Journal of Physics: Conference Series

PAPER • OPEN ACCESS

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To cite this article: Vasiliy G Baydulov *et al* 2022 *J. Phys.: Conf. Ser.* **2224** 012038

Motion of Mass Source in Stratified Fluid

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Abstract. The paper considers the problem of determining properties of an underwater moving source by analyzing the perturbation that it creates in electromagnetic or hydrodynamic fields. The computer program has been developed to simulate the spatial propagation of gravitational waves from the mass source moving along an arbitrary trajectory in a stratified fluid. The calculation results are in good agreement with analytical results obtained in the far-field approximation and with the results of experiments on the flow around underwater obstacles, moreover, the proposed technique allows to simulate any arbitrary motion of the source. Two new approaches to solving the inverse problem of determining the characteristics of the source of disturbances are proposed. The first one is based on the analysis of the signal received by radio-sensors, that scan the surface of the ocean. The second one uses data obtained from sensors installed directly in the water column.

1. Introduction

An important element in solving the problem of ocean tomography [1–3] is the development and implementation of numerical methods for modeling the motion of an underwater object. Despite of the fact that the problem of the motion of a body in a stratified fluid was solved analytically with the use of asymptotic methods, when the body was modeled by point mass sources moving horizontally [4] or at an angle to the horizon [5,6], and numerically (see, e.g., [7]), modeling was limited to the case of motion in a fixed direction with a constant velocity. This paper presents the results of numerical simulation of the propagation of gravitational waves from a mass source that moves arbitrarily in a stratified fluid. Next, two approaches are proposed to solve the inverse problem of identifying the parameters of an underwater source. In the first approach, it is proposed to solve this problem using data obtained by means of active or passive ocean radiometry. In the second one, the velocity, direction of movement, and size of the source of disturbances are determined by analyzing the signals received by underwater sensors.

2. Simulation of Internal Waves Propagation

When bodies move in a stratified fluid, they generate gravitational internal waves. The body can be modeled by point mass sources. The current section describes a numerical solution to the direct problem of modeling the propagation of internal gravitational waves generated by an arbitrarily moving underwater source.

2.1. Problem Statement

Consider the spatial propagation of internal gravitational waves generated by a mass source moving in an exponentially stratified fluid (the buoyancy frequency N is constant). The model of an ideal fluid is used. Since the changes in the density of the fluid with respect to the base stratification are small, the equations of motion are written in the Boussinesq approximation [4,8]. The body that moves with constant velocity $\mathbf{v_0}$ is modeled by a mass source $m(\mathbf{r},t) = m_0\delta(\mathbf{r} - \mathbf{v_0}t)$. Dynamic pressure $P(\mathbf{r},t)$ and fluid velocity $\mathbf{u} = (u_x, u_y, u_z)$ can be written in terms of internal potential $\psi(\mathbf{r},t)$:

$$P = -\rho_0 \left(\frac{\partial^2}{\partial t^2} + N^2\right)\frac{\partial}{\partial t}\psi, \tag{1}$$

$$\mathbf{u} = \left(\frac{\partial^2}{\partial t^2}\nabla + N^2\nabla_{\mathrm{h}}\right)\frac{\partial}{\partial t}\psi, \tag{2}$$

where $\nabla_{\mathrm{h}} = (\partial_x, \partial_y, 0)$, and the internal potential $\psi(\mathbf{r},t)$ satisfies the equation

$$\left(\frac{\partial^2}{\partial t^2}\nabla^2 + N^2\nabla_{\mathrm{h}}^2\right)\psi = m. \tag{3}$$

To simulate the motion of the underwater source, it is required to calculate how the pressure, vertical displacement fields, and the shape of the free surface change over time. In the calculations, the point mass source $m(\mathbf{r},t)$ is modelled by the function

$$f(x,y,z) = \frac{BA^3}{\sqrt{\pi^3}}\,\mathrm{e}^{\,-A^2(x^2+y^2+z^2)}.$$

Thus, in the case of rectilinear motion of the source, the equation (3) takes the form

$$\Delta\frac{\partial^2\psi}{\partial t^2} + N^2\Delta_{xy}\psi = f(\mathbf{r} - \mathbf{r_0} - \mathbf{v_0}t), \quad \mathbf{r} \in \Omega. \tag{4}$$

