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Nonlinear rendezvous guidance law design for a receiver UAV based on the Lyapunov stability theory

Hongchao Zhao[1*], Jianzhong Zhao[2], Hongmei Xu[1]

[1] College of Artificial Intelligence, Yantai Institute of Technology, Yantai, Shandong, 264005, China.

[2] College of Coastal Defence, Naval Aviation University, Yantai, Shandong, 264001, China.

[*]Corresponding author's e-mail: zhaohongchao@yitsd.edu.cn

Abstract. The rendezvous problem in the autonomous aerial refueling (AAR) was researched for the receiver unmanned aerial vehicle (UAV). The three-dimensional guidance model was constructed for the UAV to pursue the virtual tanker. In order to guarantee the zero miss distance and the intercept angle constraints, a nonlinear rendezvous guidance law was designed for the UAV based on the Lyapunov stability theory. The guidance law generated the normal acceleration command and the lateral acceleration command. The velocity control system was designed by the sliding mode control method. It controled the UAV's velocity to equal to that of the virtual tanker at the end of the rendezvous phase. The simulation results validate the effectiveness of the designed nonlinear guidance law and the velocity control system.

1. Introduction

With the rapid development of unmanned aerial vehicle (UAV) technologies, UAVs are being used to carry out various missions like long time surveillance and reconnaissance, long distance military strikes, etc., which require UAVs to continuously stay in the air for a quite long time. Accordingly, autonomous aerial refueling (AAR) becomes a key issue to be addressed for these applications of UAVs. Generally, there are two major types for AAR in operation: probe-drogue refueling (PDR) and boom receptacle refueling (BRR), and both play important roles in modern civil and military applications [1]. The PDR type is focused on in this paper. There are four main phases during AAR, i.e., rendezvous phase, docking phase, refueling phase, and dismissing phase. The rendezvous phase is studied in this paper. During rendezvous the UAV actively rendezvouses with the tanker through its guidance and flight control system, and finally it keeps the same track angle and velocity as the tanker at a certain position behind and under the tanker.

In recent decades, some scholars have researched the rendezvous problem of AAR. Xu and Luo [2] analyzed the rendezvous strategy, and then proposed a nonlinear lateral trajectory tracking guidance law to generate the UAV's lateral acceleration command. Luo et al. [3] proposed an iterative computation guidance law to compute a series of state variables for a UAV rendezvousing with a tanker. Qu et al. [4] developed a guidance law based on fractional-order sliding mode control to deal with the terminal angle and velocity constraints of a UAV in the rendezvous process. Gong et al. [5] applied the Dubins path to produce the rendezvous trajectory of the receiver and used the proportional navigation law to generate the guidance command. Wang et al. [6] used a three-dimensional (3D) proportional navigation law with angle constraint to produce the azimuth angle command, and the climb angle command was generated according to the height difference between a UAV and a tanker. Yoon et al. [7] researched the 3D path planning for the rendezvous phase, and used the pure pursuit guidance law to generate the guidance commands for multiple UAVs. Tsukerman et al. [8] designed the linear quadratic optimal guidance law for the civil AAR in the rendezvous process. Yuan et al. [9] studied the control strategy of a UAV in the rendezvous process, and applied the Dubins path to produce the rendezvous trajectory. Zhao et al. [10] analyzed the relative relationship between a virtual tanker and a UAV, and used the fast terminal sliding mode control strategy to design the UAV's guidance law and flight control law.

In the end of the rendezvous phase, the UAV must reach the certain position behind and under the tanker, at the same time, the UAV's track angles and velocity must equal to those of the tanker. Inspired by the missile interception guidance methods with intercept angle constraints [11, 12], we develop a nonlinear rendezvous guidance law for the UAV based on the Lyapunov stability theory, which can realize both zero miss distance and intercept angle constraints. In addition. The UAV's velocity control system is designed independently. The rest of this paper is organized as follows. The 3D guidance model is constructed for the UAV pursuing the virtual tanker in Section 2. In Section 3, the nonlinear guidance law are designed the UAV based on the Lyapunov stability theory. In Section 4, the UAV's velocity control system is designed to adjust the engine thrust. The numerical simulation is executed in Section 5. Finally, Section 6 concludes this paper.

