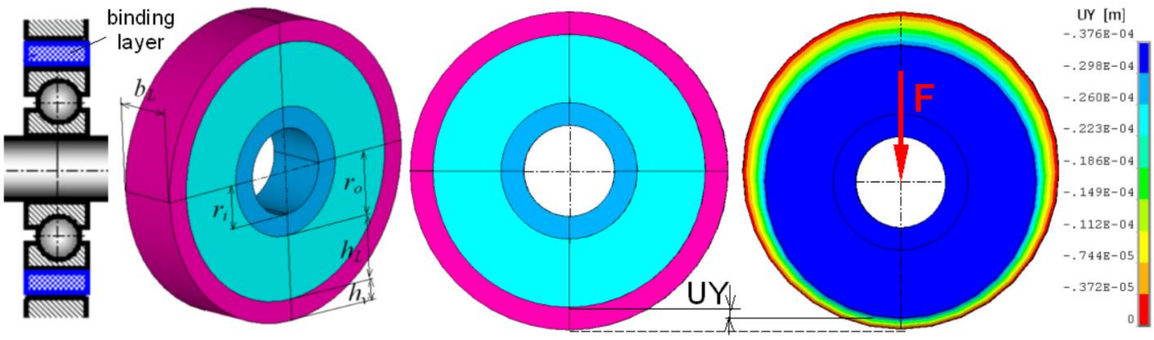

Figure 3. FE model for determining the deformation of the binding layer.

The bearing stiffness is determined from the calculated values of the binding layer deformation UY. Dependencies of bearing stiffness for different values of layer thicknesses and for different values of Young's elastic modulus are shown in figure 4.

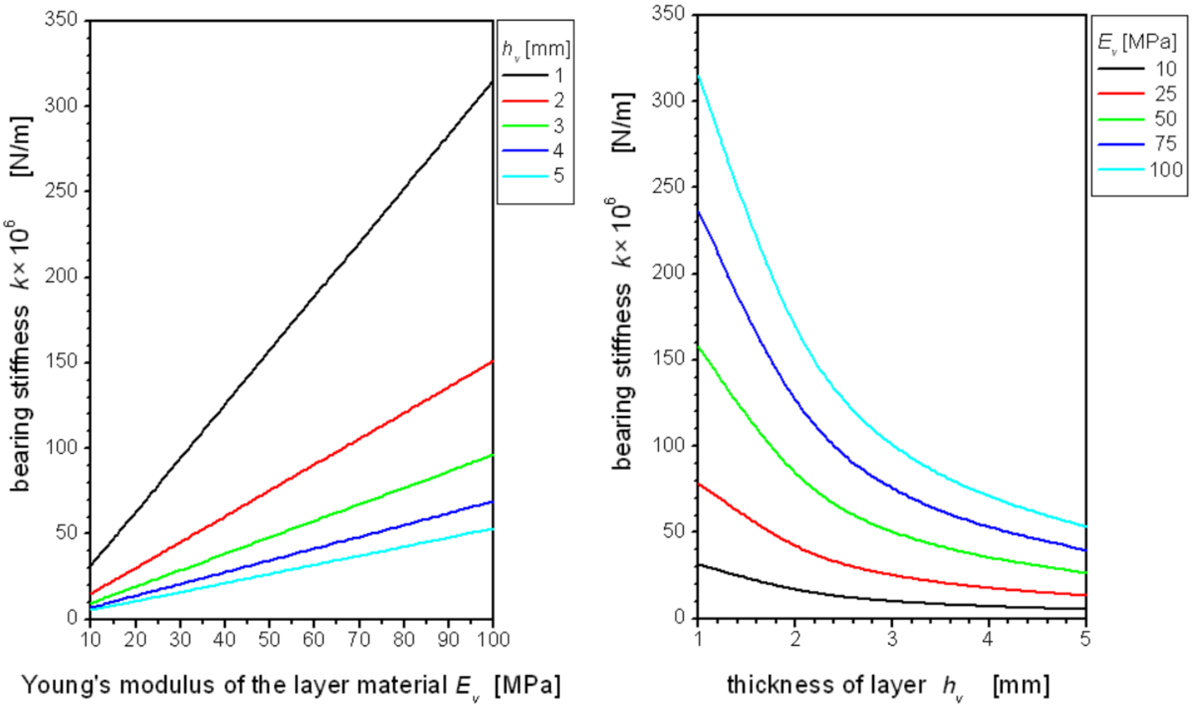

Figure 4. Dependence of bearing stiffness on layer thickness and Young's modulus of layer materials.