It is assumed that the internal potential at the boundary of the computational domain is zero,

$$\psi = 0, \quad \mathbf{r} \in \partial\Omega. \tag{5}$$

This assumption simplifies the calculation scheme but leads to the fact that the results of the calculation are reliable only in some area that moves along with the source. At the initial moment of time

$$\psi|_{t=0} = 0. \tag{6}$$

The problem (4)–(6) is numerically solved in the rectangular cuboid domain Ω. After calculating ψ, the formulas (1) and (2) are used to find the fluid velocity $\mathbf{u}$ and the pressure P. The vertical displacement of the fluid $\zeta(\mathbf{r},t)$ is related to the velocity as $\frac{\partial\zeta}{\partial t} = v_z$. Hence, considering the equation (2), we have

$$\zeta = \frac{\partial}{\partial t}\frac{\partial}{\partial z}\psi.$$

At the free surface of the fluid, we have

$$P - \zeta\rho_0 g = P_0,$$

where ρ_0 is the density of the fluid, P_0 is the atmospheric pressure, g is the acceleration of gravity. One can calculate the shape of the free surface as follows. Let the function $\zeta_{f.s.}(x,y,t)$ define a surface near the plane $z = h_f$. For a free surface, the foolowing equality would be satisfied:

$$P(x,y,h_f,t) - \zeta_{\mathrm{f.s.}}(x,y,t)\rho_0 g = P_0. \tag{7}$$

From the equations (1) and (7) we have

$$\zeta_{\mathrm{f.s.}}(x,y,t) = -\frac{1}{g}\left(\frac{\partial^2}{\partial t^2} + N^2\right)\frac{\partial}{\partial t}\psi(x,y,h_f,t) - \frac{P_0}{g\rho_0}.$$

2.2. Description of the Computations

The program is written in the C++ programming language that allows to solve the problem (4)–(6). The calculated internal potential ψ is then used to determine the pressure P, the vertical displacement of the fluid ζ, and the shape of the free surface $\zeta_{\mathrm{f.s.}}$. In the program, you can set an arbitrary law of motion of a mass source. An implicit finite-difference method is used to solve the problem (4)–(6). The domain Ω is divided into a grid that is uniform in spatial coordinates. At each time step, a system of linear algebraic equations (SLAE) is solved using the Generalized minimal residual method [9]. The GNU Scientific Library is used to perform elementary operations with sparse matrices and to solve SLAEs [10].

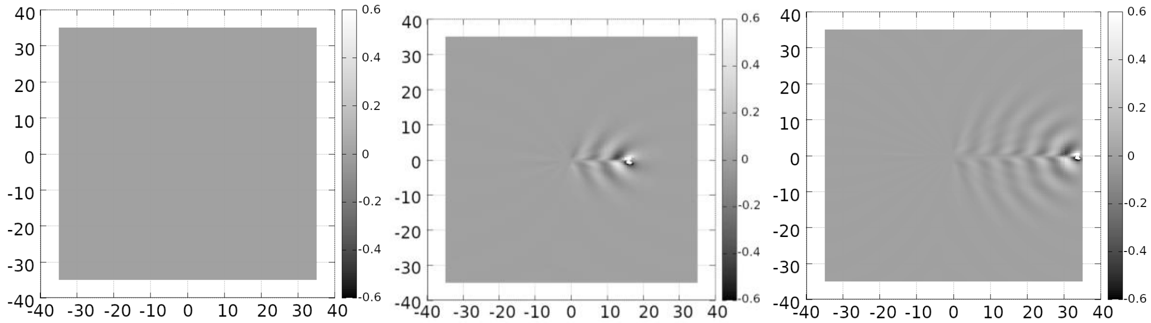

Figure 1. Vertical section of $\zeta(\boldsymbol{r}, t)$ at time instants $\mathrm{t} = 0\mathrm{s}, 17.3\mathrm{s}, 35\mathrm{s}$ (from left to right).

