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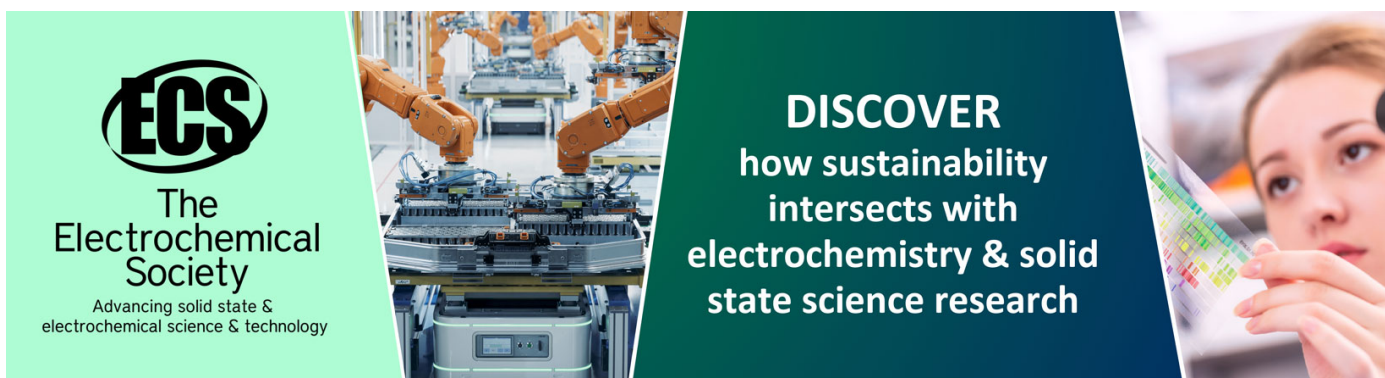
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To cite this article: Kang Yi *et al* 2022 *J. Phys.: Conf. Ser.* **2310** 012041

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Research on Distributed Optical Fiber Pipeline Micro Leakage Detection Technology Based on Recursive Least Square

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Abstract. A very weak vibration signal is generated when the pipeline leaks slightly. However, the commonly used pipeline leakage detection technology cannot obtain this kind of signal normally, thus causing large positioning error. Aiming at the above problems, a pipeline micro leakage detection technology based on Recursive Least Squares distributed optical fiber is proposed. The technology utilizes a dual Mach-Zehnder interference system with high sensitivity and simple optical path structure to monitor the interference signal caused by pipeline micro leakage at all times. Then, the adaptive filter based on Recursive Least Square method is used to denoise the interference signal. On this basis, the cross-correlation algorithm is used to obtain the delay estimation of the two test signals. This article clarifies the positioning principle of the detection technology and conducts the positioning experiment of pipeline micro leakage. The experimental results show that this technology can effectively detect the weak vibration signal generated by pipeline micro leakage and greatly improve the positioning accuracy of micro leakage points. The relative error can be controlled within 1%.

1. Introduction

Oil resources are the lifeblood of national economic development, and the main mode of oil transportation is pipeline transportation. However, in recent years, with the increasing mileage of our country's oil pipeline, the safe operation of the pipelines faces greater challenge. In the process of oil transportation, pipeline leakage accidents frequently occur due to perennial aging, human activities, natural disasters, etc. Pipeline leakage accidents will not only give rise to economic and property losses, but also pollute the environment and destroy the ecological environment. Therefore, monitoring the oil pipeline all the time is conducive to the sustainable development of economy and ecological environment.

At present, pipeline leakage detection methods mainly include negative pressure wave method, flow balance method, transient flow method, spherical internal detection method and optical fiber sensing method [1-6]. The negative pressure wave leak detection method is to locate the leak point by obtaining the time difference between the negative pressure wave reaching the pressure sensors at both ends of the pipeline, and combining the propagation speed of the negative pressure wave. The method is the most mature leak detection method applied at home and abroad, but the negative pressure wave signal generated by the micro leakage will be submerged in the noise signal, so that the negative