5. Effect of structural modifications on the modal properties of considered rotor systems

The modal properties (i.e. natural frequencies and mode shapes) of two types of structures of the rotor system (figure 5) mounted on two bearings are investigated:

- TYPE I - with an overhanging disc,
- TYPE II - with a disc located between the two bearings.

For considered types of rotor systems, the effects of bearing stiffness (A and B) on modal properties of rotor systems are investigated. Moreover, the effect of the inserted reinforcing core in the shaft body is investigated for the considered rotor systems with shafts mounted in rigid bearings.

The shaft of the rotor systems is considered to be hollow. Modal properties of considered rotor systems TYPE I and TYPE II (figure 5) are investigated for the following cases of bearing stiffness combinations:

- bearings A and B rigid (TYPE I-a, TYPE II-a),
- bearing A - flexible and bearing B - rigid (TYPE I-b, TYPE II-b),
- bearing A - rigid and bearing B - flexible (TYPE I-c, TYPE II-c),
- bearings A and B - flexible with the same stiffness (TYPE I-d, TYPE II-d),

The rotor systems in which the modifications of the shaft´s stiffness, mounted in rigid bearings, are made by inserting a reinforcing core, include the following cases:

- reinforcing core is gradually shifted into the entire length of the shaft (TYPE I-e, TYPE II-e),
- reinforcing core having a finite length L_c is gradually shifted along the entire length of the shaft and L_s is core position (TYPE I-f, TYPE II-f).

Descriptions of the rotor system components and their material properties are listed in Table 1, while the basic dimensional parameters of the considered rotor components are presented in Table 2. For the analyses, a hollow shaft with outer radius r_o and inner radius r_i are considered.

Table 1. Components of rotor systems.

Components of rotor systems	Young´s modulus [GPa]	Poisson´s ratio [-]	Density [kg.m^{-3}]
Shaft			
Disc	210.0	0.3	7800.0
reinforcing core			
binding layers	0.010; 0.025; 0.05; 0.075; 0.1	0.42	1500.0
Bearings	Type 6008 - 10		

Table 2. Geometric dimensions of rotor system components.

L_0 [m]	L_1 [m]	r_o [m]	r_i [m]	r_d [m]	b_d [m]	$h_{L1} = h_{L2}$ [m]	$b_{L1} = b_{L2}$ [m]	$h_{v1} = h_{v2}$ [mm]
0.7	0.2	0.015	0.010	0.120	0.025	0.0125	0.013	{1; 2; 3; 4; 5}