2.3. Example of Computation

To illustrate the operation of the program and verify the calculation, the propagation of internal waves from a mass source moving horizontally at a constant velocity is simulated. Figure 1 shows the function of the normalized vertical displacement of the fluid in the vertical section $y = 0$. Figure 2 shows the shape of the free surface at time t = 18s. The results of calculation of the vertical displacement of the fluid ζ agree with the analytical results from [4], where an asymptotic solution of the problem (4) is constructed in the far-field approximation and show qualitative agreement with the results of experiments on the flow of a sphere moving uniformly and rectilinearly in a stratified fluid [11]. The results of calculations of the free surface $\zeta_{f.s.}$ are qualitatively like the asymptotic results of solving similar problems, namely, the results from [12], where the free surface shape is perturbed by gravitational waves from a sphere moving under water, and the results of study of ship waves [13]. In contrast with [4,11–13], our approach allows to simulate motions along any arbitrary path with any given acceleration.

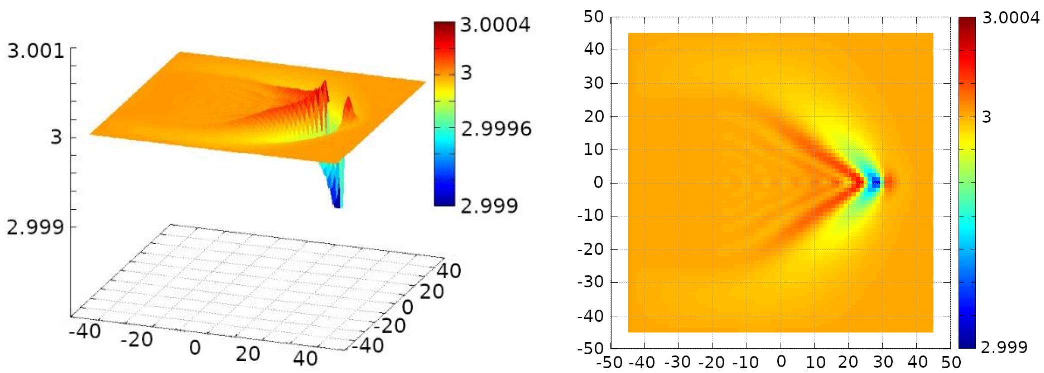

Figure 2. Free surface shape, side view (left) and top view (right).

3. Solving Inverse Problem

Let us now consider the inverse problem of obtaining information about an underwater source from the disturbance it creates. One of the approaches to sea tomography is to reconstruct the properties of an underwater process using data received from an aerial- or space-based surface radiometer [1]. In such studies, the characteristics of an underwater source of disturbances can be found by analyzing the effect of the generated gravitational waves on the surface wind wave, the change of which, in turn, can be registered by means of active or passive radio-sensing [2]. Another way to solve the inverse problem is to analyze information about hydrodynamic fields obtained from sensors installed directly in the ocean.

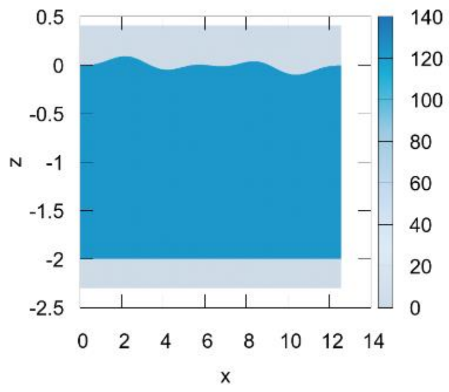

Figure 3. An example of sea surface.