pressure wave signal cannot be detected. Therefore, this method is suitable for the detection of sudden large leaks, but is not suitable for the detection of micro leakage in long pipeline; the flow balance method is based on the principle that the inflow and outflow of oil in the pipeline are equal to determine whether leakage occurs. The detection principle of this method is simple, but the current flowmeter can hardly detect the flow change caused by the micro leakage of the pipeline, which will lead to a high false alarm rate; the transient flow method establishes a mathematical model based on the principle of transient flow and thermodynamic equation, and calculates the flow field in the tube. When the calculated value exceeds the set threshold, it can be judged that leakage occurs. This method has high positioning accuracy, but the positioning model is sophisticated and mass computing, and it cannot to identify micro leakage; the spherical internal inspection method is to place a detection ball inside the pipeline, and locate the leak point by analyzing various sensor data when the pipeline leaks. This method has high sensitivity and accurate positioning, but it is pricey and prone to pipeline blockage, it is not suitable for leakage detection of small-diameter pipeline. Through the above analysis, it is so necessary to seek a real-time monitoring and positioning technology that can solve the micro leakage of the pipeline.

Distributed optical fiber sensing technology has the advantages of superior anti-electromagnetic interference and high sensitivity [7,8], it is a perfect method for detecting micro leakage in pipeline. Distributed optical fiber sensing technology can be divided into interference technology and backscattering technology. Backscatter technology requires high hardware and complex demodulation algorithm, which greatly limits its application in practical engineering. The interference technology has the advantages of wide frequency response, simple optical path structure, fast response speed, and high positioning accuracy, and can detect weak vibration signals [9,10], so as to achieve real-time monitoring of long-distance oil pipeline. With the increasing application of distributed optical fiber sensing technology in pipeline micro leakage detection, it also has higher requirements for the positioning accuracy of leakage detection.

Chen et al. directly executed cross-correlation calculation on the interference signal. In the leak positioning experiment with a length of 20 kilometers, the positioning error was 149 meters [11]. However, the direct cross-correlation operation will cause a large positioning error due to the influence of the noise signal. Zhou Yan et al. proposed a pipeline safety detection technology based on the principle of a single Mach-Zehnder interferometer [12]. However, the single interferometer has defects such as being susceptible to external interference and large positioning error. Huang Yue et al. proposed a hybrid distributed optical fiber sensing system based on Sagnac and Mach-Zehnder, and added a polarization controller on this basis, with an average positioning error of 1.44% [13].

In order to further improve the positioning accuracy of pipeline micro leakage detection, a pipeline micro leakage detection technology based on double Mach Zehnder distributed optical fiber based on Recursive Least Square method is proposed in this article. The signal-to-noise ratio of interference signal can be improved by Recursive Least Square algorithm, so as to reduce the positioning error of pipeline micro leakage.

2. Measurement methods

2.1. Working Principle of Dual Mach-Zehnder Interferometer

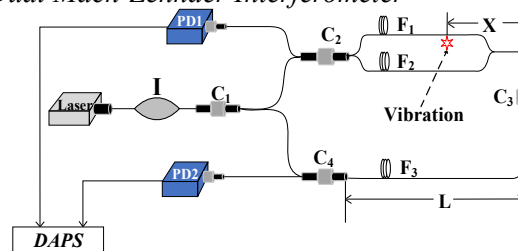


Fig 1. Schematic diagram of positioning of dual Mach-Zehnder distributed optical fiber sensing system

The schematic diagram of the dual Mach-Zehnder distributed optical fiber sensing system is shown in Figure 1. The positioning system is mainly composed of two Mach-Zehnder interferometers. The light emitted by the laser with narrow line-width is split equally when it passes through the optical isolator and reaches the fiber coupler 1. One beam is divided into two beams when it passes through the fiber coupler 2 and enters the two interference arms of the sensing fiber respectively. The interference occurs at the optical fiber coupler 3, and the interference signal is received by the photodetector 2 when it enters the optical fiber coupler 4 through the conduction optical fiber, so as to form a clockwise Mach-Zehnder interferometer; the other beam passes through the fiber coupler 4 and enters the conducting fiber. Then, when it passes through the optical fiber coupler 3, it is divided into two beams that enter the two interference arms of the sensing optical fiber respectively. The interference occurs at the optical fiber coupler 2 and is received by the photodetector 1, so as to form a counterclockwise Mach-Zehnder interferometer. The photodetector converts the detected light signal into electrical signal, and finally conducts data processing through the data acquisition and processing system.