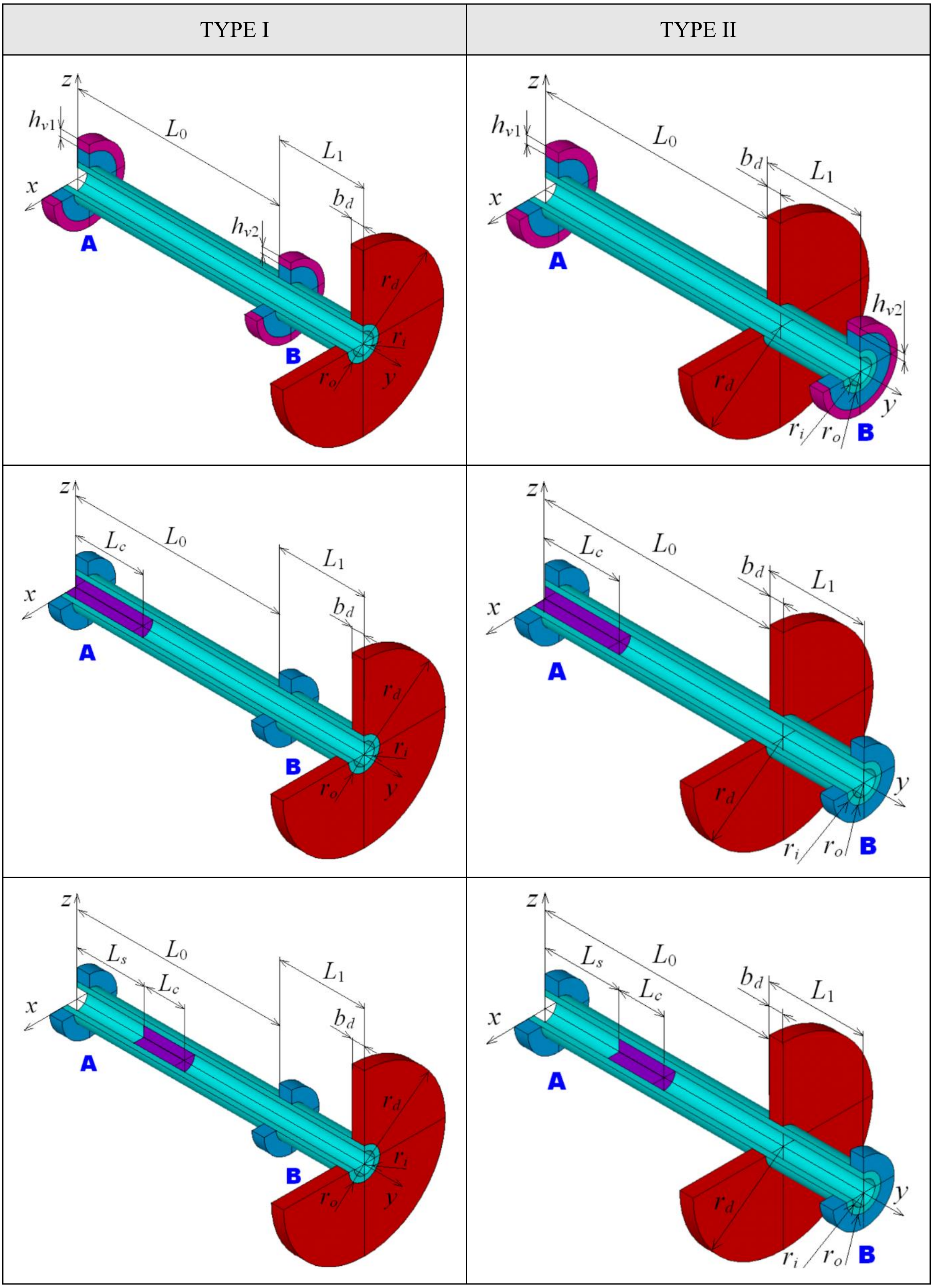

Figure 5. Models of rotor systems.

The disc position has a fundamental influence on the modal properties of rotor systems, which is evident in the case of a rotor system with an overhanging disc (TYPE-I). In the case of a rotor system with a disc between the bearings (TYPE-II), a symmetrical disc position in the center between the bearings would not provide sufficiently general knowledge about the effect of the rotor structural arrangement. Moreover, the impact of the considered structural modifications on the modal properties in such a rotor system would not yield the necessary comprehensive understanding. For this reason, the disc position on the shaft of the rotor system with the disc between the bearings is considered asymmetrically with respect to the bearings. Therefore, the disk position was chosen at the distance of $7/9\ L_0$ from bearing A and the distance from bearing B is $2/9\ L_0$.

For comparing the results and assessing the impact of bearing stiffness and structural modifications on shaft stiffness concerning the natural frequency, the deviation of the dimensionless natural frequency is introduced as follows

$$\delta_i = \frac{f_{0,m,i} - f_{0,i}}{f_{0,i}}\,, \tag{10}$$

where $f_{0,i}$ - i^{th} natural frequency of the original rotor systems (TYPE-I-a; TYPE-II-a),

$f_{0,m,i}$ - i^{th} natural frequency of the modified rotor systems (TYPE-I-b ÷ TYPE-I-f and TYPE-II-b ÷ TYP-II-f),

Computer simulations for both types of rotor systems (TYPE-I and TYPE-II) and for all considered cases of structural modifications, ranging from modification “a” to modification “f”, were conducted using the program ROTORINSA [8].