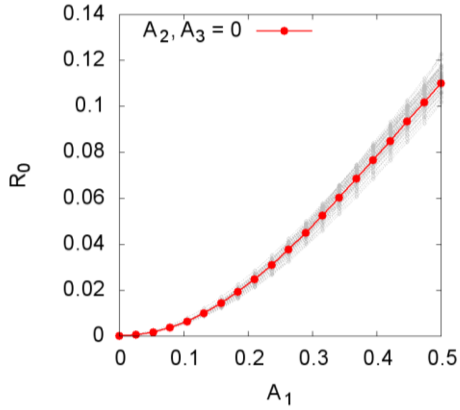

Figure 4. Simulation of active radiometry.

3.1. Sea Surface Radiometry

Sea surface radio-sensing can be active, when the sea surface is radiated by electromagnetic wave and a sensor receives a part of the scattered radiation, and passive, when a sensor receives self-radiation of the ocean [2]. We model the sea surface by a homogeneous layer of finite thickness bounded from above by the surface

$$S_1(x) = S_1^0(x) + \xi_2 \sin\left(\frac{2\pi}{2\Lambda}x\right) + \xi_3 \sin\left(\frac{2\pi}{\frac{1}{2}\Lambda}x\right),$$
$$S_1^0(x) = A_1 \sin\left(\frac{2\pi}{\Lambda}x\right). \tag{8}$$

This models the influence of long- and short-waves that have amplitudes determined by random variables ξ_2, ξ_3 on an undisturbed surface $S_1^0(x)$. An example of such layer (the section of modulus of the dielectric permittivity, $|\varepsilon(x, y, z)|$) is shown in figure 3. The blue domain shows sea water, while the light upper domain shows the air.

In passive radiometry, the signal recieved by a sensor is the fraction of the sea's own radiation in the direction of the reception. According to the reciprocity principle, the intensity of medium's own radiation in a given direction is proportional to the fraction of radiation absorbed by the medium when it is irradiated by a plane wave in the same direction — the energy defect [2]. According to the Etkin–Kravtsov effect [14], the greatest sensitivity to the self-radiation of the sinusoidal surface (8) will be observed if the following relationship between the length of the electromagnetic wave λ, the length of the sea wave Λ and the sensing angle α takes place:

$$\lambda = \Lambda(1 \pm \sin\alpha).$$

In active radiometry, the surface under study is irradiated by an electromagnetic wave and the signal received by a sensor is the fraction of radiation reflected exactly in the opposite direction. According to the Bragg–Wolf resonance scattering effect [1,15], part of the energy is reflected exactly backwards when the following relation is fulfilled:

$$\frac{\lambda}{\Lambda} = 2\sin(\alpha).$$

The results of numerous calculations on modeling the process of restoring the shape of the surface from the radiometry data have shown that the solution of this inverse problem is possible [3,16,17]. To simulate the propagation of electromagnetic radiation, approximate [18] or exact methods [19,20] can be used. The latter is an effective projection method that was used in high-resolution simulation in holography [21] and in the current research for modelling electromagnetic wave propagation both in 2d [16,17,21,22] and 3d [23] cases. Figure 4 shows an example of such calculations. Red curve shows the dependence of the level of the received signal $R_0(A_1)$ on the amplitude A_1 of the sea surface wave. Gray curves show the signal from the surfaces disturbed by short or long waves. It is monotonous over the entire range of values A_1 under consideration. Thus, in this interval it is possible to restore the shape of the surface (8) from the level of the received signal R_0.