2. Three-dimensional Guidance Model

The probe-drogue refueling mode is considered in this paper. In the docking process the receiver UAV docks with the drogue of the tanker. For simplifying the guidance problem, we omit the airflow disturbances exerting on the drogue and think that the drogue lies in a fixed position behind and under the tanker. The drogue is treated as a virtual tanker. The geometric relationship between a UAV and a virtual tanker in the rendezvous process is described in figure 1. The North-East-Down frames is the inertial frames. γ_L and χ_L are the elevation angle and the azimuth angle of line-of-sight (LOS), respectively. γ_u and χ_u are the advance elevation angle and azimuth angle of V_U relative to LOS, respectively. γ_t and χ_t are the advance elevation angle and azimuth angle of V_T relative to LOS, respectively.

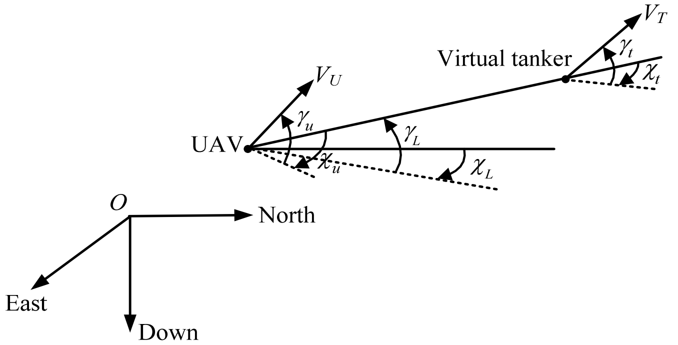

Figure 1. Geometric relationship between a UAV and a virtual tanker.

According to the geometric relationship and referring the former achievement [11], the 3D guidance model for the UAV pursuing the virtual tanker is described as the following equations,

$$\dot{R} = V_T \cos\gamma_t \cos\chi_t - V_U \cos\gamma_u \cos\chi_u \tag{1}$$

$$\dot{\chi}_L = (V_T \cos\gamma_t \sin\chi_t - V_U \cos\gamma_u \sin\chi_u)/(R\cos\gamma_L) \tag{2}$$

$$\dot{\gamma}_L = (V_T \sin\gamma_t - V_U \sin\gamma_u)/R \tag{3}$$

$$\dot{\gamma}_t = a_{T,n}/V_T - \dot{\gamma}_L \cos\chi_t - \dot{\chi}_L \sin\gamma_L \sin\chi_t \tag{4}$$

$$\dot{\chi}_t = a_{T,l}\big/(V_T \cos\gamma_t) + \dot{\chi}_L \sin\gamma_L \cos\chi_t \tan\gamma_t - \dot{\chi}_L \cos\gamma_L - \dot{\gamma}_L \sin\chi_t \tan\gamma_t \tag{5}$$

$$\dot{\gamma}_u = a_{U,n}\big/V_U - \dot{\gamma}_L \cos\chi_u - \dot{\chi}_L \sin\gamma_L \sin\chi_u \tag{6}$$

$$\dot{\chi}_u = a_{U,l}\big/(V_U \cos\gamma_u) + \dot{\chi}_L \sin\gamma_L \cos\chi_u \tan\gamma_u - \dot{\chi}_L \cos\gamma_L - \dot{\gamma}_L \sin\chi_u \tan\gamma_u \tag{7}$$

where R is the relative distance between the UAV and the virtual tanker; $a_{T,l}$ and $a_{T,n}$ are the lateral acceleration and normal acceleration of the virtual tanker, respectively; $a_{U,l}$ and $a_{U,n}$ are the UAV's lateral acceleration and normal acceleration, respectively.