The first four natural frequencies and their corresponding mode shapes for both types of rotor systems mounted in rigid bearings (TYPE-I-a and TYPE-II-a) are shown in Figure 6.

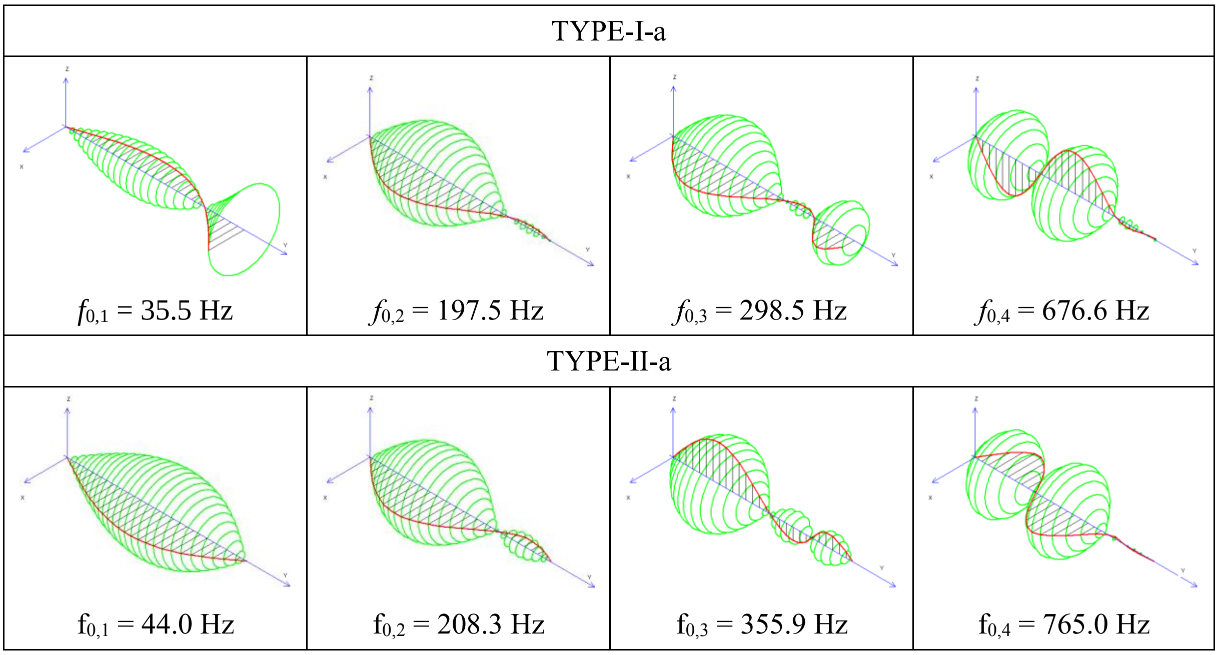

Figure 6. Natural frequencies and mode shapes for rotor systems TYPE-I-a and TYPE-II-a.

The results of the influence of the considered structural modifications (bearing stiffness, reinforcement core) on the modal properties (natural frequencies and mode shapes) are presented in the figures 7-10.