3.2. Determination of the Source Position by The Field of Internal Waves Created by It

While many papers have been devoted to the calculation of the field of attached internal waves, the inverse problem of determining the position of the source from a known wave field has not been posed. At the same time, methods of spatial signal processing are being widely developed in hydroacoustics with the subsequent solution of the problem of determining the position and parameters of the source. The construction of methods for locating the source by analyzing the field of internal waves will create additional opportunities in determining the source motion parameters, where hydroacoustic methods are ineffective. In the current section, it is proposed to solve the inverse problem of determining the properties of a source of disturbances moving in a fluid with the use of the analysis of the signal from sensors located directly in the ocean. When a body moves in a continuously stratified fluid with a constant velocity, the steady wave field moves along with the body and forms a field of so-called attached internal waves. In analytical studies, the flow impinging on the body is usually assumed to be constant. Non-stationary waves generated at the initial stage of movement are neglected. In this case, the body is modeled by point mass sources, and the wave field is located using the Green's function method, followed by using asymptotic expansions based on the stationary phase method [4].

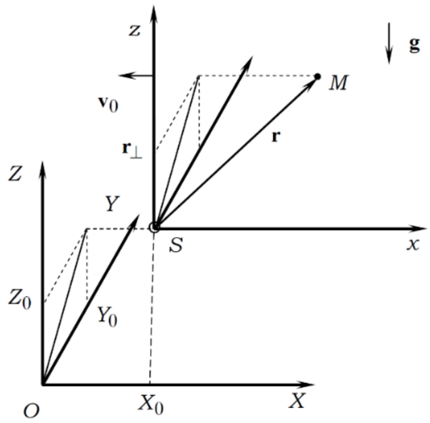

Figure 5. Geometry of the motion.

Consider the most typical horizontal movement of a body with a constant velocity. The diagram of the motion of the source of the attached internal waves is shown in figure 5. The coordinate systems (x, y, z) and (X, Y, Z) move together with the source, their axes are parallel in pairs. S stands for source, (X_0, Y_0, Z_0) are coordinates of the source in the system (X, Y, Z). For a source moving at a constant velocity, the flow field determined by the internal potential equation (3) turns out to be stationary in the coordinate system associated with the source. The asymptotic solution of the problem [4] at large distances from the source gives an expression for the vertical displacement of liquid particles in the coordinate system of the source

$$\zeta(\boldsymbol{r}, t) \sim H(x) \frac{N m_0}{2\pi v_0^2} \frac{xz}{r_\perp^3 r^2} \sqrt{x^2 y^2 + r_\perp^4} \, cos\left(\frac{N}{v_0} \frac{z}{r_\perp} r\right), \tag{9}$$

where $r_\perp^2 = y^2 + z^2$, $r^2 = x^2 + r_\perp^2$, are the coordinates of the observation point in the reference frame (x, y, z) associated with the source, $H(x)$ is the Heaviside function. The expression for vertical displacements ζ of liquid particles (9) makes it possible to calculate horizontal displacement maps at various vertical distances down from the moving source (see figure 6). These fields can be measured in the laboratory and in field experiments by traditional means of geophysical measurements. Thus, the problem of determining the position of the source by the field of vertical displacements is relevant.

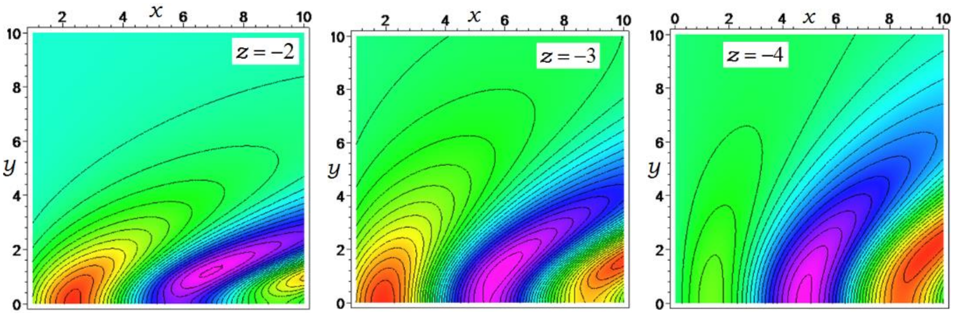

Figure 6. Horizontal maps of vertical displacements ζ at a distance $z = -2, -3, -4$.