It is assumed that the virtual tanker flies along a level straight line at a constant speed in the rendezvous process, so its elevation angle is zero, and its lateral acceleration and normal acceleration are also $a_{T,l} = 0$, $a_{T,n} = 0$. The following equations can be deduced by taking the derivative of equation (2) and equation (3), respectively, and combining with equations (1), (4)-(7),

$$\ddot{\chi}_L = 2\dot{\chi}_L\dot{\gamma}_L \tan\gamma_L - \frac{2\dot{R}}{R}\dot{\chi}_L + \frac{1}{R\cos\gamma_L}\left(a_{U,n}\sin\gamma_u \sin\chi_u - \dot{V}_U \cos\gamma_u \sin\chi_u - a_{U,l}\cos\chi_u\right) \tag{8}$$

$$\ddot{\gamma}_L = -\dot{\chi}_L^2 \sin\gamma_L \cos\gamma_L - \left(2\dot{R}\dot{\gamma}_L + \dot{V}_U \sin\gamma_u + a_{U,n}\cos\gamma_u\right)\Big/R \tag{9}$$

3. Design of nonlinear rendezvous guidance law

The autonomous rendezvous of the UAV with the virtual tanker requires that the UAV's track angles and velocity equal to those of the virtual tanker in the end of the rendezvous phase. The end is also the initial point of the docking phase. In order to satisfy the zero miss distance and the intercept angle constraints, a nonlinear rendezvous guidance law is designed for the UAV based on the Lyapunov stability theory. For the velocity constraint, the velocity control system will be designed in the next section. Applying the Lyapunov stability theory, the Lyapunov function candidate is chosen as

$$W = \frac{1}{2}(\dot{\gamma}_L)^2 + \frac{1}{2}k_1(\gamma_L - \gamma_{Lf})^2 + \frac{1}{2}(\dot{\chi}_L)^2 + \frac{1}{2}k_2(\chi_L - \chi_{Lf})^2 \tag{10}$$

where $k_1 > 0$, $k_2 > 0$, γ_{Lf} and χ_{Lf} denote the desired terminal elevation angle and terminal azimuth angle of the LOS, respectively, which represent the desired intercept angles. For the rendezvous guidance problem, $\gamma_{Lf} = \gamma_T$, $\chi_{Lf} = \chi_T$, where γ_T and χ_T are the elevation angle and the azimuth angle of the virtual tanker, respectively.

Taking the derivative of equation (10) and combining equation (8) and equation (9) give

$$\begin{aligned}
\dot{W} &= \dot{\gamma}_L\ddot{\gamma}_L + k_1(\gamma_L - \gamma_{Lf})\dot{\gamma}_L + \dot{\chi}_L\ddot{\chi}_L + k_2(\chi_L - \chi_{Lf})\dot{\chi}_L \\
&= \dot{\gamma}_L\left[k_1(\gamma_L - \gamma_{Lf}) - \dot{\chi}_L^2 \sin\gamma_L \cos\gamma_L - 2\dot{R}\dot{\gamma}_L/R - \dot{V}_U \sin\gamma_u/R - a_{U,n}\cos\gamma_u/R\right] + \\
&\dot{\chi}_L\left[k_2(\chi_L - \chi_{Lf}) + 2\dot{\chi}_L\dot{\gamma}_L \tan\gamma_L - \frac{2\dot{R}}{R}\dot{\chi}_L + \frac{a_{U,n}\sin\gamma_u}{R\cos\gamma_L}\sin\chi_u - \frac{\dot{V}_U\cos\gamma_u}{R\cos\gamma_L}\sin\chi_u - \frac{a_{U,l}\cos\chi_u}{R\cos\gamma_L}\right]
\end{aligned} \tag{11}$$