In this system, the function of the optical isolator is to allow only unidirectional transmission of light waves, which is to prevent the reverse transmission of light waves from forming a new resonance frequency, resulting in unstable periodic changes in the output interference signal. The function of the fiber coupler is to split and combine the optical signal. When the optical signal passes through the coupler, its amplitude and phase will change. For fiber couplers, without considering the external loss, the transmission coefficient of direct coupling is $\sqrt{2}/2$, The transmission coefficient of the cross-coupling is also $\sqrt{2}/2$, but it will produce a lag phase of $\pi/2$ [14]. The phase shift generated by the optical wave passing through the k-times optical fiber coupler is $k\pi/2$, and the phase shift generated by the coupler in this system is π .

2.2. Positioning Formula and Error Analysis of Micro Leakage Point

Assuming that the light intensity emitted by the laser is I_0 , and the intensity ratio of the external disturbance to the two sensing fibers is α ($0 < \alpha < 1$), according to the interference theory, The interference light intensity detected by the two photodetectors can be expressed as:

$$I_1 = \frac{1}{8}I_0 + \frac{1}{8}I_0 \cos \left\{ \frac{2\pi n \cdot \Delta L}{\lambda} + (1-\alpha) \cdot \phi \left[t - \frac{n(L-X)}{c} - \frac{n \cdot \Delta L}{c} \right] \right\} \quad (1)$$

$$I_2 = \frac{1}{8}I_0 + \frac{1}{8}I_0 \cos \left\{ \frac{2\pi n \cdot \Delta L}{\lambda} + (1-\alpha) \cdot \phi \left[t - \frac{n(L+X)}{c} \right] + \pi \right\} \quad (2)$$

where L is the length of the sensing fiber and the conducting fiber, ΔL is the length difference between the two interference arms F_1 , F_2 , X is the distance from the external disturbance point to the sensing fiber at the right end, n is the refractive index of the fiber, λ is the wavelength of the light wave, c is the propagation speed of light in vacuum, and $\phi(t)$ is the phase change caused by vibration. In order to simplify the above expression, the following assumptions are made:

$$t_1 = \frac{n(L-X)}{c} + \frac{n \cdot \Delta L}{c} \quad (3)$$

$$t_2 = \frac{n(L+X)}{c} \quad (4)$$

$$\Delta\phi_L = \frac{2\pi n \cdot \Delta L}{\lambda} \quad (5)$$

The formulas (1) and (2) can be expressed as:

$$I_1 = \frac{1}{8}I_0 + \frac{1}{8}I_0 \cos \{ \Delta\phi_L + (1-\alpha) \cdot \phi(t-t_1) \} \quad (6)$$

$$I_2 = \frac{1}{8}I_0 - \frac{1}{8}I_0 \cos\{\Delta\phi_L + (1-\alpha) \cdot \phi(t-t_2)\} \quad (7)$$

After filtering the direct current component of formulas (6) and (7), the expressions of the two reverse interference signals are as follows:

$$I_1' = \frac{1}{8}I_0 \cos\{\Delta\phi_L + (1-\alpha) \cdot \phi(t-t_1)\} \quad (8)$$

$$I_2' = -\frac{1}{8}I_0 \cos\{\Delta\phi_L + (1-\alpha) \cdot \phi(t-t_2)\} \quad (9)$$

After inverting formulas (9) can be expressed as:

$$I_2'' = \frac{1}{8}I_0 \cos\{\Delta\phi_L + (1-\alpha) \cdot \phi(t-t_2)\} \quad (10)$$

It can be observed that the similarity of I_1' and I_2'' waveforms is extremely high, which means that there is a delay difference between them. The expressions obtained from formulas (3) and (4) are:

$$\Delta t = \frac{n(2X - \Delta L)}{c} \quad (11)$$

Then the position of the external disturbance can be expressed as:

$$X = \frac{1}{2}\Delta L + \frac{c \cdot \Delta t}{2n} \quad (12)$$

Since the length difference between the two sensing fibers is relatively small, the length of ΔL can be neglected, and the positioning formula of the leak point is:

$$X = \frac{c \cdot \Delta t}{2n} \quad (13)$$

The propagation speed of light wave in the optical fiber and the refractive index of the optical fiber are definite values. Therefore, as long as the time delay difference between the two cross-correlated signals I_1' and I_2' is measured, the position of the leakage point can be calculated by the formula (13), so as to solve the positioning of the micro leakage point of the pipeline.