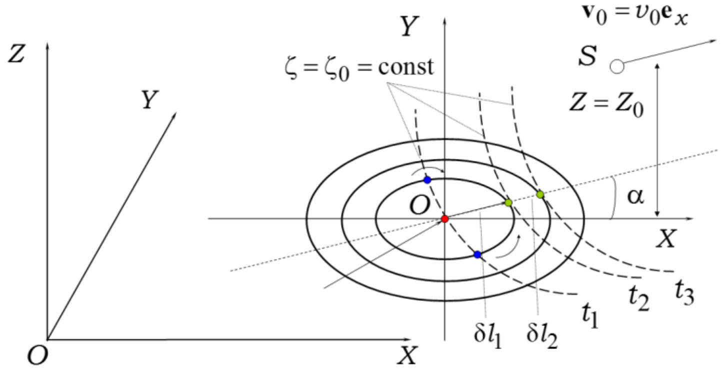

Figure 7. The scheme of measurements.

Let it be possible to measure vertical displacements in the horizontal plane over which the source is moving. From the measurement data, it is required to determine the position of the source and its velocity. Let us place measuring devices in the form of horizontal concentric circles (see figure 7). Let the vertical displacement measurements be made in the center and along the concentric circles. By measuring the displacement in the center O (red dot $\zeta = \zeta_0$) at the moment of time t_1, we find the positions of the points on the first from the center circle (distance from the center is δl_1), the displacements in which are equal to ζ_0. Then, by determining the time instant t_2 at which these points converge at one point of the first circle, we determine the direction of movement of the source and its velocity, $v_0 = \frac{\delta l_1}{t_2 - t_1}$. To determine the position of the source, we choose a frame of reference (X, Y, Z) moving with the source (see figure 5). In this case, the coordinates of the source and the observation point are (X_0, Y_0, Z_0) and $(x + X_0, y + Y_0, z + Z_0)$. Paying attention that the zeros of the vertical displacement are determined by zeros of a cosine that is in the expression [4] for the first and second zeros, we get $\frac{z_1}{r_{\perp,1}} r_1 = \frac{v_0}{N}\frac{\pi}{2}$ and $\frac{z_2}{r_{\perp,2}} r_2 = 3\frac{v_0}{N}\frac{\pi}{2}$. That is, the points with coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2), in which ζ vanishes, satisfy these relations.

Note that the relation $r / r_\perp$ turns out to be invariant with respect to stretching transformations $(x, y, z) \to p(x, y, z)$. Then the coordinates of the first and second zeros are connected by the relations

$x_2 = 3x_1, y_2 = 3y_1, z_2 = 3z_1$, which makes it possible to determine the position of the source, when two measurement points are used:

$$X_0 = \frac{1}{2}(3X_1 - X_2), \quad Y_0 = \frac{1}{2}(3Y_1 - Y_2), \quad Z_0 = \frac{1}{2}(3Z_1 - Z_2). \tag{10}$$

Thus, knowing the displacement field in a fixed reference frame (X', Y', Z'), we should determine the positions of the zeros of ζ. Using the known velocity of the source, the coordinates of the zeros (X_i', Y_i', Z_i') are recalculated into the coordinates of the zeros in the moving reference frame (X_i, Y_i, Z_i), from which, in turn, the position of the wave source can be determined using the relations (10).

4. Conclusion

The paper considers the problem of ocean tomography. The C++ program has been developed and verified, that simulates propagation of gravitational waves from a moving underwater mass source. Two approaches to solution of the inverse problem of determining the characteristics of the source are proposed. One is based on radiometry of the sea surface; in the other, a previously unconsidered problem of determining the position of the source and its velocity by the measurements of hydrophysical fields at a known buoyancy frequency of the medium is solved. Results of modelling the motion of the underwater source is in good agreement with both the analytical and experimental results from [4,12,13] and [11] respectively, while the developed method allows to simulate any arbitrary motion of the source in contrast to the uniform straight motions studied in the cited papers. Methodology of solving the inverse problem proposed in the current paper is new and was not previously considered.