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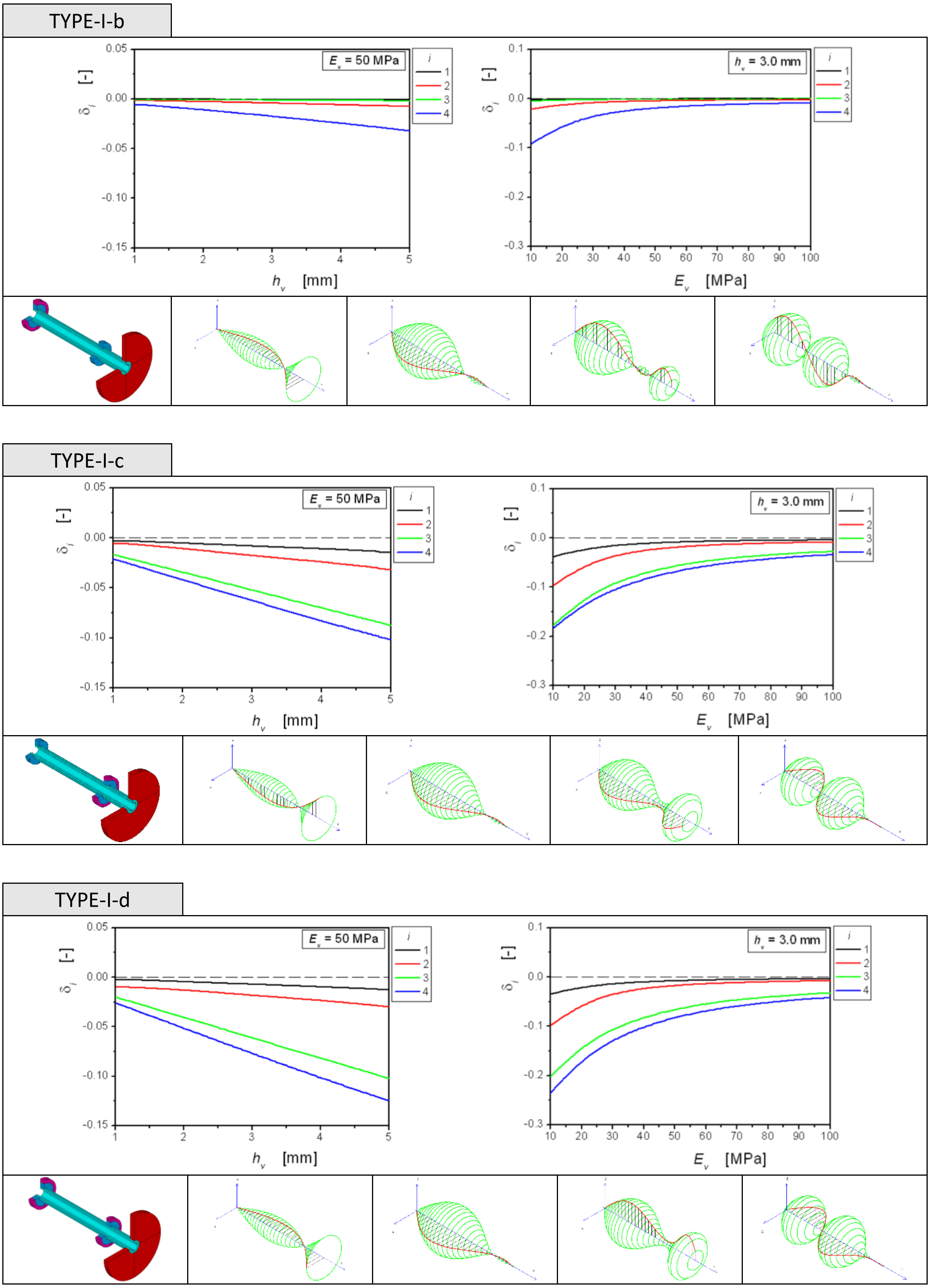

Figure 7. Dependences of rotor modal properties (TYPE-I) on the bearing stiffness.