Since the interference signals detected by the photodetector are generated by the same light source and the same vibration, the two interference signals have strong correlation. The time delay difference of the two interference signals can be obtained by using the cross-correlation algorithm. The delay difference is the product of the sampling period and the number of intervals, if the sampling period of the data is T_s , the expression of the delay difference is $\Delta t = N \cdot T_s$ (N is a positive integer), and the positioning accuracy resolution of system leakage detection can be expressed as:

$$\Delta x = ((N+1)\Delta t)x - (N\Delta t)x \quad (14)$$

$$= \frac{c}{2n} \cdot T_s \quad (15)$$

From equation (15), it can be known that the sampling period of the data can indirectly affect the positioning accuracy of the micro-leakage point. For a single-mode fiber with a wavelength of 1550 nm the refractive index of the fiber is about 1.5, and the propagation speed of the light wave is about $2.0 \times 10^8 \text{ m/s}$. When the sampling frequency of the Data Acquisition Board is 1MHz, the resolution of the positioning accuracy is 66.7meters calculated by substitution; when the sampling frequency of the Data Acquisition Board is 5MHz, the resolution of the positioning accuracy is 13.3meters calculated by substitution; when the sampling frequency of the Data Acquisition Board is 10MHz, the resolution of the positioning accuracy is 6.67meters calculated by substitution. In order to reduce the ultimate positioning error of micro leakage point, the sampling frequency of the Data Acquisition Board used in this article is 10MHz.

2.3. Adaptive Filtering Theory

In the practical real-time monitoring of pipeline leakage, due to the interference of the external environment and the sensor itself, the acquired interference signal is a non-linear non-stationary random signal accompanied by high-frequency noise signal. However, the noise signal leads to large error on the result of the cross-correlation operation, so it is necessary to de-noise the acquired interference signal. Adaptive filtering is a filtering method developed in recent years. The basic principle is to automatically adjust the parameters of the filter at the current moment through the parameters of the filter at the previous moment, so that the cost function is minimized. Because it does not need to know the prior characteristics of the interference signal in advance, it can adaptively adjust to the optimal filtering state, and has been widely used in practical signal processing [15,16].

At present, the widely applied adaptive filtering algorithms mainly consist of Least Mean Square algorithm [17] and Recursive Least Square algorithm [18]. Compared with the Least Mean Square algorithm, the Recursive Least Squares algorithm has the advantages of fast convergence speed and high stability, and is more attention in practical applications. The structure of the adaptive filter system is shown in Figure 2.

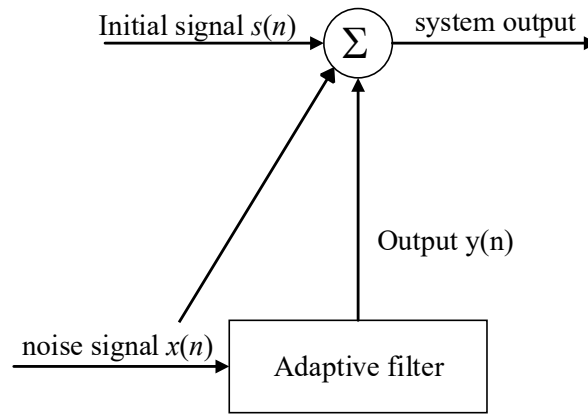


Fig 2. Adaptive filter system structure diagram

2.4. Recursive Least Squares Algorithm

The basic concept of the Recursive Least Squares algorithm is to adaptively select the weight coefficients of the filter in the iterative process, so that the deviation between the output signal and the expected signal is as far as possible [19]. The implementation process of the adaptive filtering algorithm based on Recursive Least Squares is as follows:

Assuming that the input signal of the filter is $u(i)$ and the tap weight vector is $w(i)$, then the output of the filter can be expressed as:

$$y(i) = w^H(n)u(i) \quad (16)$$

where the weighted conjugate matrix is w^H and the tap weight vector is $w(n)$.