The computations are performed using both personal computers and supercomputer resources. Results shown in figure 5 are obtained in 50 min. on 10 nodes of MVS-100K cluster (20 processors, 80 computing cores) installed at Joint Supercomputer Center of the Russian Academy of Sciences (JSCC RAS), Moscow, Russia. When using a computational grid of 300x300x300, the results shown in figures 1 and 2 require 12 days of computation on one node of Govorun supercomputer installed at Information Technology Laboratory (LIT) of the Joint Institute for Nuclear Research (JINR), Dubna, Russia.

Acknowledgments

The authors express their gratitude to the management and staff of JSCC RAS and LIT JINR who kindly provided the opportunity and technical support for the computations performed on the clusters. The work was supported by the Russian Science Foundation, project no. 21-11-00151, https://rscf.ru/en/project/21-11-00151/.

References

[1] Nesterov S V, Shamaev A S and Shamaev S I 1996 *Methods, Algorithms and Tools of Aerospace Computer Tomography of Near-Surface Layer of the Earth* (Moscow: Nauchnij mir) p 272 (in Russian)
[2] Ulaby F T, Moore R K and Fung A K 1981 *Microwave Remote Sensing. Active and Passive* (Massachusetts: Addison-Wesley Publishing Company) p 456
[3] Gavrikov A, Knyazkov D, Romanova A, Chernik V and Shamaev A 2016 *Program Systems: Theory and Applications* 7**(**2) 73–84
[4] Voisin B 1994 *J. Fluid Mech.* **261** 333–374
[5] Scase M M and Dalziel S B 2004 *J Fluid Mech* **498** 289–313
[6] Scase M M and Dalziel S B 2006 *J Fluid Mech* **564** 305–331
[7] Matyushin P V 2019 *Fluid Dyn* **54** 374–388
[8] Bulatov V V and Vladimirov Yu V 2019 *A general Approach to Ocean Wave Dynamics Research: Modelling, Asymptotics, Measurements* (Moscow: OntoPrint) p 587
[9] Saad Y, Schultz M H 1986 *SIAM Journal on Scientific and Statistical Computing.* **3** 856–869
[10] Galassi M et al *GNU Scientific Library Reference Manual (3rd Ed.)* p 592
[11] Chashechkin Y, Gumennik E, Sysoeva E 1995 *J Appl Mech Tech Phy* **36**. 19–29

[12] Voisin B 2007 Lee waves from a sphere in a stratified flow *J. Fluid Mech.* **574** 273–315
[13] Arzhannikov A V and Kotelnikov I A 2016 *Phys. Rev.* E. **94** 023103
[14] Kravtsov Yu A, Mirovskaya Ye A, Popov A E, Troitskiy I A, Etkin V S 1978 *Izvestiya of the Academy of Sciences of the USSR. Atmospheric and Oceanic Physics* **14**(7) 522–526
[15] Bass F G and Fuks I M 1979 *Wave Scattering from Statistically Rough Surfaces* (Pergamon) p 540
[16] Knyazkov D 2017 *Proc. of 2017 Days on Diffraction (DD), 19–23 June 2017, Saint-Petersburg, Russia* pp 180–185
[17] Knyazkov D 2017 *AIP Conf. Proc.* **1863** 370006
[18] Knyaz'kov D Yu, Romanova A V, Shamaev A S 2016 *Proc. Steklov Inst. Math.* **295** 168–178
[19] Il'inskiy A S 1974 *USSR Computational Mathematics and Mathematical Physics* **14**(4) 242–6
[20] Bykov A A and Il'inskii A S 1979 *USSR Computational Mathematics and Mathematical Physics* **19**(3) 74–82
[21] Chernik V V 2018 *J. Phys.: Conf. Ser.* **1096** 012147
[22] Knyazkov D 2017 *Lobachevskii J Math* **38**(5) 936–939
[23] Knyazkov D 2018 *AIP Conf. Proc.* **1978** 470075