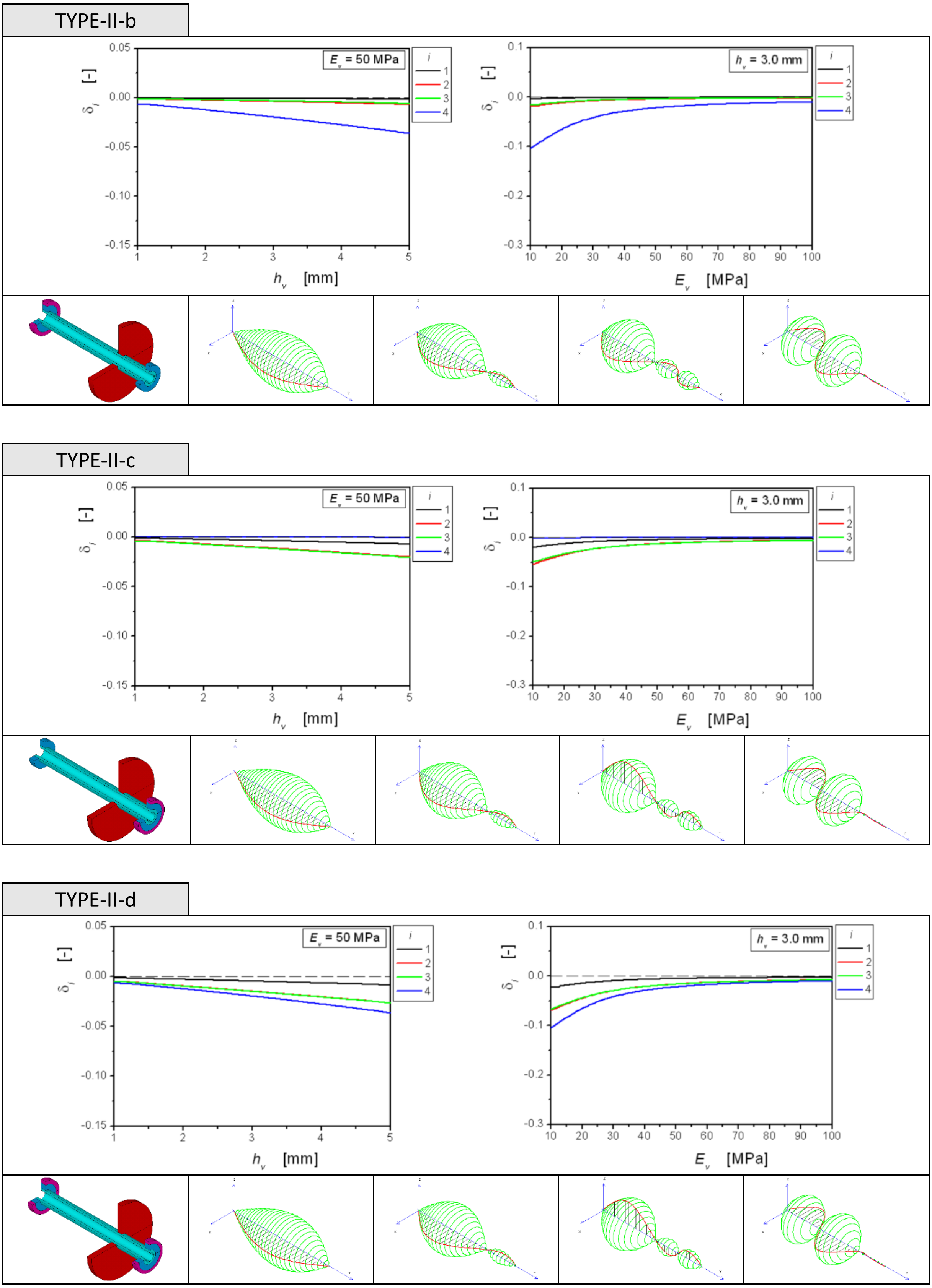

Figure 8. Dependences of rotor modal properties (TYPE-II) on the bearing stiffness.