Define the difference between the output of the filter and the expected response as $e(i)$, namely:

$$e(i) = d(i) - w^H(n)u(i) \quad (17)$$

The minimum objective cost function and exponential weighting factor are introduced. The core of the Least Square Method is to select the optimal tap weight vector to minimize the cost function, the cost function is defined as:

$$\varepsilon(w_0, \dots, w_{M-1}) = \sum_{i=0}^n \lambda^{n-i} |e(i)|^2 \quad (0 < \lambda < 1) \quad (18)$$

Where λ is the forgetting factor, which is used to ensure that the observed data are forgotten in the past period, so that the filter can track the statistical changes of the observed data at all times.

Derivative the tap weight vector to minimize the objective cost function, namely:

$$\frac{\partial J(n)}{\partial w} = 0 \quad (19)$$

Solution:

$$w(n) = R^{-1}(n)r(n) \quad (20)$$

where $R(n)$ and $r(n)$ are defined as two new matrices.

In order to normalize the estimated value, the prior information related to the mapping relationship between input and output needs to be considered. Therefore, the cost function needs to be extended to the sum of two parts [20], namely:

$$\varepsilon(n) = \sum_{i=0}^n \lambda^{n-i} |e(i)|^2 + \delta \lambda^n \|w(n)\|^2 \quad (21)$$

Then, according to the inverse matrix lemma, the recursive formula for updating the tap weight vector is obtained as follows:

$$\hat{w}(n) = \hat{w}(n-1) + k(n)\xi^*(n) \quad (22)$$

$$\xi(n) = d(n) - \hat{w}^H(n-1)u(n) \quad (23)$$

$$e(n) = d(n) - \hat{w}^H(n)u(n) \quad (24)$$

$$k(n) = \frac{P(n-1)u(n)}{\lambda + u^H(n)P(n-1)u(n)} \quad (25)$$

Where $\xi(n)$ is the a priori estimation difference, which is the error estimation value before updating the tap weight vector. It should be noted that the a priori estimation difference is different from the a posteriori estimation difference $e(n)$. $k(n)$ is the gain vector and $P(n)$ is the inverse matrix of $R(n)$. Equation (22) illustrates the recursive process of the algorithm, and equation (23) illustrates the filtering process of the algorithm.

3. Experiment Procedure

3.1. Basic Conditions of The Experiment

In this experiment, a laser with a wavelength of 1550nm and an output power of 23mW was used as the light source. Three dual-core single-mode optical fibers are deployed outside the oil pipeline with a distributed manner. The length of the dual-core single-mode optical fiber is 20 kilometers, the refractive index of the optical fiber is 1.468, and the propagation speed of light wave in the optical fiber is $2.0436 \times 10^8 m/s$. Due to the long-distance transmission of laser, in order to ensure that the photodetector can still maintain good detection effect, Therefore, a photodetector is selected with an acceptance sensitivity of negative 60dBm and a wavelength response range of 800nm to 1700nm. According to the analysis in Section 2, the NI PCI-5114 Acquisition Board with a sampling frequency of 10MHz should be selected for data acquisition.

3.2. Micro Leakage Points Positioning Process

The acquired data is denoised by the adaptive filter based on Recursive Least Squares. On this basis, the cross-correlation algorithm is used to estimate the delay of the denoised data, and the positioning calculation of the micro leakage point is carried out. Finally, draw a conclusion after analyzing the positioning calculation results. The flow chart of positioning calculation is shown in Figure 3:

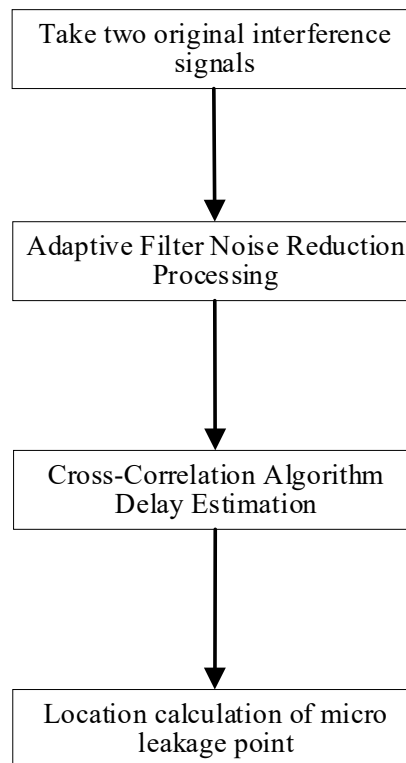


Fig 3. The flow chart of pipeline micro leakage points positioning calculation

4. Results and discussion

During the experiment, the leakage point is 5350m away from the end of the oil pipeline, the leakage aperture is 3mm, and the leakage flow rate is controlled to 1L/h, which accords with the conditions of micro leakage. The collected original interference signal is shown in Figure 4:

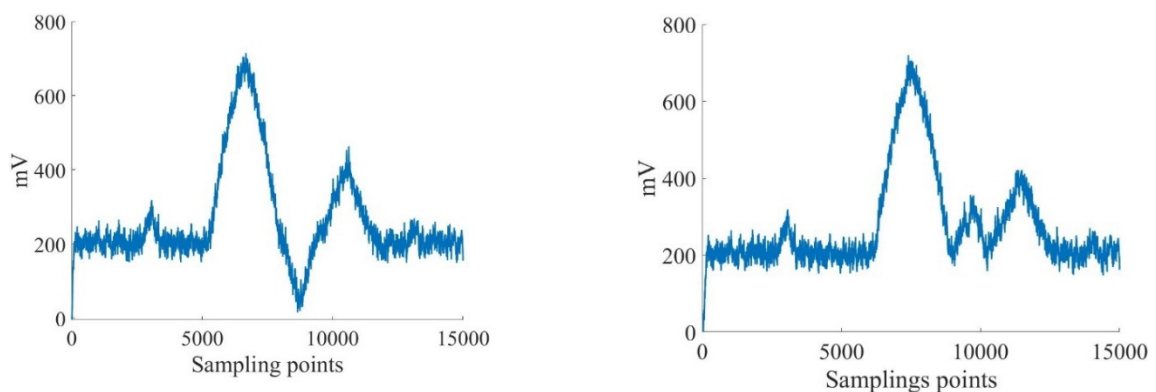


Fig 4. Waveform of original interference signal of pipeline micro leakage

The adaptive filter is used to denoise the original interference signal with noise. The waveform of the denoised interference signal is shown in Figure 5.

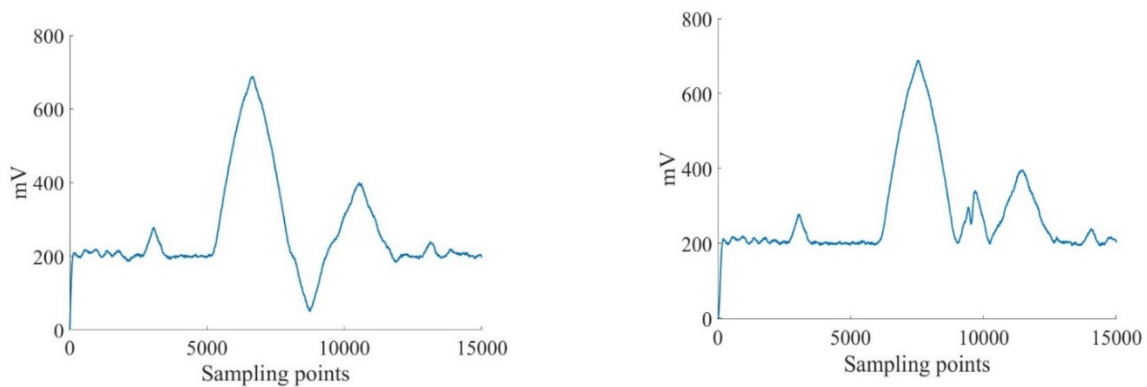


Fig 5. Denoising diagram of the original interference signal

According to the comparison between Figure 4 and Figure 5, in the process of light interference, the waveform of the high-frequency noise signal generated by the interference of noise and the error of the sensor tends to be smooth after passing through the adaptive filter. It can be concluded from Fig. 5 that the waveforms of the two collected original interference signals are extremely similar, and there is a certain delay, indicating that the two interference signals have strong cross-correlation.

According to the analysis in Section 2, it is only necessary to measure the time delay difference between the two cross-correlated signals to locate the micro-leakage point of the pipeline. The cross-correlation operation is executed on the two interference signals after adaptive filtering, and the operation result is shown in Figure 6.