In order to ensure $\dot{W}$ to be negative definite, the normal acceleration command and the lateral acceleration command can be designed as follows, respectively,

$$a_{U,n} = \left[(k_\gamma R - 2\dot{R})\dot{\gamma}_L + k_1 R(\gamma_L - \gamma_{Lf}) - R\dot{\chi}_L^2 \sin\gamma_L \cos\gamma_L - \dot{V}_U \sin\gamma_u\right]\Big/\cos\gamma_u \tag{12}$$

$$\begin{aligned}
a_{U,l} = &\left[(k_\chi R - 2\dot{R})\dot{\chi}_L \cos\gamma_L + k_2 R\cos\gamma_L(\chi_L - \chi_{Lf}) + \right. \\
&\left. 2R\dot{\gamma}_L\dot{\chi}_L \sin\gamma_L + a_{U,n}\sin\gamma_L \sin\chi_u - \dot{V}_U \cos\gamma_u \sin\chi_u\right]\Big/\cos\chi_u
\end{aligned} \tag{13}$$

where $k_\gamma > 0$, $k_\chi > 0$. The guidance performance analysis is expressed as the following theorem.

Theorem 1: For the rendezvous guidance system described as equations (1)-(7), If the normal acceleration and lateral acceleration are designed as equation (12) and equation (13), respectively, then the zero miss distance and the intercept angles constraints can be guaranteed.

Proof. The Lyapunov function candidate is chosen as (10), and its derivative is expressed as equation (11). Substituting equation (12) and equation (13) into equation (11), we can get

$$\dot{W} = -k_\gamma R(\dot{\gamma}_L)^2 - k_\chi R\cos\gamma_L(\dot{\chi}_L)^2 \tag{14}$$

In the rendezvous process $R > 0$, $\gamma_L < \pi/2$, so $\dot{W} < 0$ as $\dot{\gamma}_L \neq 0$ or $\dot{\chi}_L \neq 0$, which means that W decreases always as $\dot{\gamma}_L \neq 0$ or $\dot{\chi}_L \neq 0$. In other words, $\dot{\gamma}_L = 0$ and $\dot{\chi}_L = 0$ are the conditions of the equilibrium point of the guidance system. According to the guidance theory, that the LOS rates $\dot{\gamma}_L$ and $\dot{\chi}_L$ are zeros can guarantee the zero miss distance.

Can the intercept angle constraints be guaranteed? We substitute equation (12) and equation (13) into equation (9) and equation (8), respectively, the following equations can be given,

$$\ddot{\gamma}_L = -k_\gamma\dot{\gamma}_L - k_1(\gamma_L - \gamma_{Lf}) \tag{15}$$

$$\ddot{\chi}_L = -k_\chi R\cos\gamma_L\dot{\chi}_L - k_2 R\cos\gamma_L(\chi_L - \chi_{Lf}) \tag{16}$$

From equation (15), if $\dot{\gamma}_L = 0$, but $\gamma_L - \gamma_{Lf} \neq 0$, then $\ddot{\gamma}_L \neq 0$. $\dot{\gamma}_L = \int_0^t \ddot{\gamma}_L d\tau$, so $\dot{\gamma}_L$ will depart form 0, i.e. $\dot{\gamma}_L \neq 0$. From equation (16), if $\dot{\chi}_L = 0$, but $\chi_L - \chi_{Lf} \neq 0$, then $\ddot{\chi}_L \neq 0$, $\dot{\chi}_L \neq 0$. $\dot{\gamma}_L = 0$ and $\dot{\chi}_L = 0$ represent the unstable equilibrium point. As a result $\dot{W} < 0$ as $\gamma_L - \gamma_{Lf} \neq 0$ or $\chi_L - \chi_{Lf} \neq 0$, which means that W decreases always as $\gamma_L - \gamma_{Lf} \neq 0$ or $\chi_L - \chi_{Lf} \neq 0$. Finally W will decrease to its minimum 0. $\dot{\gamma}_L = 0$, $\dot{\chi}_L = 0$, $\gamma_L - \gamma_{Lf} = 0$ and $\chi_L - \chi_{Lf} = 0$ are the whole conditions of the stable equilibrium point. Therefore the intercept angle constraints are guaranteed. This completes the proof.