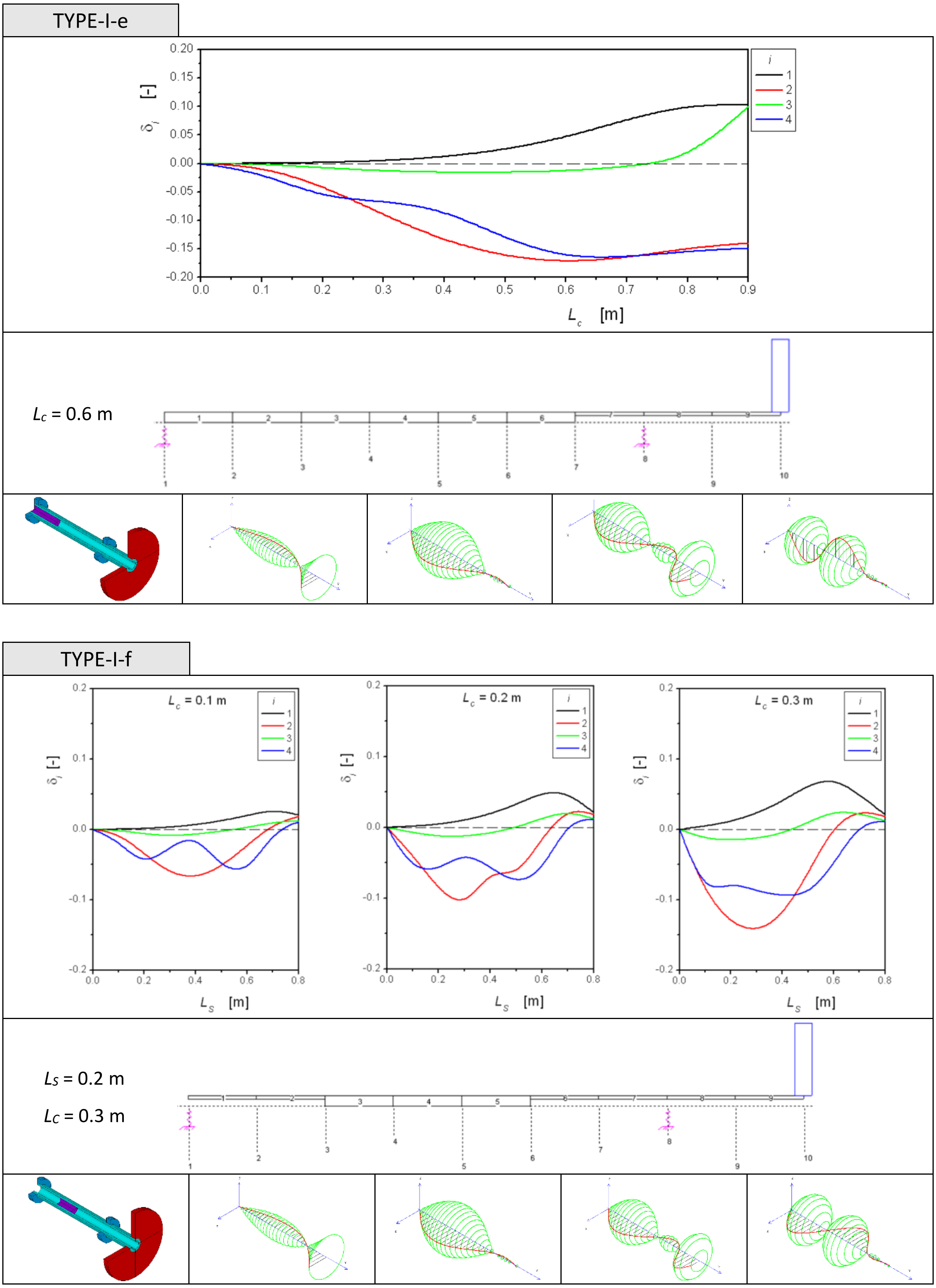

Figure 9. Dependences of rotor modal properties (TYPE-I) on the reinforcing core.