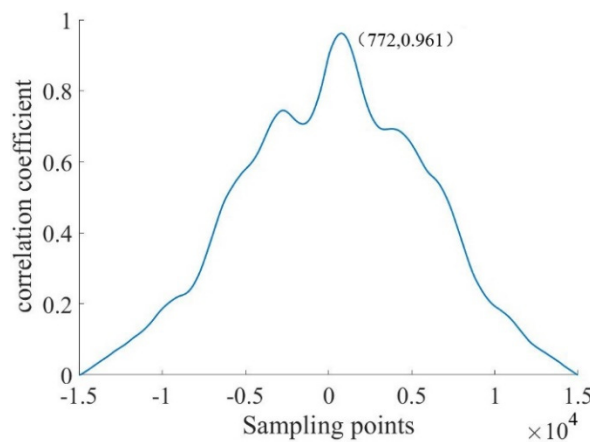


Fig 6. The cross-correlation calculation curve of the two interference signals

The time delay difference of the two interference signals is the abscissa corresponding to the highest point, and the correlation coefficient is the ordinate corresponding to the highest point. It can be concluded from Figure 5 that the abscissa of the peak value of the cross-correlation operation curve of the two interference signals is 7720 and the ordinate is 0.961. The time delay difference of interference signal is substituted into formula (13) to calculate the positioning distance of micro leakage point, the calculation result is 5373.49m. Compared with the physical distance, the absolute error is 23.49m and the relative error is 0.44%; If the original interference signal is not denoised, the result of direct cross-correlation operation is 5554.47m. Compared with the physical distance, the absolute error is 204.47m and the absolute error is 3.8%.

In order to ensure the reliability of the experiment, several positioning experiments are carried out, and the results calculated by the direct cross-correlation method and the algorithm in this paper are compared. The comparison results are shown in Figure 7:

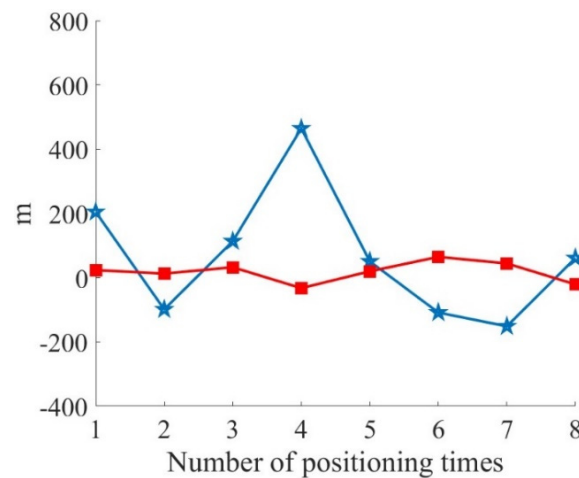


Fig 7. Comparison of positioning results of two different algorithms

The positioning error is shown in Table 1:

Table 1. Positioning Error Comparison

Positioning Methods	Direct cross- correlation method	Proposed method
Maximum Error(m)	464.4	64.7
Average Error(m)	156.5	31.4
Relative Error (%)	2.93	0.58

The above calculation results show that the direct cross-correlation delay calculation of the interference signal will generate a large error, and the positioning error after denoising with the adaptive filter is reduced significantly. The average positioning error is reduced by about 5 times. In the process of long-distance oil transportation in oil pipeline, when micro leakage occurs, if the positioning error is controlled within a certain range, combined with manual inspection method, the location of leakage point can be found quickly.

5. Conclusion

The pipeline micro leakage detection technology based on dual Mach Zehnder distributed optical fiber sensor has the advantages of high positioning accuracy, rapid response, simple optical path structure and extremely sensitive response to the weak vibration signal generated from the outside. This technology can detect the micro leakage point of pipeline with slow flow and small leakage aperture. The leak positioning experiment was carried out on the oil pipeline with a leak diameter of 3mm, a leak flow of 1L/h and a total length of 20km. Experiments show that the leakage point positioning accuracy of this technology can reach 30m, and the relative positioning error can be controlled within 1%, which can accord with the detection requirements of pipeline micro leakage in the process of practical production and transportation. This technology is a real-time monitoring and positioning technology that can solve the micro leakage of pipeline. Most important of all, this technology can not only recover the economic loss caused by leakage, but also protect the ecological environment and reduce the loss of energy. It has great practical significance in ecological protection.

Acknowledgements

This research was supported by General program of National Natural Science Foundation of China (62173049).

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