In this paper the guidance law and the flight control system are designed separately. The flight control system is the same as that of the reference [10]. The desired commands required by the flight control system are the attitude angle commands. The transformation relationships are as follows,

$$\alpha_d = (ma_{U,n} - T\sin\alpha + mg\cos\gamma_U)/(QSC_{L\alpha}) - \left[C_{L0} + C_{Lq}\,qc/(2V_U) + C_{L\delta}\delta_e\right]/C_{L\alpha} - \alpha_0 \tag{17}$$

$$\beta_d = 0 \tag{18}$$

$$\phi_d = \arctan\left[a_{U,l}/(a_{U,n} + g\cos\gamma_U)\right] \tag{19}$$

where α_d, β_d and ϕ_d are the commands of the UAV's angle of attack, sideslip angle and roll angle, respectively; m and g are the mass and gravity acceleration, respectively; T is the thrust; γ_U is the elevation angle; Q is the dynamic pressure; S and c are the wing area and the mean aerodynamic chord, respectively; C_{L0}, $C_{L\alpha}$, C_{Lq} and $C_{L\delta}$ denote the lift coefficients; q is the pitch rate; δ_e is the elevator deflection; α_0 is the zero-lift angle of attack.

4. Velocity Control System Design

The UAV's velocity control system adjusts its flight velocity by adjusting the engine thrust. In order to ensure that the UAV's velocity equals to that of the virtual tanker at the end of the rendezvous phase, the velocity control system should also control the relative distance R, whose dynamic equation is equation (1). The dynamic equation of the velocity state is expressed as follows,

$$\dot{V}_U = (T\cos\alpha\cos\beta - D)/m - g\sin\gamma_U \tag{20}$$

where T is the thrust, D is the drag. The velocity control system is designed by the sliding mode control method, and the sliding surface is chosen as

$$s_V = R + k_V(V_T - V_U) \tag{21}$$

where $k_V > 0$. Adopting the power reaching law $\dot{s}_V = -k_s |s_V|^{a_1} \operatorname{sign}(s_V)$ with $k_s > 0$, $1 > a_1 > 0$, the desired thrust is designed as follows,

$$T = \frac{m}{k_V \cos\alpha \cos\beta}\left[V_T \cos\gamma_t \cos\chi_t - V_U \cos\gamma_u \cos\chi_u + \frac{D}{m} + g\sin\gamma_U + k_s |s_V|^{a_1} \operatorname{sign}(s_V)\right] \tag{22}$$

5. Numerical Simulation

The numerical simulation is performed to illustrate the effectiveness of the designed guidance law and velocity control system in the UAV's rendezvous phase. The initial states of the virtual tanker and the UAV are set as follows, $x_T = 9000\text{m}$, $y_T = 2000\text{m}$, $h_T = 6000\text{m}$, $V_T = 180\text{m/s}$, $\gamma_T = 0$, $\chi_T = 5^{\circ}$, $x_U = y_U = 0$, $h_U = 4000\text{m}$, $V_U = 240\text{m/s}$, $\gamma_U = \chi_U = 0$. By simulation trial, the UAV's guidance and control parameters are chosen as $k_\gamma = 0.18$, $k_1 = 0.01$, $k_\chi = 0.38$, $k_2 = 0.05$, $k_V = 6.5$, $k_s = 1.0$, $a_1 = 0.6$. The simulation results are shown in figures 2-5.