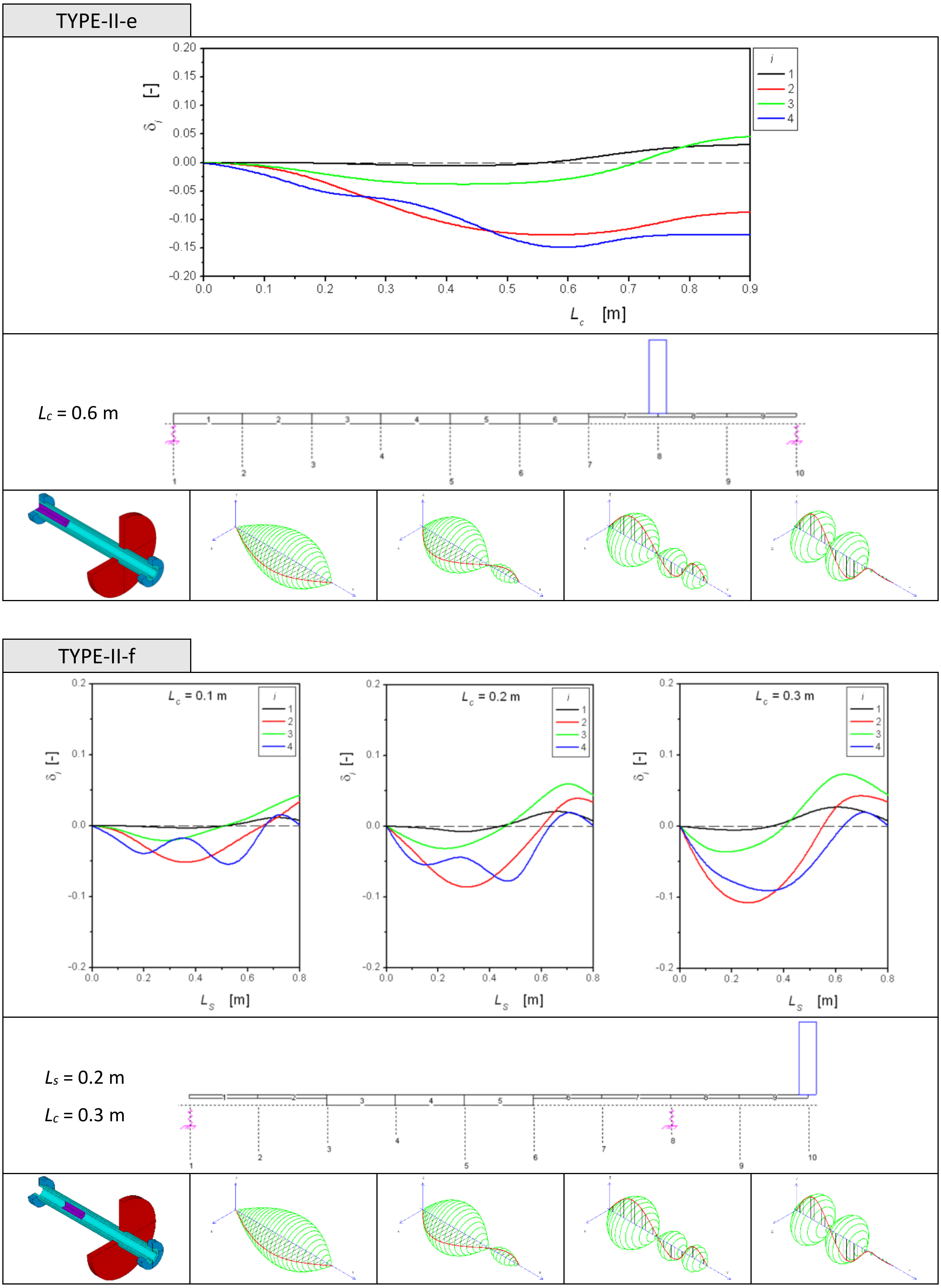

Figure 10. Dependences of rotor modal properties (TYPE-II) on the reinforcing core.

6. Conclusions

The dependencies of modal properties of selected types of rotor systems (rotor system with an overhanging disk, rotor system with a disk between bearings) on the material properties and geometrical parameters of basic structural components of rotor systems are studied in this paper. The results of modal analyses obtained from numerical simulations using the "ROTORINSA" program lead to the conclusion that alterations in bearing stiffness and considered modifications in the mass and stiffness distribution of the shaft affect the modal properties of the rotor system.

The changes in the bearing stiffness are caused by a change in the parameters of the binding layers in which the bearings are inserted. These parameters are Young's modulus of the layer material and layer thickness. Increasing in the thickness of binding layer and decreasing in the value of the Young's modulus of elasticity of the binding layer material lead to the decrease in the bearing stiffness. As a result, the total stiffness of the rotor systems decreases, and since there is no change in the mass properties, these facts logically lead to a decrease in the values of natural frequencies. These conclusions are also confirmed by the presented results of numerical simulations.

The different problem appears in the case of modification of the rotor system by inserting the reinforcing core into the hollow shaft. Two concepts of modifying shaft properties are investigated in the paper. The first case is based on the inserting of the reinforcing core into the entire shaft length. In the second case, the reinforcing core having a finite length is gradually moved in the shaft length. It is obvious, these cases with an inserted reinforcing core cause changes in the mass and stiffness distribution of the rotor system, which lead to the changes in the natural frequencies. In contrast to the bearing stiffness changes, in these cases of rotor modification, there is an increase and decrease in the value of natural frequencies, which of course depends on the distribution of mass and stiffness parameters of the modified rotor system.

The structural modifications of rotor systems presented in this paper provide suitable methods for "tuning" their modal properties. Using these methods the reduction or elimination of resonance states of rotor systems, which are undesirable in operational modes, can be achieved.

Acknowledgements
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