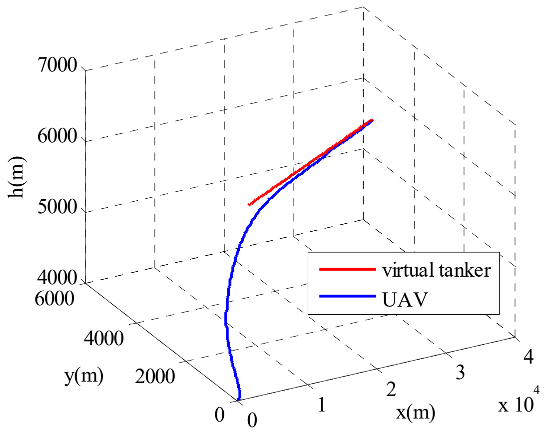

Figure 2. 3D trajectories of the UAV and the virtual tanker.

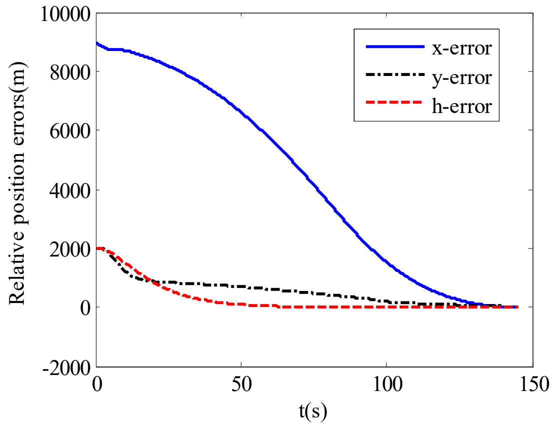

Figure 3. Relative position errors of the UAV.

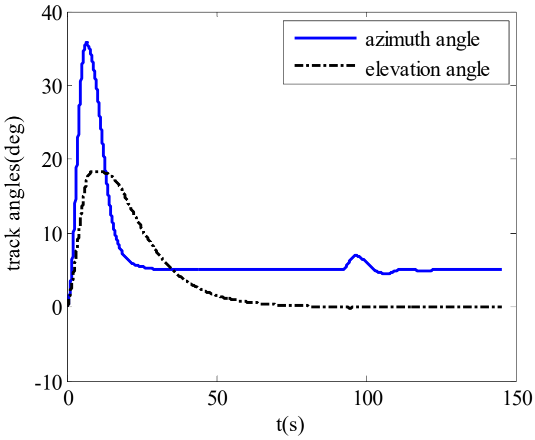

Figure 4. Variations of the UAV's azimuth angle and elevation angle.

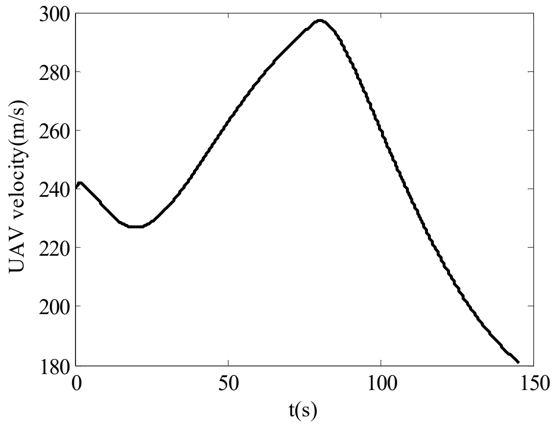

Figure 5. Velocity variation of the UAV.

Figure 2 depicts the 3D trajectories of the UAV and the virtual tanker, respectively. Figure 3 depicts the relative position errors of the UAV with the virtual tanker in x-direction, y-direction and height, respectively. Figure 4 depicts the variations of the UAV's elevation angle and azimuth angle, respectively. Figure 5 depicts the velocity variation of the UAV in the rendezvous phase.

It can be seen form figures 2 and 3 that the UAV's relative position errors in x-direction, y-direction and height reduce to the little neighborhood of zero in the end of the rendezvous phase. It can be seen form figure 4 that the UAV's elevation angle and azimuth angle coincide with those of the virtual tanker finally. It can be seen form figure 5 that the UAV accelerates from 20s to 80.5s to pursue rapidly the virtual tanker, then it slows down gradually, and finally the UAV's velocity reduces to 180.6m/s at 145.5s, meanwhile, the relative distance between the UAV and the virtual tanker reduces to 0.5m. The UAV's velocity error is smaller than 1m/s and the relative distance is smaller than 1m, which satisfies the conditions of ending the rendezvous phase. The simulation results validate the effectiveness of the designed guidance law and the velocity control system.

6. Conclusions

In this paper the 3D guidance model is constructed for the UAV pursuing the virtual tanker in the rendezvous phase. The nonlinear guidance law based on the Lyapunov stability theory is designed to control the UAV's normal acceleration and lateral acceleration, which can guarantee the intercept angle constraints. The UAV's velocity control system is designed by the sliding mode control method, and it controls both the relative distance and the UAV's velocity. The numerical simulation is performed to illustrate the effectiveness of the designed nonlinear guidance law and the velocity control system. The simulation results show that the designed guidance law and the velocity control system can guarantee the UAV to successfully rendezvous with the virtual tanker.

References

[1] Su Z., Wang H., Li N., Yu Y., Wu J. (2018) Exact docking flight controller for autonomous aerial refueling with back-stepping based high order sliding mode. Mechanical Systems and Signal Processing, 101: 338-360.
[2] Xu W., Luo D., Yang L., Wu S. (2013) Guidance law research for autonomous aerial refueling of UAV. In: Int. Conf. on Cyber Technology in Automation, Control and Intelligent Systems. Nanjing. pp. 440-445.
[3] Luo D., Xie R., Duan H. (2014) A guidance law for UAV autonomous aerial refueling based on the iterative computation method. Chinese Journal of Aeronautics, 27: 875-883.
[4] Qu Y., Yu Z., Zhang Y. (2015) Fractional-order sliding mode guidance control for rendezvous of unmanned aerial vehicles during air refueling. Control Theory & Applications, 32: 1464-1469.
[5] Gong Q., Yuan S., Zhang J. (2016) Guidance and control of autonomous rendezvous in aerial refueling based on Dubins path. Journal of Harbin Engineering University, 37: 1081-1088.
[6] Wang J., Zhang T., Li L. (2017) Guidance and control of UAV autonomous rendezvous in aerial refueling. Flight Dynamics, 35: 39-43.
[7] Yoon Y., Kim M., Kim Y. (2018) Three-dimensional path planning for aerial refueling between one tanker and multiple UAVs. International Journal of Aeronautical and Space Sciences, 19: 1027-1040.
[8] Tsukerman A., Weiss M., Shima T., Löbl D., Holzapfel F. (2018) Optimal rendezvous guidance laws with application to civil autonomous aerial refueling. Journal of Guidance Control and Dynamics, 41: 1164-1174.
[9] Yuan L., Wu X., Qi S. (2021) Research on control strategy of autonomous aerial refueling in rendevous phase. Flight Dynamics, 39: 68-74.
[10] Zhao W., Duan Y., Yu Z., Qu Y., Zhang Y. (2021) Integrated guidance and control for autonomous rendezvous of unmanned aerial vehicle during aerial refueling. In: Int. Conf. on Unmanned Aircraft Systems. Athens. pp 814-820.
[11] Zhao H., Qi Q., Zhang W. (2013) Three-dimensional extended propotional navigation guidance law of anti-warship missile's large-airspace varying trajectory. Command, Control & Simulation, 35: 102-105.
[12] Dong C., Yao T., Wang S., Yang M. (2014) Longitudinal guidance law with constraints on impact angle and control. Journal of Solid Rocket Technology, 37: 285-290.